Decoupled Search: A New Form of State-Space Exploration

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AALBORG UNIVERSITY

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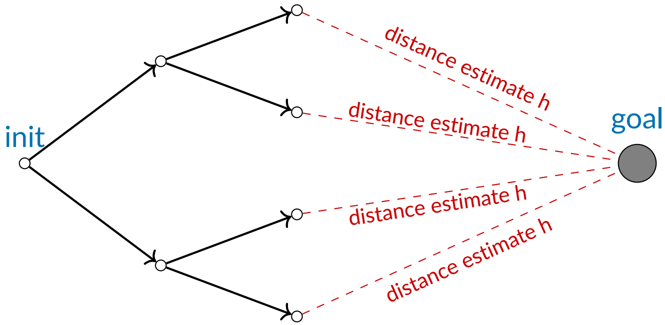
What's this About

- Decoupled Search:
  - New technique for state-space exploration in AI-planning and model-checking

Daniel Gnad (Gnad (2021))

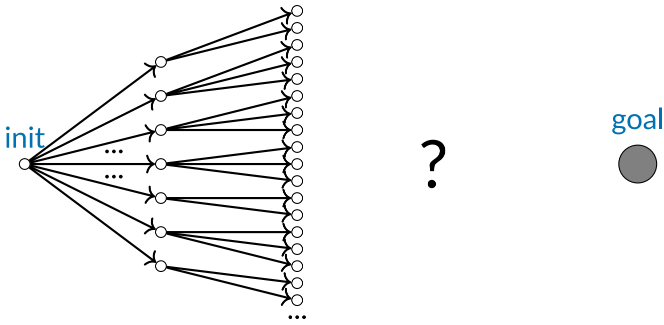
Joerg Hoffmann

A Successful Approach in General: Heuristic Search



$\rightarrow$ State space search with heuristic function h maps states s to an estimate $h(s)$ of goal distance.

Heuristic Search – Limitations



State explosion problem:
State space of a planning task is exponential in the number of variables.

Domain-independent planning versus domain-dependent solvers

Finding Optimal Solutions to Rubik's Cube Using Pattern Databases

Richard E. Korf
Computer Science Department
University of California, Los Angeles
Los Angeles, Ca. 90095
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Abstract

We have found the first optimal solutions to random instances of Rubik's Cube. The median optimal solution length appears to be 18 moves. The algorithm used is iterative-deepening-A* (IDA*), with a lower-bound heuristic function based on large memory-based lookup tables, or "pattern databases" (Culberson and Schaeffer 1996). These tables store the exact number of moves required to solve various subgoals of the problem, in this case subsets of the individual movable cubies. We characterize the effectiveness of an admissible heuristic function by its expected value, and hypothesize that the overall performance of the program obeys a relation in which the product of the time and space used equals the size of the state space. Thus, the speed of the program increases linearly with the amount of memory available. As computer memories become larger and cheaper, we believe that this approach will become increasingly cost-effective.

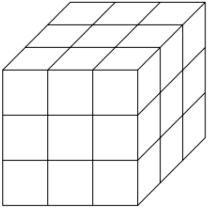

Figure 1: Rubik's Cube

The dream: reduce the gap to a point where domain-independent planners are as efficient than

Domain-independent vs domain-dependent

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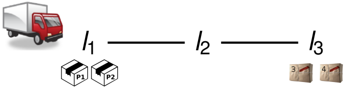

- $V = \{t, p_1, \dots p_N\}$ with $D_t = \{l_1, l_2, l_3, l_4\}$ and $D_{p_i} = \{t, l_1, l_2, l_3, l_4\}$.
- $I = \{(t, l_1), (p_1, l_1), (p_2, l_1), (p_3, l_3), (p_4, l_3)\}$
- A = $\{load(p_i, x), unload(p_i, x), drive(x, x')\}$, where:
  $pre_{load(p_i,x)} = \{(t, x), (p_i, x)\}$ and
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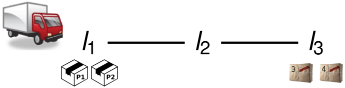

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- bool `isGoal` (s)
- int `heuristic` (s)

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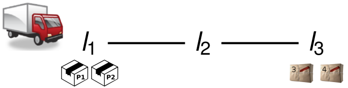

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- State `init` (s) $\rightarrow$return I
- set(A) `applicable` (s) $\rightarrow$return $\{a \mid s \models pre(a)\}$
- State `apply` (s, a) $\rightarrow$return $s[a]$
- bool `isGoal` (s) $\rightarrow$return $s \models G$
- int `heuristic` (s) $\rightarrow$Any domain-independent planning heuristic

Exercise: Pick-up and Delivery

We have M trucks and N packages across L locations. Trucks drive around to pick and deliver the packages. We want to compute a (optimal) route. How do you design the search space? States? Actions?

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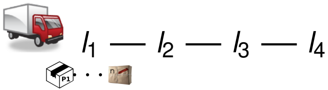
Option 3: Truck-centered

- State: truck routes, whether packages have been delivered
- Actions: drive-to(t, l)

State Explosion Problem – Example



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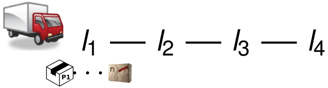

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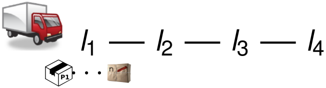

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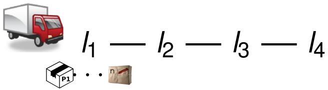

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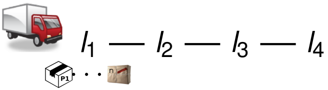

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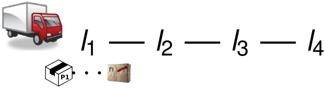

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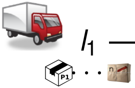

l_1 — l_2 — l_3 — l_4

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Can this be avoided? What is the connection between the packages?

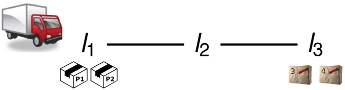
Exponential Reduction of the State Space

Domain	Reachable State Space. Right: Average over Instances Commonly Built Success Std	POR	Unfold.	Decoupled	Representation Size (in Thousands) Std	POR	Decoupled
Solvable Benchmarks: From the International Planning Competition (IPC)							
Depots	4	4	2	5	30,954.8	30,954.8	3,970.0
Driverlog	5	5	3	10	35,632.4	35,632.4	127.2
Elevators	21	17	3	41	22,652.1	22,651.1	186.7
Logistics	12	12	11	27	3,793.8	3,793.8	8.2
Miconic-STRIPS	50	45	30	145	52,728.9	52,673.1	2.4
Nomystery	11	11	7	40	29,459.3	25,581.5	10.0
Pathways	4	4	3	4	54,635.5	1,229.0	11,211.9
PSR	3	3	3	3	39.4	33.9	11.1
Rovers	5	6	4	5	98,051.6	6,534.4	4,032.9
Satellite	5	5	5	4	2,864.2	582.5	352.7
TPP	5	5	4	11	340,961.5	326,124.8	.8
Transport	28	23	11	34	4,958.6	4,958.5	173.3
Woodworking	11	20	22	16	438,638.5	226.8	9,688.9
Zenotravel	7	7	4	7	17,468.0	17,467.5	99.4
Unsolvable Benchmarks: Extended from Hoffmann and Nebel (2001)							
Nomystery	9	8	4	40	85,254.2	65,878.2	3.8
Rovers	4	4	0	4	697,778.9	302,608.9	20,924.4
$\sum$	186	181	116	398			

Agenda

Decoupled Search – Intuition

Running Example:



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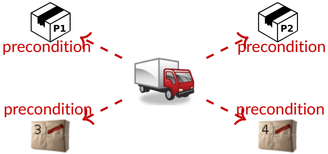
Causal Graph: **Dependencies across (components of) state variables.**



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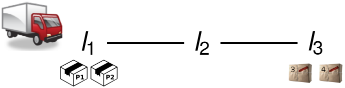

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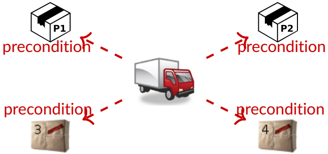

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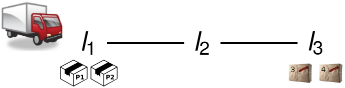

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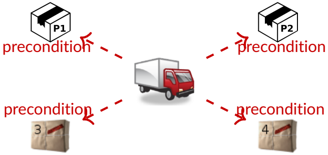

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Search over global actions; handle each leaf component separately.

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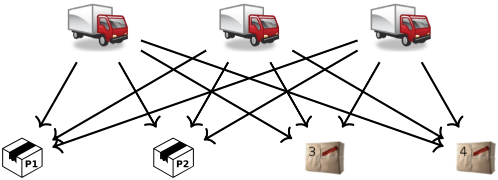

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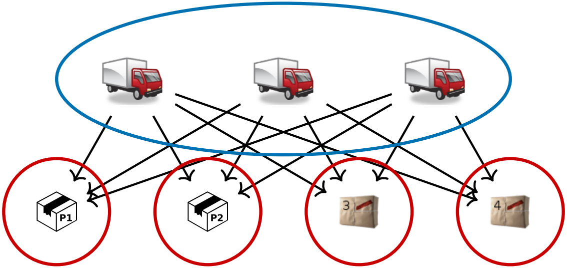

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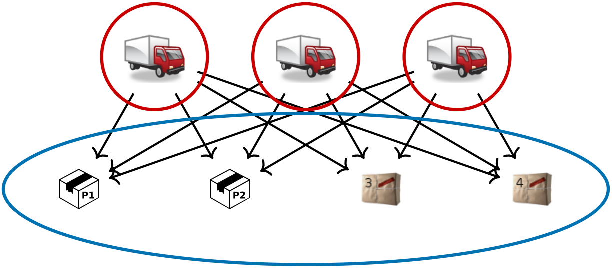

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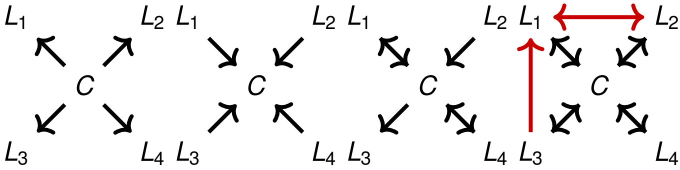
Examples:

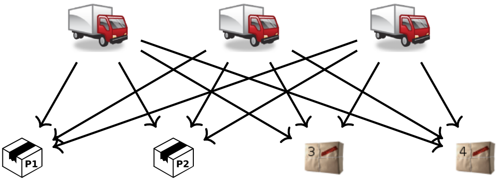


Fork Inverted-Fork Strict-Star Star

Dividing the Actions

Given a Factoring $\mathcal{F} = \{C, L_1, \ldots, L_n\}$, the set of actions is divided into $n+1$ *subsets*:

- Internal (leaf-only) Actions A^L: affect only one leaf $L \in \mathcal{L}$, $a \in A^L \Leftrightarrow V[\mathit{eff}_a] \subseteq L \wedge V[\mathit{pre}_a] \cup V[\mathit{eff}_a] \subseteq C \cup L$.
- Global Actions A^C: those that are not leaf actions, e.g.:
  - have an effect on a center variable
  - have effects and/or preconditions on two leaves



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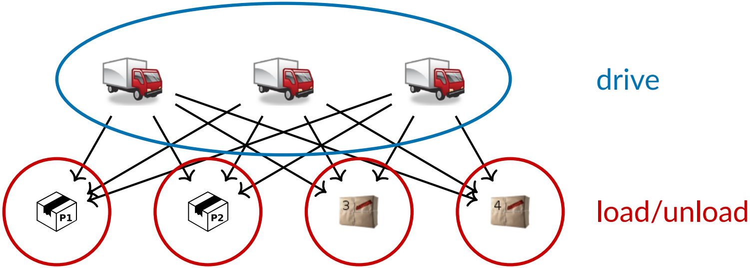

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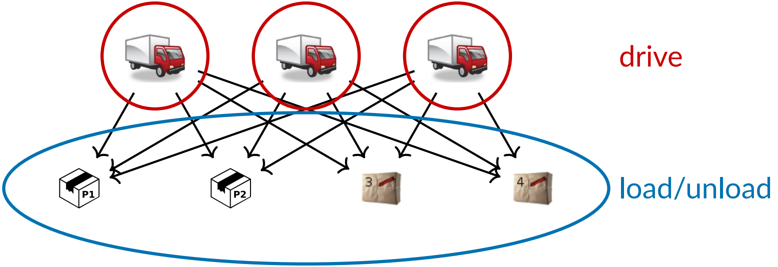
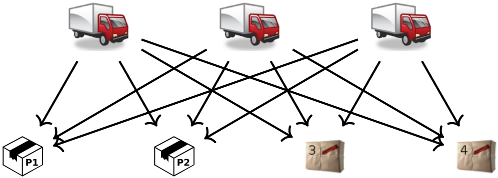


Applying a Factoring to a Planning Task

Given a Factoring $\mathcal{F} = \{C, L_1, \ldots, L_n\}$, we define

- Center States $s^C \in S^C$: complete assignment to C
- Leaf States $s^L \in S^L$: complete assignment to an $L \in \mathcal{L}$



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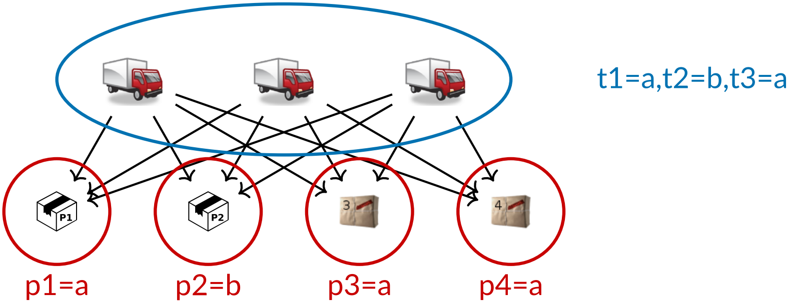

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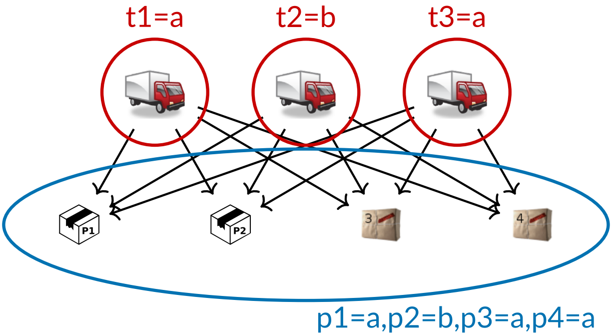
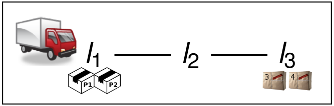


Decoupled Search – Intuition II



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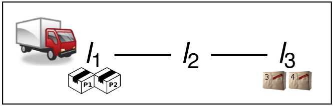

Center path:

$$l_1 \xrightarrow{\text{drive}(l_1, l_2)} l_2 \xrightarrow{\text{drive}(l_2, l_3)} l_3 \xrightarrow{\text{drive}(l_3, l_2)} l_2 \xrightarrow{\text{drive}(l_2, l_1)} l_1$$

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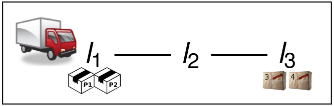

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:

(a) $l_1 \xrightarrow{\text{load}(p_1, l_1)} t \xrightarrow{\text{unload}(p_1, l_2)} l_2$

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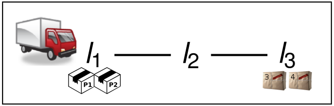

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:

(a) $l_1 \xrightarrow{\text{load}(p_1, l_1)} t \xrightarrow{\text{unload}(p_1, l_2)} l_2$

(b) $l_1 \xrightarrow{\text{load}(p_1, l_1)} t \quad \xrightarrow{\text{unload}(p_1, l_3)} l_3$

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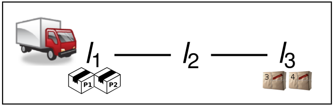

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:

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(b) $l_1 \xrightarrow{load(p_1, l_1)} t \qquad \xrightarrow{unload(p_1, l_3)} l_3$

:

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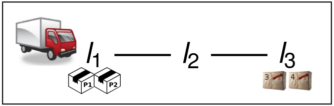
Decoupled Search – Intuition II



Center path:

$$l_1 \xrightarrow{\text{drive}(l_1, l_2)} l_2 \xrightarrow{\text{drive}(l_2, l_3)} l_3 \xrightarrow{\text{drive}(l_3, l_2)} l_2 \xrightarrow{\text{drive}(l_2, l_1)} l_1$$

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:

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(b) $l_3 \xrightarrow{\text{load}(p_3, l_3)} t \qquad \xrightarrow{\text{unload}(p_3, l_1)} l$

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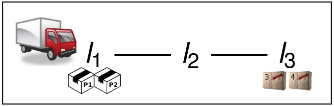

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:

(a) $l_1 \xrightarrow{\text{load}(p_1, l_1)} t \xrightarrow{\text{unload}(p_1, l_2)} l_2$

(b) $l_1 \xrightarrow{\text{load}(p_1, l_1)} t \qquad \xrightarrow{\text{unload}(p_1, l_3)} l_3$

:

(a) $l_3 \xrightarrow{\text{load}(p_3, l_3)} t \xrightarrow{\text{unload}(p_3, l_2)} l_2$

(b) $l_3 \xrightarrow{\text{load}(p_3, l_3)} t \qquad \xrightarrow{\text{unload}(p_3, l_1)} l$

We can choose (a) or (b) for each of p_1 and p_3 independently

$\Longrightarrow$ **Maintain the compliant paths for each leaf separately.**

Decoupled Search – Intuition III

	l_1	t	l_2	l_3	l_4
p_1	0	1	2	2	∞

	l_1	t	l_2	l_3	l_4
p_2	0	1	∞	2	2

$t = l_3$

	l_1	t	l_2	l_3	l_4
p_3	∞	1	∞	2	0

	l_1	t	l_2	l_3	l_4
p_4	∞	1	∞	2	0

Decoupled State $s^{\mathcal{F}}$

Decoupled Search – Intuition III

	l_1	t	l_2	l_3	l_4
p_1	0	1	2	2	∞

	l_1	t	l_2	l_3	l_4
p_2	0	1	∞	2	2

$t = l_3$

	l_1	t	l_2	l_3	l_4
p_3	∞	1	∞	2	0

	l_1	t	l_2	l_3	l_4
p_4	∞	1	∞	2	0

Decoupled State $s^{\mathcal{F}}$

$tl_3, p_1 t, p_2 l_3, p_3 l_3, p_4 l_3$ 7
$tl_3, p_1 l_3, p_2 l_3, p_3 t, p_4 t$ 6
$tl_3, p_1 t, p_2 t, p_3 t, p_4 l_3$ 5
$tl_3, p_1 l_3, p_2 t, p_3 t, p_4 l_3$ 6
$tl_3, p_1 t, p_2 l_3, p_3 t, p_4 t$ 5
...

Hypercube $[s^{\mathcal{F}}]$ (144 member states!

Decoupled Search – Intuition III

	l_1	t	l_2	l_3	l_4
p_1	0	1	2	2	∞

	l_1	t	l_2	l_3	l_4
p_2	0	1	∞	2	2

$t = l_3$

	l_1	t	l_2	l_3	l_4
p_3	∞	1	∞	2	0

	l_1	t	l_2	l_3	l_4
p_4	∞	1	∞	2	0

Decoupled State $s^{\mathcal{F}}$

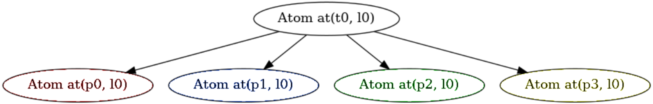
$$tl_3, p_1t, p_2l_3, p_3l_3, p_4l_3\ 7$$
$$tl_3, p_1l_3, p_2l_3, p_3t, p_4t\ 6$$
$$tl_3, p_1t, p_2t, p_3t, p_4l_3\ 5$$
$$tl_3, p_1l_3, p_2t, p_3t, p_4l_3\ 6$$
$$tl_3, p_1t, p_2l_3, p_3t, p_4t\ 5$$
$$\dots$$

Hypercube $[s^{\mathcal{F}}]$ (144 member states!

Every member state annotated with its price in $s^{\mathcal{F}}$.

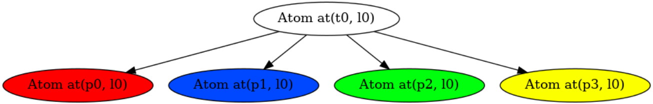
Hypercube dimensions = Leaves; Axis values = Leaf States.

Examples



Logistics

Examples



Logistics

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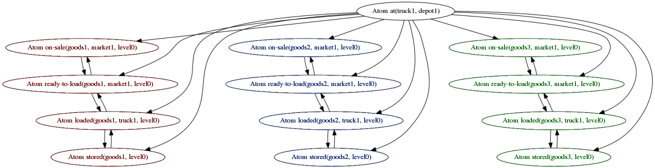
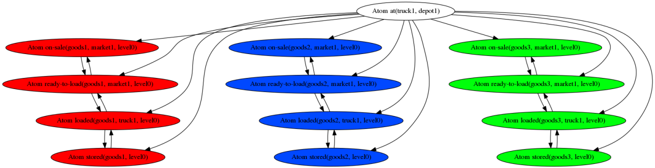


TPP

Examples



TPP

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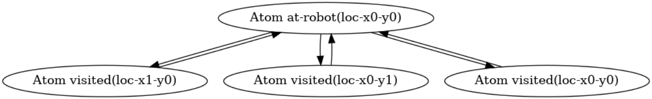
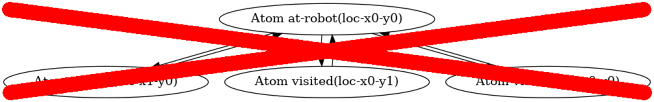


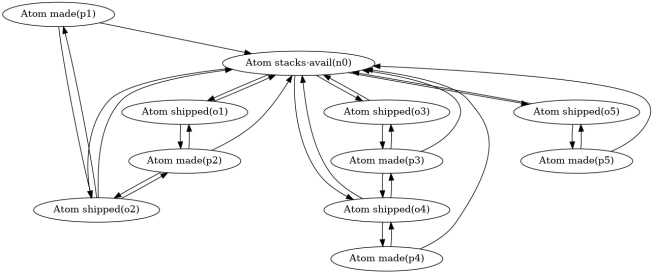
Visit-All

Examples



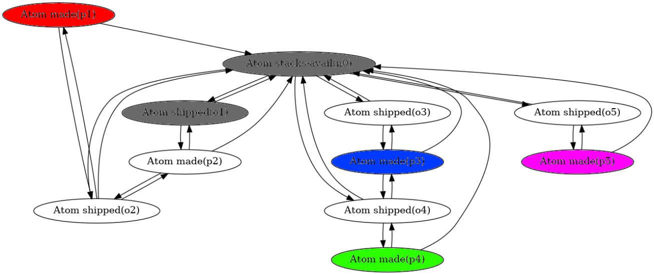
Visit-All

Examples



Openstacks

Examples

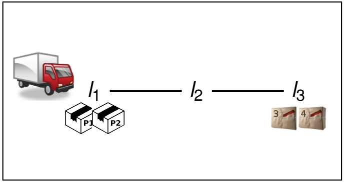


Openstacks

Initial Decoupled State

- Center variables get their value from the explicit state

Example:



Explicit initial state

	l_1	t	l_2	l_3		l_1	t	l_2	l_3
p_1	∞	∞	∞	∞	p_2	∞	∞	∞	∞

$t = l_1$

	l_1	t	l_2	l_3		l_1	t	l_2	l_3
p_3	∞	∞	∞	∞	p_4	∞	∞	∞	∞

Decoupled initial state

Initial Decoupled State

- Center variables get their value from the explicit state
- Set price of 0 for the leaf state that holds in the initial state

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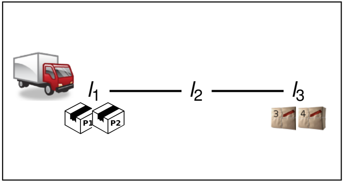

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p_1	0	∞	∞	∞

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p_3	∞	∞	∞	0

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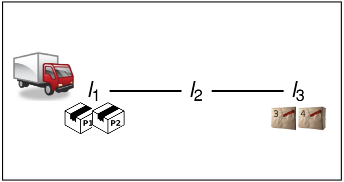

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p_1	0	∞	∞	∞

	l_1	t	l_2	l_3
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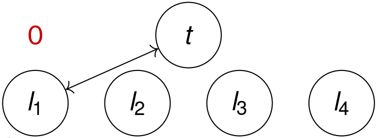

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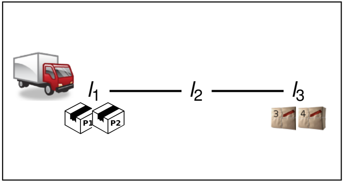

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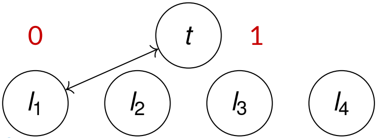

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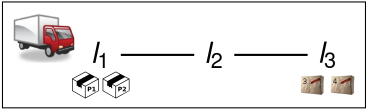
(Optimal planning: minor modifications required)

Decoupled Search – Fork (Truck-centered)



	l_1	t	l_2	l_3
p_1	0	1	∞	∞

	l_1	t	l_2	l_3
p_2	0	1	∞	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

drive(l_1, l_2)

	l_1	t	l_2	l_3
p_1	0	1	**2**	∞

	l_1	t	l_2	l_3
p_2	0	1	**2**	∞

$t = l_2$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

drive(l_2, l_1)

	l_1	t	l_2	l_3
p_1	0	1	**2**	∞

	l_1	t	l_2	l_3
p_2	0	1	**2**	∞

$t = l_1$

	l_1	t	l_2	l_3
p_3	∞	∞	∞	0

	l_1	t	l_2	l_3
p_4	∞	∞	∞	0

drive(l_2, l_3)

	l_1	t	l_2	l_3
p_1	0	1	2	**2**

	l_1	t	l_2	l_3
p_2	0	1	2	**2**

$t = l_3$

	l_1	t	l_2	l_3
p_3	∞	**1**	∞	0

	l_1	t	l_2	l_3
p_4	∞	**1**	∞	0

Solution Reconstruction

For every member state $s \in [s^{\mathcal{F}}]$ of a decoupled state $s^{\mathcal{F}}$, we can construct a **global plan** reaching s from the initial state I.

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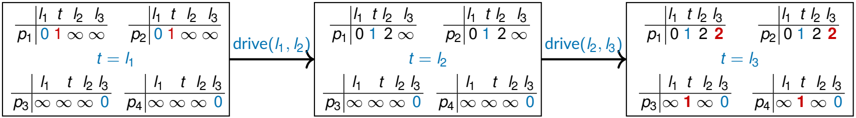
1. Extract global plan following parent pointers

Solution Reconstruction

For every member state $s \in [s^{\mathcal{F}}]$ of a decoupled state $s^{\mathcal{F}}$, we can construct a **global plan** reaching s from the initial state I.

Approach:

1. Extract global plan following parent pointers
2. For every step in the global plan, each leaf adds actions (by independence, actions of different leaves can be applied in any order)



Solution Reconstruction

For every member state $s \in [s^{\mathcal{F}}]$ of a decoupled state $s^{\mathcal{F}}$, we can construct a **global plan** reaching s from the initial state I.

Approach:

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Decoupled state 1 ($t = l_1$)

	l_1	t	l_2	l_3
p_1	0	1	∞	∞
p_2	0	1	∞	∞
p_3	∞	∞	∞	0
p_4	∞	∞	∞	0

$\xrightarrow{\text{drive}(l_1, l_2)}$

Decoupled state 2 ($t = l_2$)

	l_1	t	l_2	l_3
p_1	0	1	2	∞
p_2	0	1	2	∞
p_3	∞	∞	∞	0
p_4	∞	∞	∞	0

$\xrightarrow{\text{drive}(l_2, l_3)}$

Decoupled state 3 ($t = l_3$)

	l_1	t	l_2	l_3
p_1	0	1	2	2
p_2	0	1	2	2
p_3	∞	1	∞	0
p_4	∞	1	∞	0

unload(p_1, l_3)

unload(p_2, l_3)

load(p_3, l_3)

load(p_4, l_3)

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For every member state $s \in [s^{\mathcal{F}}]$ of a decoupled state $s^{\mathcal{F}}$, we can construct a **global plan** reaching s from the initial state I.

Approach:

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$t = l_1$

	l_1	t	l_2	l_3
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p_4	∞	∞	∞	0

$\xrightarrow{drive(l_1, l_2)}$

$t = l_2$

	l_1	t	l_2	l_3
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p_3	∞	∞	∞	0
p_4	∞	∞	∞	0

$\xrightarrow{drive(l_2, l_3)}$

$t = l_3$

	l_1	t	l_2	l_3
p_1	0	1	2	**2**
p_2	0	1	2	**2**
p_3	∞	**1**	∞	0
p_4	∞	**1**	∞	0

load(p_1, l_1)

load(p_2, l_1)

unload(p_1, l_3)

unload(p_2, l_3)

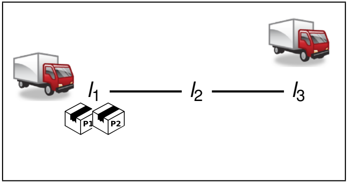
load(p_3, l_3)

load(p_4, l_3)

Initial Decoupled State – Inv Fork (Package–centered)

- Center variables get their value from the explicit state

Example:



Explicit initial state

	l_1	l_2	l_3
t_1	∞	∞	∞

	l_1	l_2	l_3
t_2	∞	∞	∞

$p_1 = l_1, p_2 = l_1$

Decoupled initial state

Initial Decoupled State – Inv Fork (Package-centered)

- Center variables get their value from the explicit state
- Set price of 0 for the leaf state that holds in the initial state

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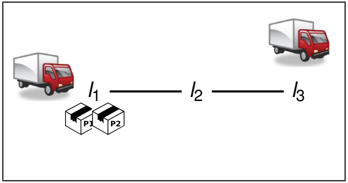

Explicit initial state

$$\begin{array}{c|ccc} & l_1 & l_2 & l_3 \\ \hline t_1 & 0 & \infty & \infty \end{array} \qquad \begin{array}{c|ccc} & l_1 & l_2 & l_3 \\ \hline t_2 & \infty & \infty & 0 \end{array}$$

$$p_1 = l_1, p_2 = l_1$$

Decoupled initial state

Initial Decoupled State – Inv Fork (Package–centered)

- Center variables get their value from the explicit state
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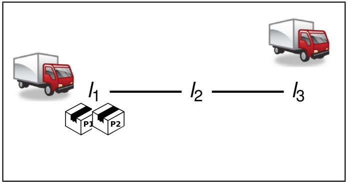

Explicit initial state

	l_1	l_2	l_3
t_1	0	1	

	l_1	l_2	l_3
t_2		1	0

$p_1 = l_1, p_2 = l_1$

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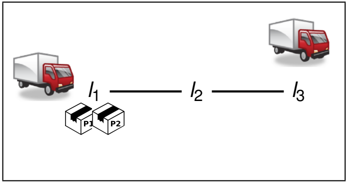


Explicit initial state

	l_1	l_2	l_3
t_1	0	1	2

	l_1	l_2	l_3
t_2	2	1	0

$p_1 = l_1, p_2 = l_1$

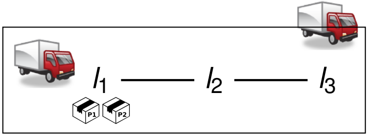
Decoupled initial state

Decoupled Search – InvFork (Package-centered)

	l_1	l_2	l_3
t_1	0	1	2

	l_1	l_2	l_3
t_2	2	1	0

$p_1 = l_1, p_2 = l_1$



$\text{load}(p_1, t_2, l_1)$

	l_1	l_2	l_3
t_1	0	1	2

	l_1	l_2	l_3
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$p_1 = t_2, p_2 = l_1$

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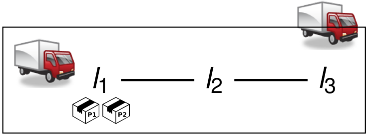

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	l_1	l_2	l_3
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	l_1	l_2	l_3
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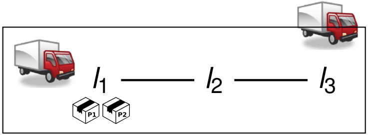


Decoupled Search – InvFork (Package-centered)

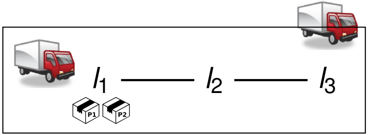
$$\begin{array}{c|ccc} & l_1 & l_2 & l_3 \\ \hline t_1 & 0 & 1 & 2 \end{array} \qquad \begin{array}{c|ccc} & l_1 & l_2 & l_3 \\ \hline t_2 & 2 & 1 & 0 \end{array}$$

$p_1 = l_1, p_2 = l_1$

$\xrightarrow{\text{load}(p_1, t_2, l_1)}$

$$\begin{array}{c|ccc} & l_1 & l_2 & l_3 \\ \hline t_1 & 0 & 1 & 2 \end{array} \qquad \begin{array}{c|ccc} & l_1 & l_2 & l_3 \\ \hline t_2 & 2 & 3 & 4 \end{array}$$

$p_1 = t_2, p_2 = l_1$

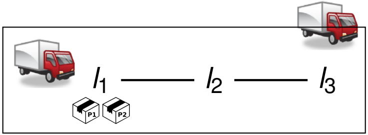


Decoupled Search – InvFork (Package-centered)

	l_1	l_2	l_3
t_1	0	1	2

	l_1	l_2	l_3
t_2	2	1	0

$p_1 = l_1, p_2 = l_1$



load(p_1, t_2, l_1)cost=3

	l_1	l_2	l_3
t_1	0	1	2

	l_1	l_2	l_3
t_2	0	1	2

$p_1 = t_2, p_2 = l_1$

Ensuring optimallity

All search algorithms can directly be applied in the decoupled search space

- Complete
- Optimal

Minor technical detail: in optimal planning, stop when min_f open $\geq$ current solution cost

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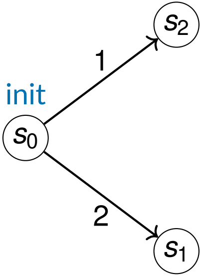
Minor technical detail: in optimal planning, stop when min_f open $\geq$ current solution cost

A^* cannot stopped when expanding a goal decoupled state

Reason: decoupled states contain multiple states, so the state with minimum f and the goal state could be two different ones

How to Eliminate Previously Seen States?



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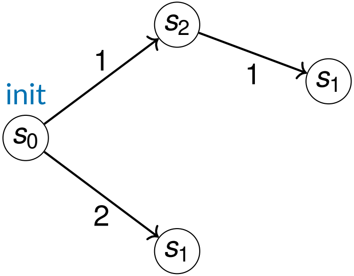

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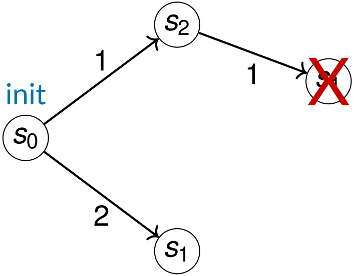

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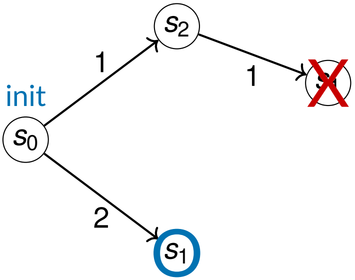

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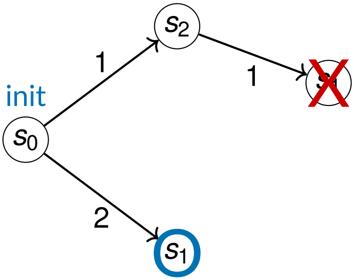

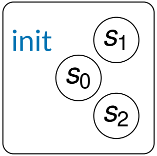

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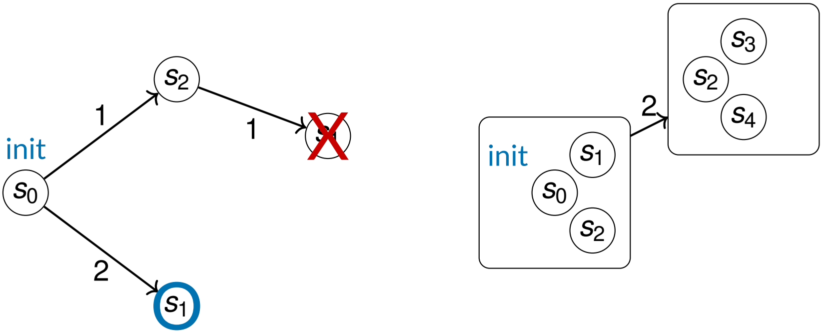

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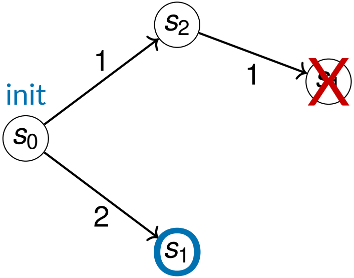

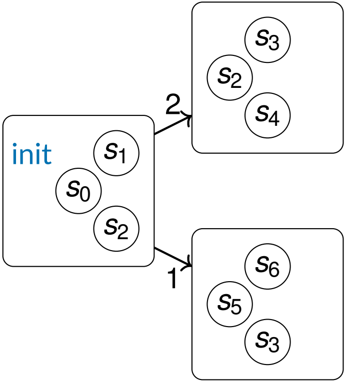

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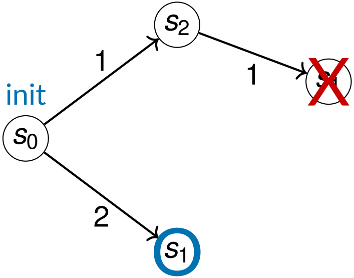

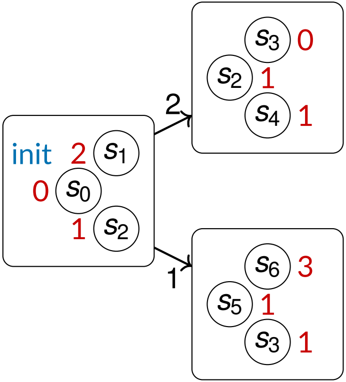

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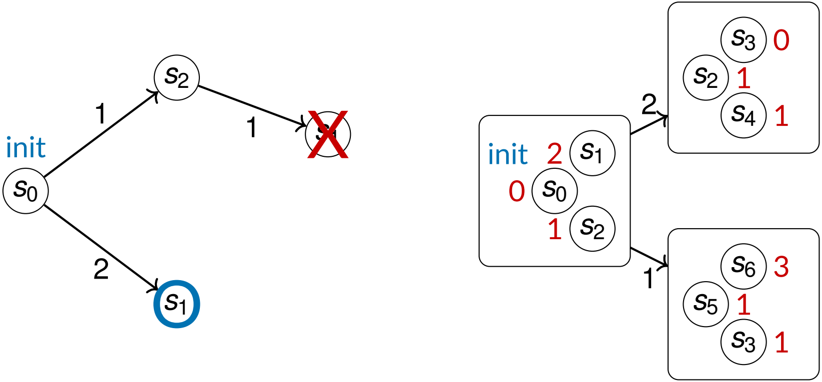

How powerful is exact duplicate checking for decoupled search?

Dominance Pruning for Decoupled States

Definition Dominance Pruning. *A decoupled state* $s^{\mathcal{F}}$ *dominates another state* $t^{\mathcal{F}}$*, denoted* $t^{\mathcal{F}} \preceq s^{\mathcal{F}}$*, if the center state is the same, i.e.* $s^{C}(s^{\mathcal{F}}) = s^{C}(t^{\mathcal{F}})$*, and* ***for all leaf states*** s^{L}*:*
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1. Dominance pruning can be exponentially stronger than exact duplicate checking.
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Practical Issues?

Exact duplicate checking is extremely efficient $\rightarrow$ hashing.

$\rightarrow$ For dominance pruning, we need to compare a new decoupled state to all previously seen states with the same center state.

Heuristics for Decoupled States

Heuristic: $h(s) : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$

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	l_1	t	l_2	l_3	l_4
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$t = l_3$

	l_1	t	l_2	l_3	l_4
p_3	∞	1	∞	2	0

	l_1	t	l_2	l_3	l_4
p_4	∞	1	∞	2	0

Decoupled State $s^{\mathcal{F}}$

$tl_3, p_1 t, p_2 l_3, p_3 l_3, p_4 l_3$ 7
$tl_3, p_1 l_3, p_2 l_3, p_3 t, p_4 t$ 6
$tl_3, p_1 t, p_2 t, p_3 t, p_4 l_3$ 5
$tl_3, p_1 l_3, p_2 t, p_3 t, p_4 l_3$ 6
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...

Hypercube $[s^{\mathcal{F}}]$ (144 member states!)

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Definition (Decoupled Heuristic). $h : S^{\mathcal{F}} \mapsto \mathbb{R} \cup \{\infty\}$

Star-perfect heuristic: $h^*_{\mathcal{F}}(s^{\mathcal{F}}) := \min_{s \in [s^{\mathcal{F}}]} prices(s^{\mathcal{F}}, s) + h^*(s)$

$h_{\mathcal{F}}$ is star-admissible if $h_{\mathcal{F}} \leq h^*_{\mathcal{F}}$

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$\rightarrow$ Pricing function is taken into account.

Planning Heuristics I: Naive Method

Given any planning heuristic $h_\Pi(s) : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$,
How to use $h_\Pi(s)$ to compute $h_\mathcal{F}(s^\mathcal{F})$?

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Pros:

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Cons:

- Decompresses the decoupled state, losing all the gains that decoupled search has

$\rightarrow$ So, we need better ways to compute or approximate this

Planning Heuristics II: Buy-leaves compilation

Buy-Leaves compilation: compute $h_{\Pi'}(s)$ on a different planning task Π', which is equal to Π but with additional actions:

- buy-p1-l1: eff: $p_1 = l_1$, cost=0,
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- Heuristic may approximate buying leaf states

Heuristics Part III: Precomputed heuristics

Given a precomputed abstraction heuristic (PDB, ADD, M&S) can we compute $h_{\mathcal{F}}(s^{\mathcal{F}})$ efficiently?

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Open Question: How to approximate additive abstractions in more general ways?

Pruning Methods

- Symmetry breaking (Gnad *et al.* (2017))
  $\rightarrow$ Permute prices and/or center state
- Dominance pruning with dominance analysis (for forks) (Torralba *et al.* (2016))
  $\rightarrow$ Propagate prices from better to worse leaf states
- Partial order reduction (Gnad *et al.* (2019))
  $\rightarrow$ over global actions

Recharging Robots

- Running Example from (Gnad *et al.* (2022))
- Submitted to the International Planning Competition
- IPC Organizers improved the domain (so, the version here is substantially different from the IPC version).

Decoupled Search – The Story so far..

- **Beating LM-cut with hmax (Sometimes) – Fork-Decoupled State-Space Search**
  G, Hoffmann, ICAPS'15.

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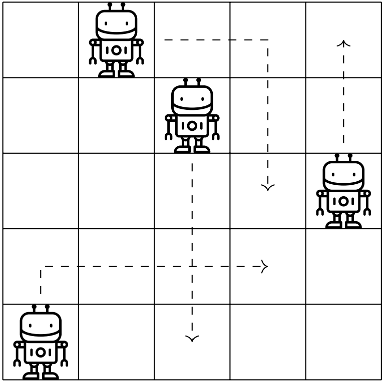
- How to obtain Star Factorings? IJCAI'17, ICAPS'19.

Collaborative Robots – Where is the center?



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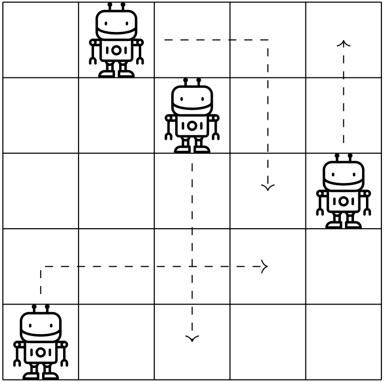

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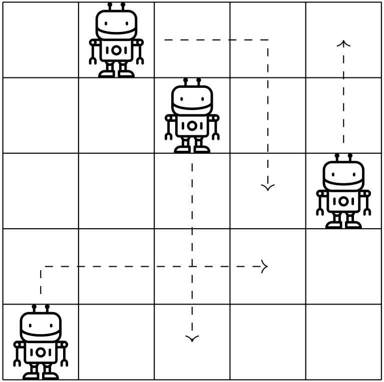

Causal Graph:

$$\begin{tikzcd}
& R_1 \leftrightarrow B_1 \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] \arrow[dd, leftrightarrow] & \\
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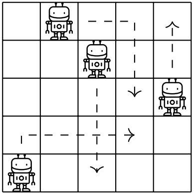
Factoring in the Recharging Robots



$move(R_i, B_i, l_x, l_y)$:

$$pre = \{R_i = l_x, B_i = b\},$$
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$charge(R_i, B_i, R_j, B_j)$:

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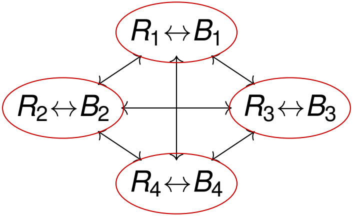

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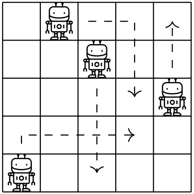

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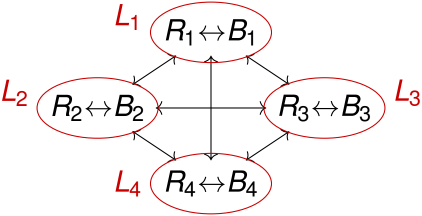
$\mathbf{C} = \emptyset$



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- **Require minimum flexibility** $\{0\%, 5\%, \ldots 100\%\}$.
- Leaf candidates: **action effect schemas** $vars(\mathit{eff}\, a)$ and **SCCs of CG**.

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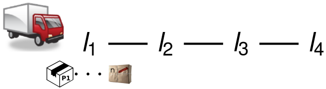

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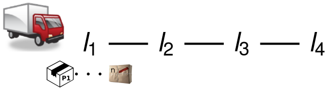

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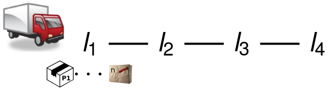

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- **Leaf mobility:**

  A leaf factor $L \in \mathcal{L}$ is *mobile*, if it has only-leaf actions

  $\rightarrow$ Leaves that are not mobile do not contribute to the search-space reduction

Maximizing the Number of Leaves – Complexity

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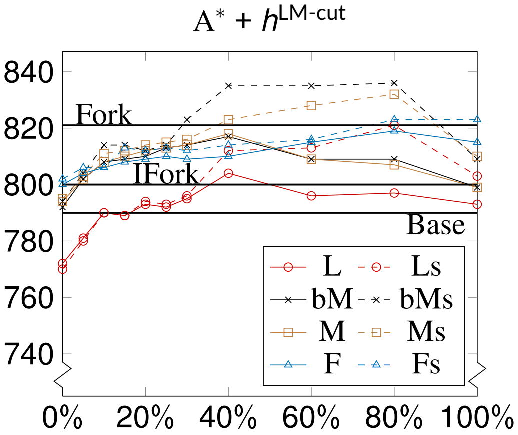
Practical Approaches:

- Compute MIS of CG $\rightarrow$ strict-star factorings,
- Analyze strongly-connected components in CG
  $\rightarrow$ (inverted-)fork factorings,
- Greedy selection of center variables based on CG connectivity
  $\rightarrow$ strict-star factorings,
- Encode factoring as Integer Linear Program $\rightarrow$ star factorings.

Enforce minimum Leaf Fact Flexibility



Fork: fork factorings, IFork: inverted-forks, Base: explicit-state search.

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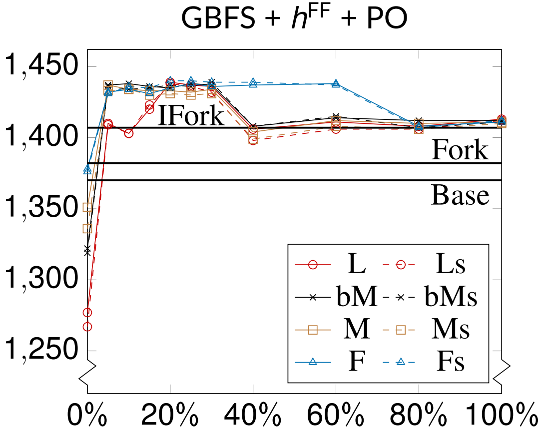

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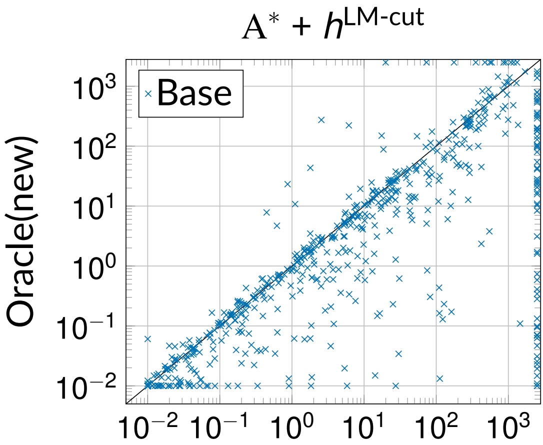
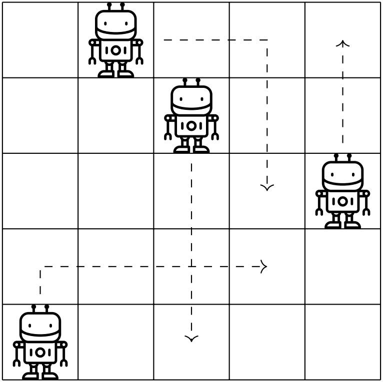
Runtime Scatterplot – LM-cut



Collaborative Robots – Deja vu?



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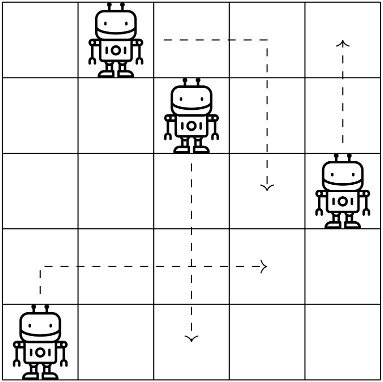

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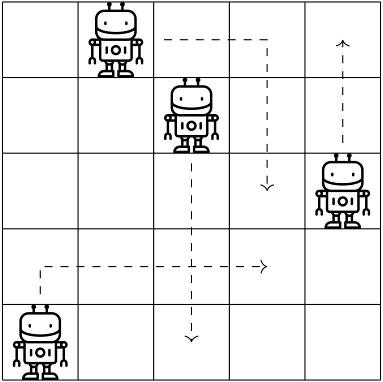
Multi-Agent Pathfinding

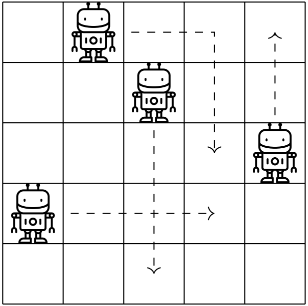


Actions: $move(R_i, l_x, l_y)$

- Constraint: Two agents cannot be in the same cell at the same time
- Metric: Minimize Makespan

Conflict-Based Search (Sharon *et al.* (2015))

1. Each agent plans their own shortest path
2. If there is no conflict, done



Conflict-Based Search (Sharon *et al.* (2015))

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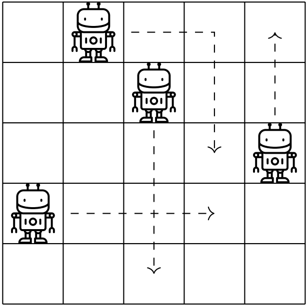
3. If there is a conflict, branch adding constraints that resolve it



R2 cannot be at $(3, 2)$ at $t = 2$

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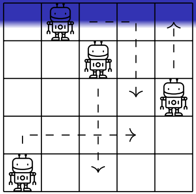
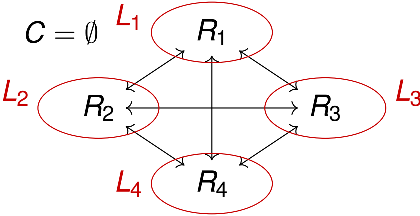
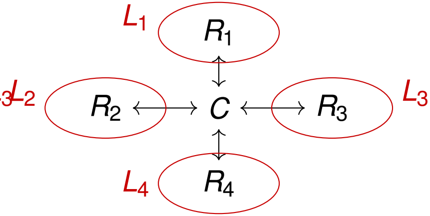


$move(R_i, B_i, l_x, l_y)$: $\rightarrow$ **internal(leaf-only) actions**

$book\_space(R_i, l_y, t)$: $\rightarrow$ **global actions**





Parallels between Decoupled Search and CBS

Decoupled Search for planning:	CBS for MAPF:
Leafs are conditionally independent	Agents are conditionally independent
Search over center actions	Search over conflict resolution
Handle conflicts eagerly	Handle conflicts lazily

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→ Can we transfer ideas?

Conclusion

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- The success of heuristic search heavily depends on the definition of the search space
- Decoupled Search:
  - State-space reduction method (reduce the search space by orders of magnitude)
  - Define the search space
  - Exploit the task structure (conditional independence)
  - Each search node in the new search space represents many states of the planning task
- Properties:
  - captures the reachability of all states of a planning task and preserves optimality for any optimal search algorithm
  - Decoupled search can be combined with (in principle) any known AI Planning heuristic, making available highly informed search guidance techniques for decoupled states.
- Still lots of things to do!

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Daniel Gnad, Álvaro Torralba, Alexander Shleyfman, and Jörg Hoffmann. Symmetry breaking in star-topology decoupled search. In Laura Barbulescu, Jeremy Frank, Mausam, and Stephen F. Smith, editors, *Proceedings of the Twenty-Seventh International Conference on Automated Planning and Scheduling (ICAPS 2017)*, pages 125–134. AAAI Press, 2017.

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