# From Qualitative to Quantitative Dominance Pruning for Optimal Planning

Álvaro Torralba
Saarland University

HSDIP Workshop
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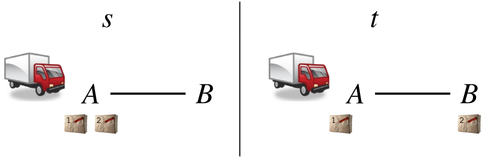
# Outline

# Dominance




Compare states: Which one is better?

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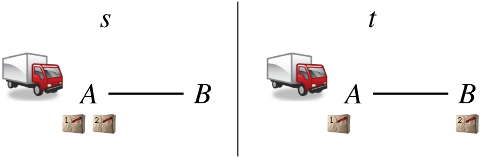


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**Compare states: Which one is better?**


: $A \preceq T \preceq B$

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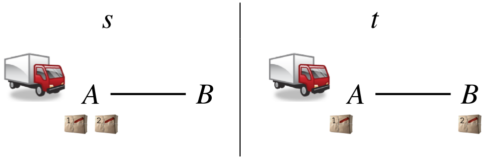


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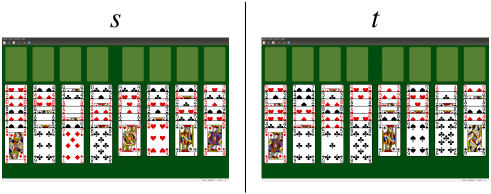

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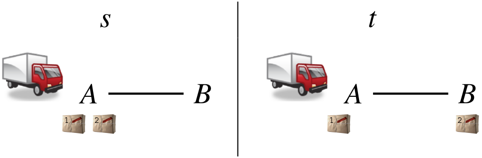


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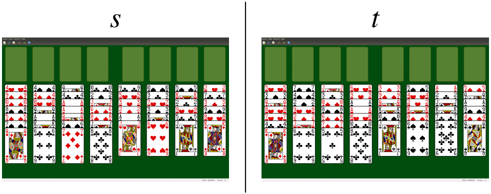


$t$ does not dominate $s$: $s \not\preceq t$

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Does $t$ dominate $s$? $\rightarrow$Yes/No answer

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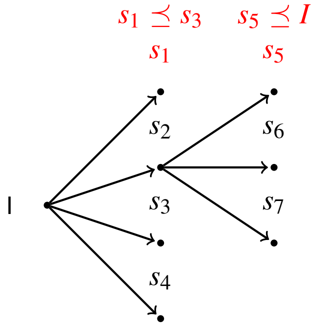
Prune $n_s$ if there exists $n_t$ s.t.

$$g(n_t) \leq g(n_s) \text{ and } s \preceq t$$

- Open or closed list



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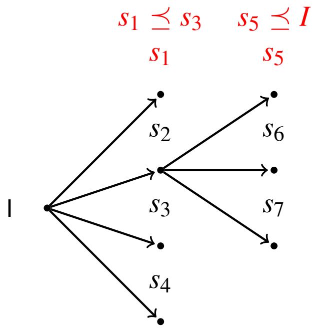
- Open or closed list
- Closed list
- Parent $\rightarrow$Never unload a package in any location other than its destination!



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By how much $t$ dominates $s$? $\rightarrow$function $\mathcal{D} : S \times S \rightarrow \mathbb{R} \cup \{-\infty\}$

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$$\mathcal{D}(s,t) = \begin{cases} C & t \text{ is strictly closer to the goal than s (by at least } C) \\ 0 & t \text{ is at least as close as } s \\ -C & \text{t is at most C units of cost farther than } s \\ -\infty & \text{we know nothing} \end{cases}$$

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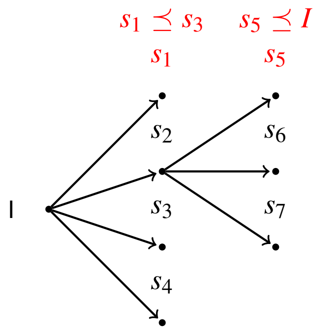
$\rightarrow$ Qualitative dominance is a special case if we use only $0$ or $-\infty$

# Leveraging Quantitative Dominance

| | |
|---|---|
| | Prune $n_s$ if there exists $n_t$ s.t. |
| Qualitative | $g(n_t) \leq g(n_s)$ and $s \preceq t$ |
| Quantitative | |



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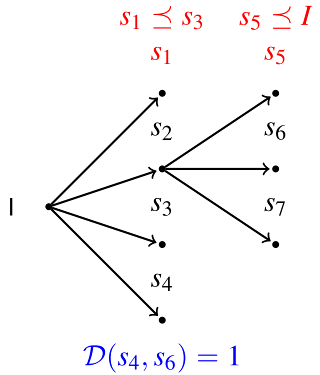
| Qualitative | $g(n_t) \leq g(n_s)$ and $s \preceq t$ |
| Quantitative | $\mathcal{D}(s,t) + g(n_s) - g(n_t) \geq 0$ if $\mathcal{D}(s,t) \geq 0$ |



# Leveraging Quantitative Dominance

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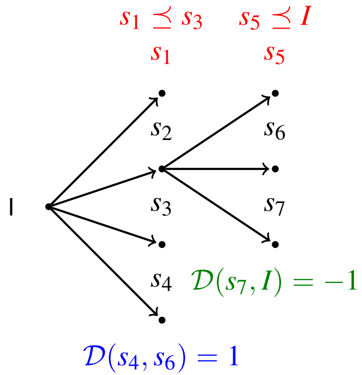
| Qualitative | $g(n_t) \leq g(n_s)$ and $s \preceq t$ |
| Quantitative | $\mathcal{D}(s,t) + g(n_s) - g(n_t) \geq 0$ if $\mathcal{D}(s,t) \geq 0$ |
| | $\mathcal{D}(s,t) + g(n_s) - g(n_t) > 0$ if $\mathcal{D}(s,t) < 0$ |



# Outline

# Compositional Approach

Consider a partition of the problem: $\Theta_1, \ldots, \Theta_k$

$\{\preceq_1, \ldots, \preceq_k\}$ is a label-dominance simulation if, whenever $s \preceq_i t$:

- Goal-respecting: $s \in S_i^G$ implies that $t \in S_i^G$
- For all $s \xrightarrow{l} s'$ in $\Theta_i$, there exists $t \xrightarrow{l'} t'$ in $\Theta_i$ s.t.:
  1. $s' \preceq_i t'$,
  2. $c(l') \leq c(l)$, and
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$\rightarrow s \preceq t$ iff $\forall i \in [1, k]\ s_i \preceq_i t_i$

# Quantifying Label-Dominance Simulation

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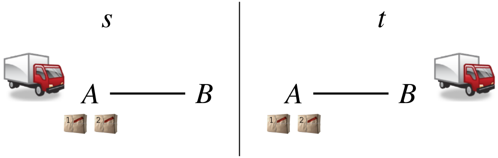
$\{\mathcal{D}_1, \dots, \mathcal{D}_k\}$ is a quantitative LD simulation for $\{\Theta_1, \dots, \Theta_k\}$ if:

$$\mathcal{D}_i(s,t) \leq \min_{s \xrightarrow{l} s'} \max_{t \xrightarrow{l'} t'} \mathcal{D}_i(s',t') + c(l) - c(l') + \sum_{j \neq i} \mathcal{D}_j^L(l,l')$$

$$\mathcal{D}(s,t) = \sum_{i \in [1,k]} \mathcal{D}_i(s_i,t_i)$$

# Discovering negative dominance

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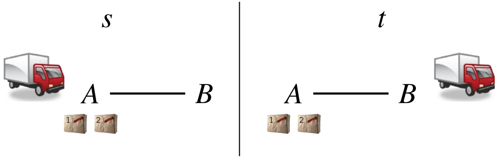


We can always drive between $s$ and $t$: $\mathcal{D}(t,s) = \mathcal{D}(s,t) = -1$

$\tau$-label: no preconditions or negative side effects elsewhere

$s \xrightarrow{l} s'$ can be simulated by a path $t \xrightarrow{\tau}{}^* u \xrightarrow{l'} u' \xrightarrow{\tau}{}^* t'$

: $\mathcal{D}_P(A,T) = \mathcal{D}_P(T,B) = +1$

: $\mathcal{D}_T(A,B) = \mathcal{D}_T(B,A) = -1$

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If $s \xrightarrow{a} s'$ and $\mathcal{D}(s, s') \geq c(a)$ then $a$ starts an optimal plan from $s$.

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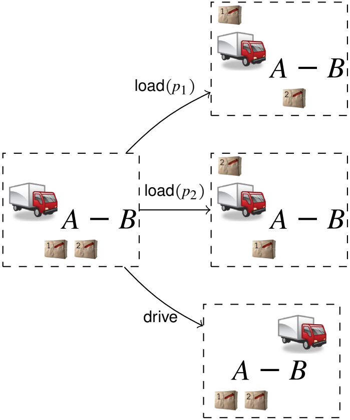

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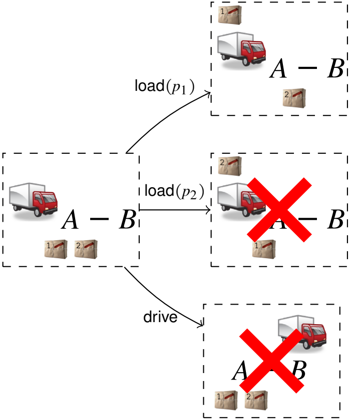


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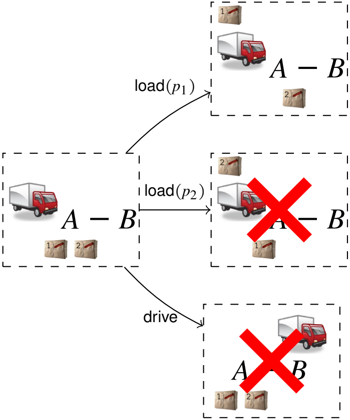
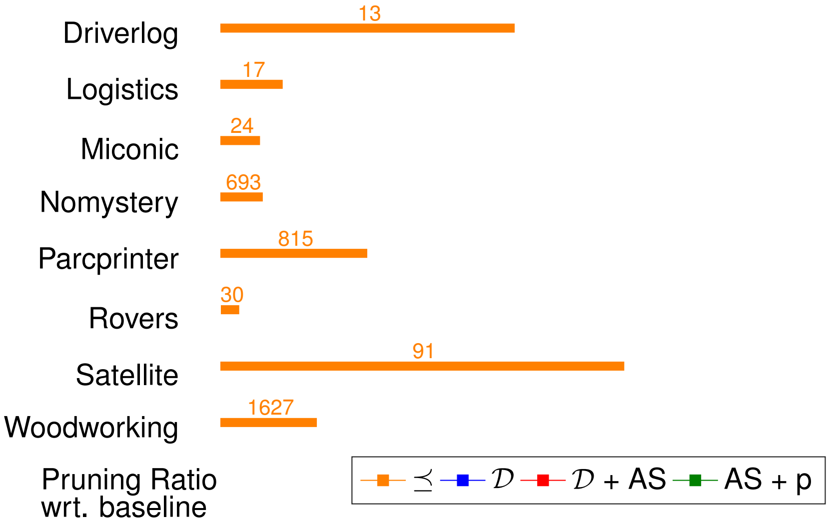



- Prune every other successor
- Reduce branching factor to $1$!
- In our example. If possible:
  - load a package
  - unload a package in its destination

→Branch only over drive actions!

# Outline

# Dramatic Pruning Power in Blind Search!



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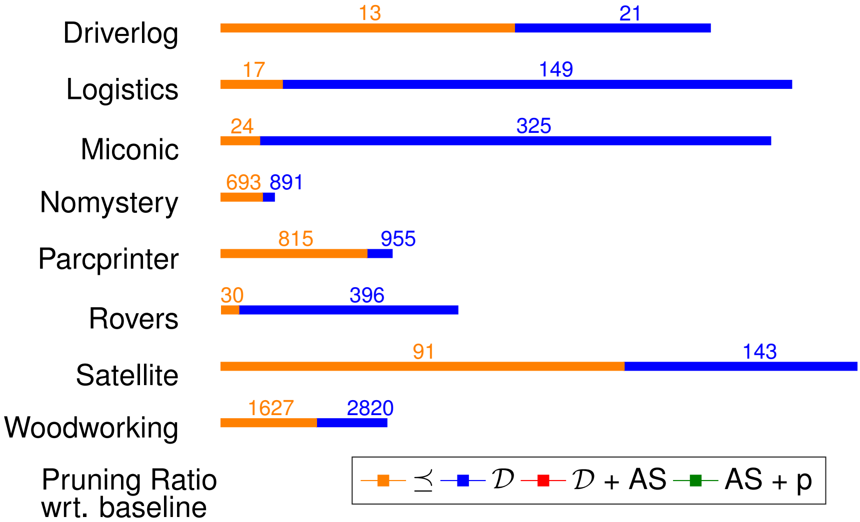

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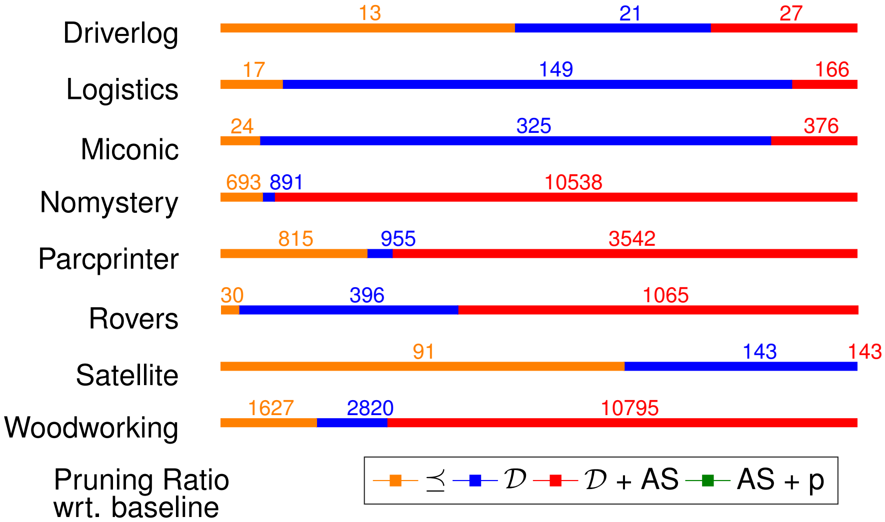

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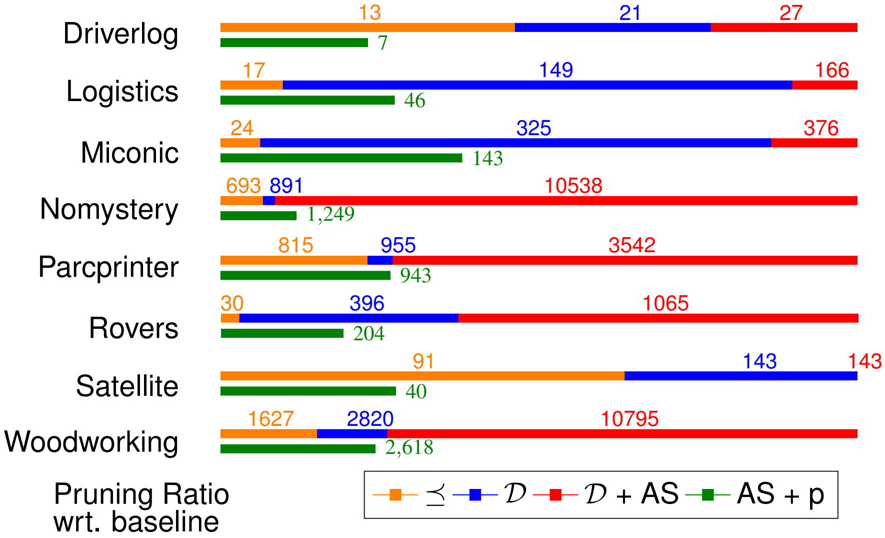
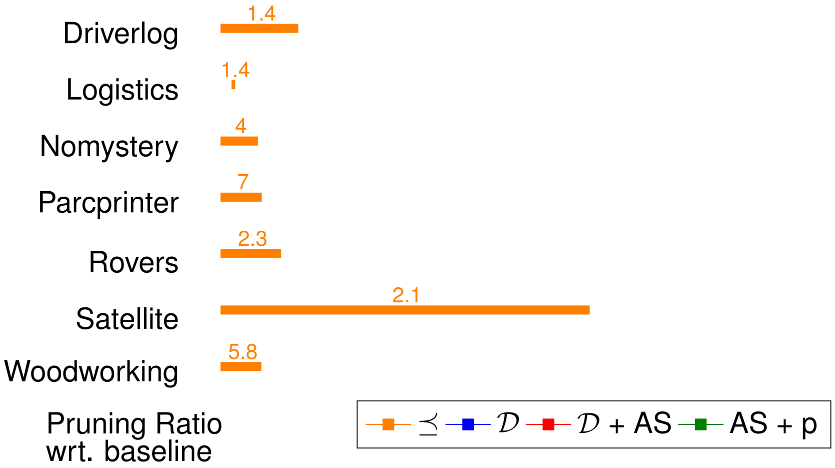


# Great Pruning Power in LM-Cut!



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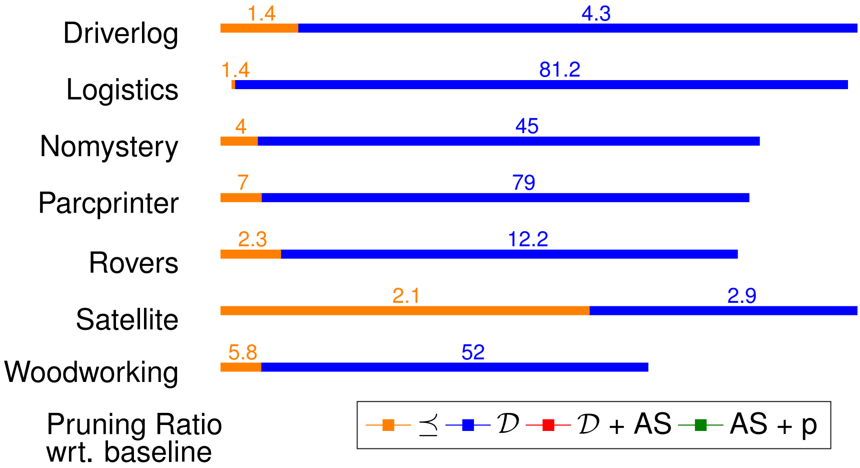

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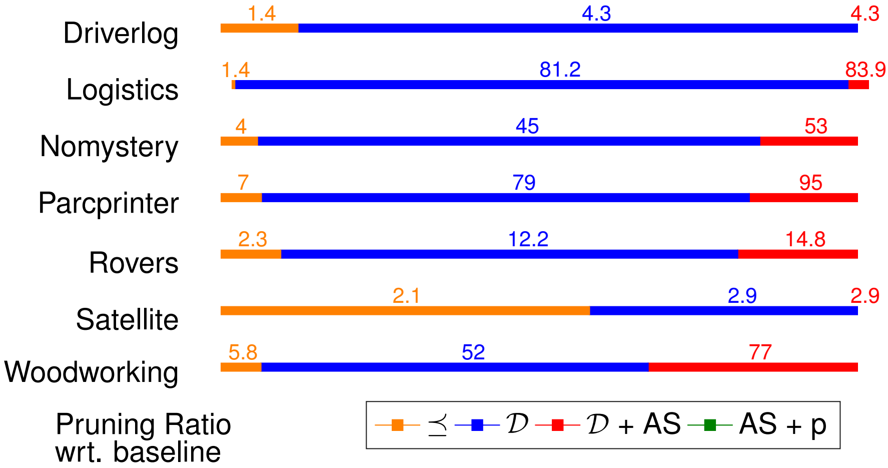

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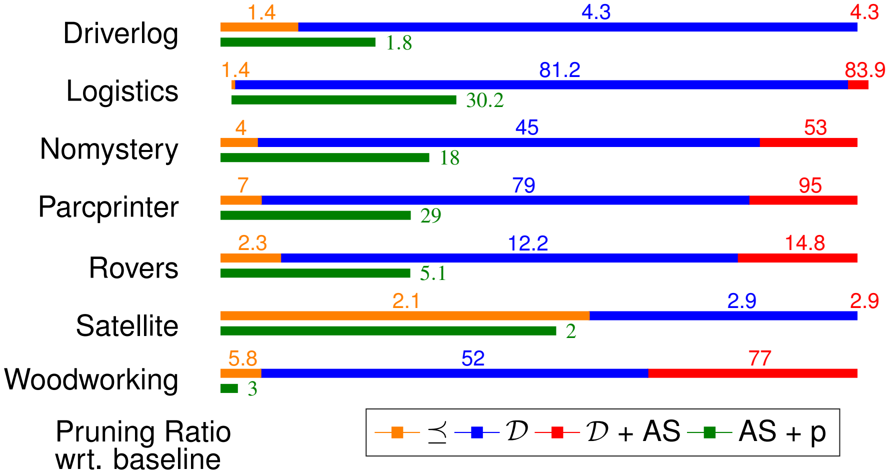

# Coverage

| | | Blind | | | | LM-cut | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $B$ | $\preceq$ | AS + p | POR | $B$ | $\preceq$ | AS + p | POR |
| Driverlog | 20 | 7 | 9 | **10** | 7 | 13 | 13 | 13 | 13 |
| Floortile | 40 | 2 | 11 | **16** | 2 | 13 | **16** | **16** | 13 |
| Logistics | 63 | 12 | 21 | **27** | 12 | 26 | 26 | **33** | 27 |
| Miconic | 150 | 55 | 60 | **77** | 50 | 141 | 141 | **142** | 141 |
| Nomystery | 20 | 8 | 16 | **20** | 8 | 14 | **20** | **20** | 14 |
| Openstacks | 100 | 49 | 51 | **55** | 50 | 47 | 51 | **52** | 49 |
| Parcprinter | 50 | 16 | 32 | 44 | **50** | 31 | 35 | 48 | **50** |
| Pathwaysnoneg | 30 | 4 | 4 | **5** | 4 | 5 | 5 | 5 | 5 |
| Rovers | 40 | 6 | **8** | **8** | 7 | 7 | 9 | **10** | **10** |
| Satellite | 36 | 6 | 6 | 6 | 6 | 7 | 10 | **12** | **12** |
| Sokoban | 50 | 41 | **43** | **43** | 39 | **50** | 49 | 49 | **50** |
| TPP | 30 | 6 | 6 | 6 | 6 | 7 | 7 | **8** | 6 |
| Trucksstrips | 30 | 6 | **8** | **8** | 6 | 10 | 10 | 10 | 10 |
| Visitall | 40 | 12 | **13** | 12 | 12 | 15 | **16** | 15 | 15 |
| Woodworking | 50 | 11 | 30 | **38** | 24 | 29 | 48 | **50** | 46 |
| Zenotravel | 20 | 8 | **9** | **9** | 8 | 13 | 13 | 13 | 13 |
| Total | 1612 | 610 | 659 | **738** | 613 | 835 | 856 | **896** | 881 |

# Outline

# Conclusions

- Quantitative Dominance:
  - Bound difference in goal distance between states
  - Useful for dominance and action selection pruning
  - Good results and even more potential to be unleashed!
- Future work:
  - New ways to discover (quantitative) dominance
  - More efficient ways to perform dominance pruning
  - New uses for dominance