Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Conclusions

On the Optimal Efficiency of A* with Dominance Pruning

Álvaro Torralba



AALBORG UNIVERSITET

AAAI'2021

イロト イポト イヨト イヨト 二日

1/32

On the Optimal Efficiency of A* with Dominance Pruning

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
Motivation					

- A*: canonical choice for solving shortest path problems
- A* is optimally efficient in node expansions (Dechter and Pearl, 1985)

・ロット (日) ・ (日) ・ (日)

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
Motivation					

- A*: canonical choice for solving shortest path problems
- A* is optimally efficient in node expansions (Dechter and Pearl, 1985)
- Dominance pruning methods

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
Motivation					

- A*: canonical choice for solving shortest path problems
- A* is optimally efficient in node expansions (Dechter and Pearl, 1985)
- Dominance pruning methods \rightarrow new source of information!

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
Motivation					

- A*: canonical choice for solving shortest path problems
- A* is optimally efficient in node expansions (Dechter and Pearl, 1985)
- Dominance pruning methods→ new source of information!
- We use dominance pruning in A*:
 - Is this a good choice?
 - Could we achieve more pruning with different expansion orders?
 - What tie-breaking strategies are good for dominance pruning?

A D N A D N A D N A D N B

Stackelberg Planning ●000000	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
Outline					

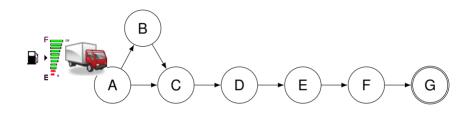
Stackelberg Planning

- 2 Solving Stackelberg Tasks: Previous Work
- Symbolic Leader Search
- 4 Net-Benefit Stackelberg Planning
- 5 Empirical Results
- 6 Conclusions

On the Optimal Efficiency of A* with Dominance Pruning

Stackelbe o●oooo	0	ling	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
_							

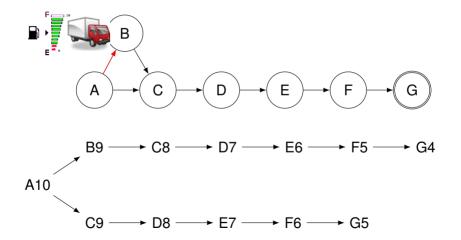
Running Example



<ロ> <四> <四> <三</p>

Stackelberg Planning o●ooooo	Solving	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions

Running Example



Stackelberg Planning oo●oooo	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
DXBB Algori	thms				

5/32

UDXBB: Unidirectional, Deterministic, Expansion-based, Black Box

Access to the state space Θ only via node expansions

Additionally the algorithm is given an admissible heuristic function $h \rightarrow h(s)$ estimates the distance from *s* to the goal $h^*(s)$, $h(s) \leq h^*(s)$

<mark>h2</mark> A10

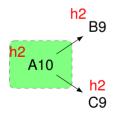
On the Optimal Efficiency of A* with Dominance Pruning

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
DXBB Algori	thms				

UDXBB: Unidirectional, Deterministic, Expansion-based, Black Box

Access to the state space Θ only via node expansions

Additionally the algorithm is given an admissible heuristic function $h \rightarrow h(s)$ estimates the distance from *s* to the goal $h^*(s)$, $h(s) \leq h^*(s)$

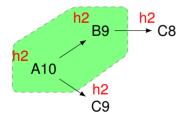


Stackelberg Planning oo●oooo	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
DXBB Algori	thms				

UDXBB: Unidirectional, Deterministic, Expansion-based, Black Box

Access to the state space Θ only via node expansions

Additionally the algorithm is given an admissible heuristic function $h \rightarrow h(s)$ estimates the distance from *s* to the goal $h^*(s)$, $h(s) \leq h^*(s)$

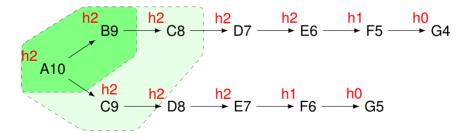


Stackelberg Planning oo●oooo	Solving	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions 00
DXBB Alao	rithms				

UDXBB: Unidirectional, Deterministic, Expansion-based, Black Box

Access to the state space Θ only via node expansions

Additionally the algorithm is given an admissible heuristic function $h \rightarrow h(s)$ estimates the distance from *s* to the goal $h^*(s)$, $h(s) \leq h^*(s)$



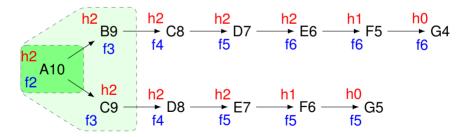
On the Optimal Efficiency of A* with Dominance Pruning

3

Stackelberg Planning 000●000	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
A*					

A*: Expand nodes based on *f*-value: $f(n_s) = g(n_s) + h(s)$

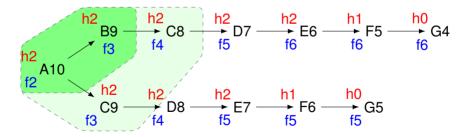
Family of algorithms: tie-breaking strategy may pick any node with minimum *f*-value



Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Conclusions
000●000	0000		0000	0000000	00
A*					

A*: Expand nodes based on *f*-value: $f(n_s) = g(n_s) + h(s)$

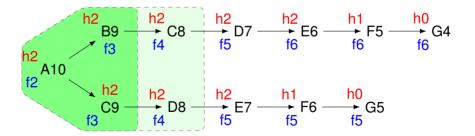
Family of algorithms: tie-breaking strategy may pick any node with minimum *f*-value



Stackelberg Planning 000●000	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
A*					

A*: Expand nodes based on *f*-value: $f(n_s) = g(n_s) + h(s)$

Family of algorithms: tie-breaking strategy may pick any node with minimum *f*-value



イロン 不良 とくほう イロン 一日

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Co
0000000					

Generalized BF Search Strategies and the Optimality of A*

Admissible Globally Compatible Boet-Firet if h<h* with A* And Aac Ant A** is 3-optimal Admissible A* is 1-ontimal A* is 1-optimal IAD. No 2-optimal exists No 0-optimal exists No 0-optimal exists Admissible and A* is 2-optimal A* is 0-optimal A * is 0-optimal non pathological No 1-optimal exists Domain 170 Problem Instances Consistent A* is 1-optimal A* is 1-optimal A* is 1-optimal No 0-optimal exists No 0-optimal exists No 0-optimal exists I CON Consistent nonnathological A* is 0-optimal A* is 0-optimal A* is 0-optimal 1CON

Class of Algorithms

531

On the Optimal Efficiency of A* with Dominance Pruning

01

Conclusions

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Conclusions
0000000					

Generalized BF Search Strategies and the Optimality of A*

531

7/32

			_	
		Admissible if h≦h* A _{ad}	Globally Compatible with A* Agc	Best-First A br
Domain of Problem Instances	Admissible I _{AD}	A** is 3-optimal No 2-optimal exists	A* is 1-optimal No 0-optimal exists	A* is 1-optimal No 0-optimal exists
	Admissible and nonpathological I _{ÃD}	A* is 2-optimal No 1-optimal exists	A* is 0-optimal	A * is 0-optimal
	Consistent CON	A* is 1-optimal No 0-optimal exists	A* is 1-optimal No 0-optimal exists	A* is 1-optimal No 0-optimal exists
of A [*] with Down	Consistent nonpathological ICON	A* is 0-optimal	A* is 0-optimal	A* is 0-optimal

Class of Algorithms

On the Optimal Efficiency of A* with Dominance Pruni

Stackelberg Planning 00000€0	Solving	Symbolic Leader Search	Net-Benefit	Results 0000000	Conclusions

A* is 1-optimal on consistent instances

Let *N* be the set of states expanded by any admissible UDXBB algorithm, then there exists a tie-breaking of A^{*} that expands subset of *N*.

Stackelberg Planning 00000●0	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions

A* is 1-optimal on consistent instances

Let *N* be the set of states expanded by any admissible UDXBB algorithm, then there exists a tie-breaking of A^{*} that expands subset of *N*.

Consistent Heuristic: $h(s) - h(t) \le c(s, t)$

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Conclusions
0000000					

A* is 1-optimal on consistent instances

Let *N* be the set of states expanded by any admissible UDXBB algorithm, then there exists a tie-breaking of A^{*} that expands subset of *N*.

Consistent Heuristic: $h(s) - h(t) \le c(s, t)$

Nodes are expanded with their optimal g-value (no re-expansions)

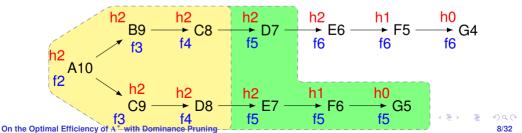
Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Conclusions
0000000					

A* is 1-optimal on consistent instances

Let *N* be the set of states expanded by any admissible UDXBB algorithm, then there exists a tie-breaking of A^{*} that expands subset of *N*.

Consistent Heuristic: $h(s) - h(t) \le c(s, t)$

Nodes are expanded with their optimal g-value (no re-expansions)
 Must-expand nodes: *f*(*n*) < *f**



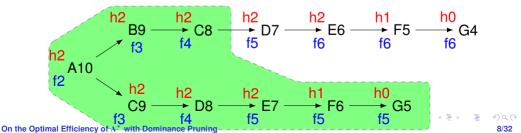
Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Conclusions
0000000					

A* is 1-optimal on consistent instances

Let *N* be the set of states expanded by any admissible UDXBB algorithm, then there exists a tie-breaking of A^{*} that expands subset of *N*.

Consistent Heuristic: $h(s) - h(t) \le c(s, t)$

Nodes are expanded with their optimal g-value (no re-expansions)
 Must-expand nodes: *f*(*n*) < *f**



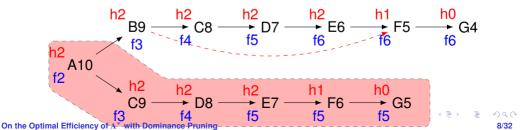
Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Conclusions
0000000					

A* is 1-optimal on consistent instances

Let *N* be the set of states expanded by any admissible UDXBB algorithm, then there exists a tie-breaking of A^{*} that expands subset of *N*.

Consistent Heuristic: $h(s) - h(t) \le c(s, t)$

Nodes are expanded with their optimal g-value (no re-expansions)
 Must-expand nodes: *f*(*n*) < *f**



Stackelberg Planning	Solving ●000	Symbolic Leader Search	Net-Benefit	Results 0000000	Conclusions
Outline					

- Stackelberg Planning
- 2 Solving Stackelberg Tasks: Previous Work
- 3 Symbolic Leader Search
- 4 Net-Benefit Stackelberg Planning
- 5 Empirical Results
- Conclusions

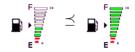
On the Optimal Efficiency of A* with Dominance Pruning

Stackelberg Planning	Solving o●oo	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
Dominance					

t dominates *s* ($s \leq t$) implies that $h^*(t) \leq h^*(s)$

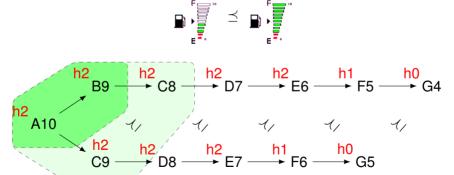
Stackelberg Planning	Solving o●oo	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions 00
Dominance					

t dominates *s* ($s \leq t$) implies that $h^*(t) \leq h^*(s)$



Stackelberg Planning	Solving ○●○○	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
Dominance					

t dominates *s* (*s* \leq *t*) implies that $h^*(t) \leq h^*(s)$

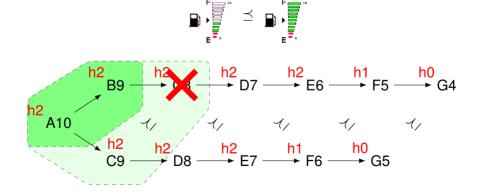


On the Optimal Efficiency of A* with Dominance Pruning

イロン 不良 とくほう イロン しゅう

Stackelberg Planning	Solving ○●○○	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
Dominance					

t dominates *s* (*s* \leq *t*) implies that $h^*(t) \leq h^*(s)$



On the Optimal Efficiency of A* with Dominance Pruning

イロン 不良 とくほう イロン 一日

Stackelberg Planning	Solving ○○●○	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions 00
UDXBB with	Domina	nce Pruning			

$UDXBB_{pr}$

UDXBB algorithms that can prune a node n_s whenever another n_t has been seen such that $g(n_t) \le g(n_s)$ and t dominates s.

Stackelberg Planning	Solving ○○●○	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
UDXBB with	Domina	nce Pruning			

UDXBB_{pr}

UDXBB algorithms that can prune a node n_s whenever another n_t has been seen such that $g(n_t) \le g(n_s)$ and t dominates s.

No access to \preceq : can only use dominance for pruning according to the rule above

Stackelberg Planning	Solving oo●o	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
UDXBB with	Domina	nce Pruning			

UDXBB_{pr}

UDXBB algorithms that can prune a node n_s whenever another n_t has been seen such that $g(n_t) \le g(n_s)$ and t dominates s.

No access to \leq : can only use dominance for pruning according to the rule above

- A^* with dominance pruning (A_{pr}^*) :
 - Expand nodes based on *f*-value: $f(n_s) = g(n_s) + h(s)$
 - Prune any node that can be pruned

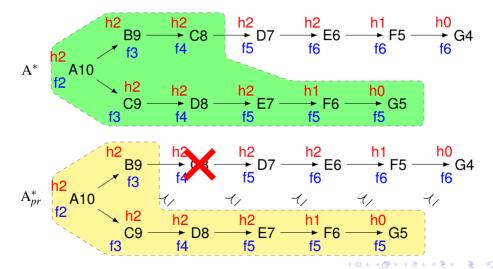
Stackelberg Planning	Solving	Symbolic Leader Search ●oooooo	Net-Benefit 0000	Results 0000000	Conclusions
Outline					

- Stackelberg Planning
- 2 Solving Stackelberg Tasks: Previous Work
- Symbolic Leader Search
 - 4 Net-Benefit Stackelberg Planning
 - 5 Empirical Results
 - Conclusions

On the Optimal Efficiency of A* with Dominance Pruning



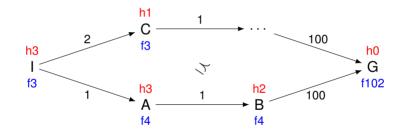
A_{pr}^* is 1-optimal over A^* (until last f-layer)



On the Optimal Efficiency of A* with Dominance Pruning



 \rightarrow The expansion order of A^* may not be optimal

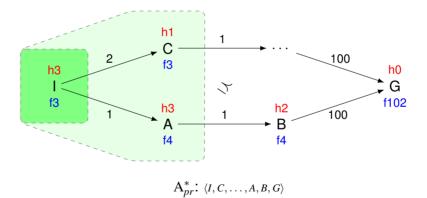


On the Optimal Efficiency of A* with Dominance Pruning

A D N A B N A B N A B N



 \rightarrow The expansion order of A^* may not be optimal

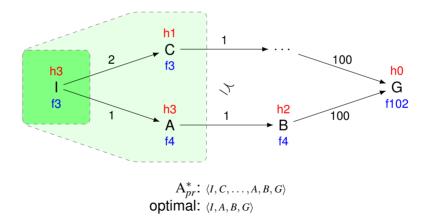


On the Optimal Efficiency of A* with Dominance Pruning

A D N A D N A D N A D N



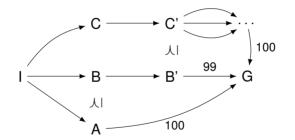
 \rightarrow The expansion order of A^* may not be optimal



A D N A D N A D N A D N



 \rightarrow Sometimes it could be worth it to expand a node even if it can be pruned

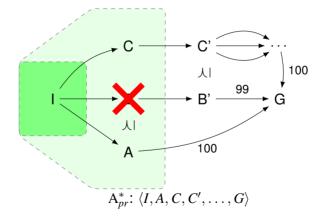


化口水 化固水 化医水化医水



A_{pr}^{*} is not optimally efficient in general (take 2)

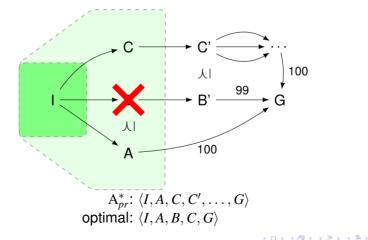
 \rightarrow Sometimes it could be worth it to expand a node even if it can be pruned





A_{pr}^{*} is not optimally efficient in general (take 2)

 \rightarrow Sometimes it could be worth it to expand a node even if it can be pruned



Stackelberg Planning	Solving	Symbolic Leader Search oooo●oo	Net-Benefit 0000	Results 0000000	Conclusions
Consistenc	V				

Consistent instances

An instance $I = \langle \Theta, h, \preceq \rangle$ is consistent if:

- h is consistent
- $\ge \ \le$ is a transitive cost-simulation relation
- \bigcirc \preceq is consistent with *h*: $s \preceq t \implies h(t) \leq h(s)$

Stackelberg Planning	Solving	Symbolic Leader Search 00000€0	Net-Benefit	Results 0000000	Conclusions
Consistent	Demine	naa Dalatiana			

 \preceq is a transitive cost-simulation relation

Stackelberg Planning	Solving	Symbolic Leader Search 00000●0	Net-Benefit	Results 0000000	Conclusions
Consistent	Domina	nce Relations			

 \preceq is a transitive cost-simulation relation

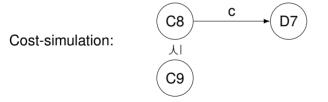
Transitive: $C \leq B$ and $B \leq A$ implies $C \leq A$

(日)

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions
	Denstree	na Dalationa			

\preceq is a transitive cost-simulation relation

Transitive: $C \leq B$ and $B \leq A$ implies $C \leq A$

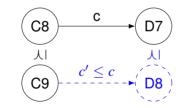


Stackelb 000000	erg Planning	olving	Symbolic Leader Search	Net-Benefit	Results 0000000	Conclusions
<u> </u>	1. A.		D L U			

\preceq is a transitive cost-simulation relation

Transitive: $C \leq B$ and $B \leq A$ implies $C \leq A$

Cost-simulation:



Stackelbe 000000	erg Planning	Solving	Symbolic Leader Search 00000€0	Net-Benefit	Results 0000000	Conclusions
~	1. A.		D I I I			

\preceq is a transitive cost-simulation relation

Transitive: $C \leq B$ and $B \leq A$ implies $C \leq A$

 $\rightarrow \mbox{State-of-the-art}$ methods on dominance pruning derive transitive cost-simulation relations

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results	Conclusions
		000000			

 \leq is consistent with *h*: $s \leq t \implies h(t) \leq h(s)$

Stackelbe	e rg Planning D	Solving	Symbolic Leader Search ○○○○○○●	Net-Benefit	Results 0000000	Conclusions

 \leq is consistent with *h*: $s \leq t \implies h(t) \leq h(s)$

If h(s) < h(t) then $h(t) \le h^*(t) \le h^*(s)$, so we can raise h(s) to h(t)!

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results 0000000	Conclusions

 \leq is consistent with *h*: $s \leq t \implies h(t) \leq h(s)$

If h(s) < h(t) then $h(t) \le h^*(t) \le h^*(s)$, so we can raise h(s) to h(t)!

Thm: If \leq is consistent with *h*, whenever n_s can be pruned by n_t then $f(n_t) \leq f(n_s)$

On the Optimal Efficiency of A* with Dominance Pruning

イロン 不良 とくほう イロン 一日

Stackelberg Planning	Solving	Symbolic Leader Search 000000●	Net-Benefit	Results 0000000	Conclusions

 \leq is consistent with *h*: $s \leq t \implies h(t) \leq h(s)$

If h(s) < h(t) then $h(t) \le h^*(t) \le h^*(s)$, so we can raise h(s) to h(t)!

Thm: If \leq is consistent with *h*, whenever n_s can be pruned by n_t then $f(n_t) \leq f(n_s)$

Conjecture: Most consistent heuristics and dominance relations are consistent unless dominance considers larger subsets of variables

イロン 不良 とくほう イロン 一日

Stackelberg Planning	Solving 0000	Symbolic Leader Search 000000●	Net-Benefit	Results 0000000	Conclusions

 \leq is consistent with *h*: $s \leq t \implies h(t) \leq h(s)$

If h(s) < h(t) then $h(t) \le h^*(t) \le h^*(s)$, so we can raise h(s) to h(t)!

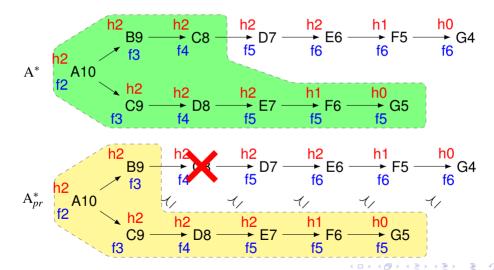
Thm: If \leq is consistent with *h*, whenever n_s can be pruned by n_t then $f(n_t) \leq f(n_s)$

Conjecture: Most consistent heuristics and dominance relations are consistent unless dominance considers larger subsets of variables

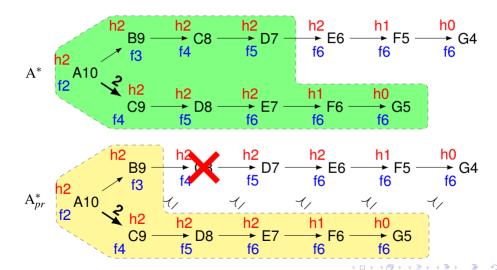
The information of *h* and \leq is still complementary!

- When h(s) < h(t), s is more promising but there is no guarantee
- When $t \leq s$, we are certain that t is as good as s (but if $s \not\leq t$ we know nothing)

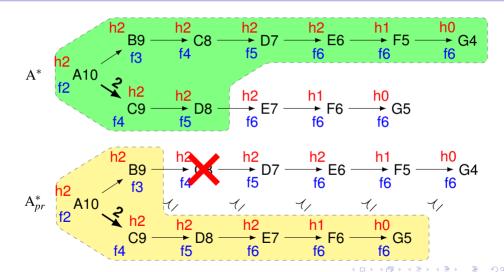








Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results 0000000	Conclusions
A* is NOT	1_optim	al on consistan	t instance	e	



Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit 0000	Results 0000000	Conclusions 00
A_{nr}^* is #-opt	imal				

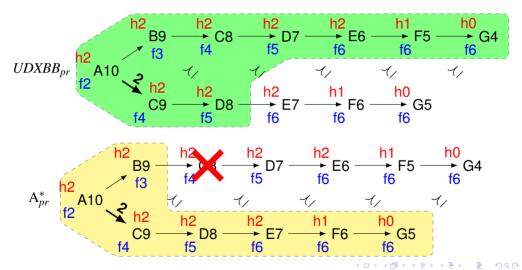
\mathbf{A}_{pr}^* is **#-optimal** on consistent instances wrt. UDXBB_{pr}

Let *N* be the set of states expanded by any admissible $UDXBB_{pr}$ algorithm, then there exists a tie-breaking of A_{pr}^* that expands *N'* with $|N'| \le |N|$.

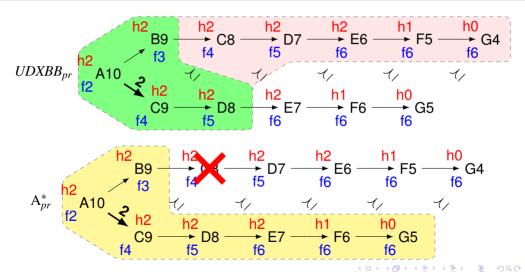
 \rightarrow It may not expand a subset of nodes but it will expand the least amount of nodes!

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit	Results 0000000	Conclusions

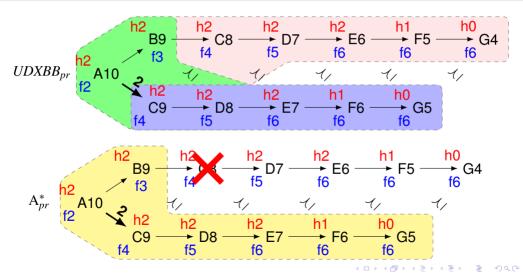
A_{pr}^{*} is #-optimal on consistent instances











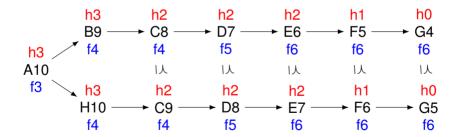
Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit ●000	Results 0000000	Conclusions
Outline					

23/32

- Stackelberg Planning
- 2 Solving Stackelberg Tasks: Previous Work
- 3 Symbolic Leader Search
- 4 Net-Benefit Stackelberg Planning
 - 5 Empirical Results
 - Conclusions

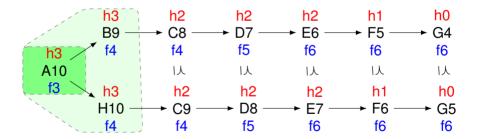
Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit o●oo	Results 0000000	Conclusions
Tie-Breaking	J				

- In A* tie-breaking is only relevant in the last *f*-layer
- In A_{pr}^* , tie-breaking is relevant in all layers



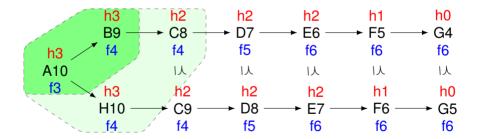
Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit o●oo	Results 0000000	Conclusions
Tie-Breaking	J				

- In A* tie-breaking is only relevant in the last *f*-layer
- In A_{pr}^* , tie-breaking is relevant in all layers



Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit o●oo	Results 0000000	Conclusions
Tie-Breaking	J				

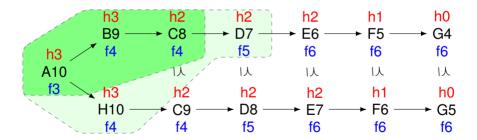
- In A* tie-breaking is only relevant in the last *f*-layer
- In A_{pr}^* , tie-breaking is relevant in all layers



(D) (A) (A) (A) (A) (A)

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit o●oo	Results 0000000	Conclusions 00
Tie-Breaking	J				

- In A* tie-breaking is only relevant in the last *f*-layer
- In A_{pr}^* , tie-breaking is relevant in all layers



A (B) > A (B) > A (B) >

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit oo●o	Results 0000000	Conclusions
Tie-Breaking	Strateg	ies			



Prefer nodes with lower h value

- Standard in most implementations of A*
- Advantage: follow heuristic in the last *f*-layer

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit ○○●○	Results 0000000	Conclusions
Tie-Breaking	Strate	gies			



Prefer nodes with lower h value

- Standard in most implementations of A*
- Advantage: follow heuristic in the last *f*-layer

 \rightarrow We prove that it is not optimally efficient until the last *f*-layer

Stackelberg Planning	Solving	Symbolic Leader Search	Net-Benefit ○○●○	Results 0000000	Conclusions
Tie-Breakin	a Strate	aies			

くロン 人間 とくほ とくほとう

25/32



Prefer nodes with lower h value

- Standard in most implementations of A*
- Advantage: follow heuristic in the last *f*-layer

 \rightarrow We prove that it is not optimally efficient until the last *f*-layer



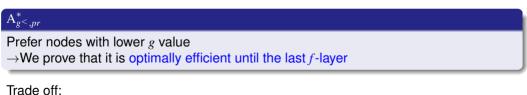
 \rightarrow We prove that it is optimally efficient until the last *f*-layer

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit oo●o	Results 0000000	Conclusions 00
Tie-Breakir	na Strate	aies			



Prefer nodes with lower h value

- Standard in most implementations of A*
- Advantage: follow heuristic in the last *f*-layer
- \rightarrow We prove that it is not optimally efficient until the last *f*-layer



follow \leq up to the last f-laver

A D A A B A A B A A B A B B

25/32

• follow *h* in the last f-layer

Stackelberg Planning	Solving 0000	Symbolic Leader Search	Net-Benefit 000●	Results 0000000	Conclusions
Conclusion					

- Dominance pruning introduces a new source of information for heuristic search algorithms
- Consistent instances:
 - Consistent heuristic
 - Dominance relation is a transitive cost-simulation relation
 - Heuristic and dominance relation are consistent with each other
- A^{*}_{pr} is #-optimally efficient on consistent instances
- Until last layer is better to break ties in favor of minimum g-value

A D N A D N A D N A D N B