Faster Stackelberg Planning via Symbolic Search and Information Sharing

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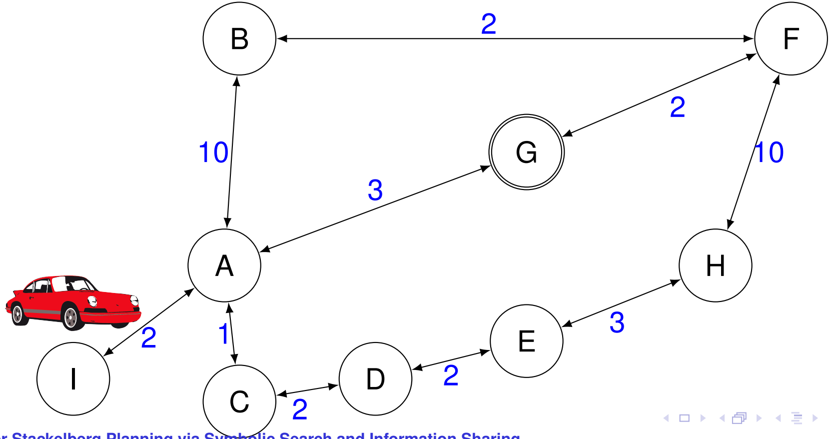
Outline

Classical Planning

- Optimal Planning: Find a minimum-cost plan to the goal



Classical Planning

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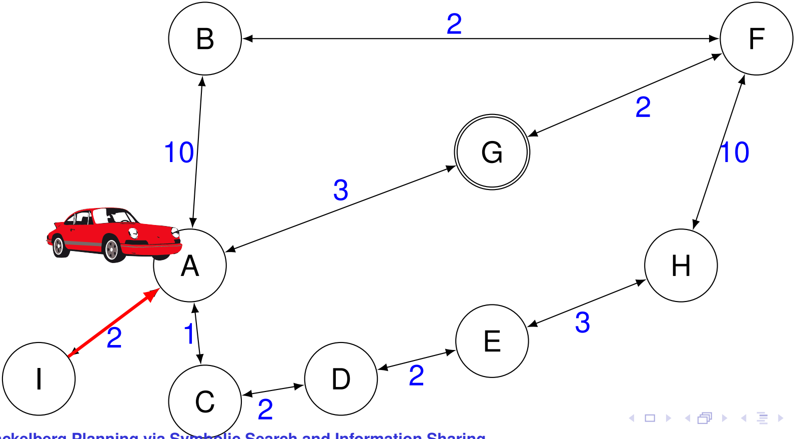

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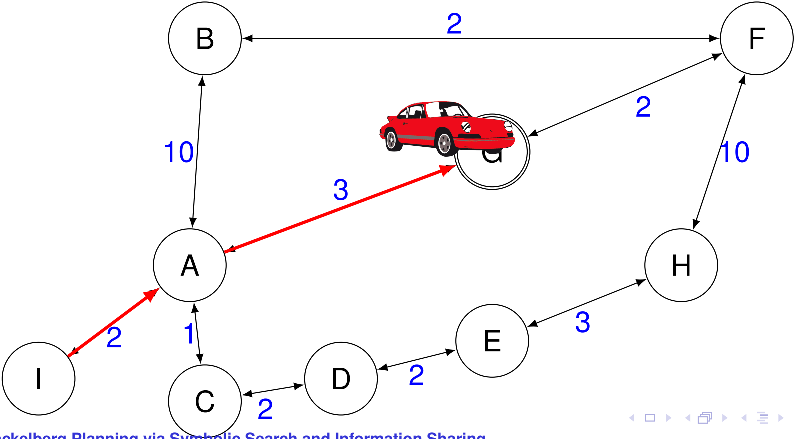


Classical Planning

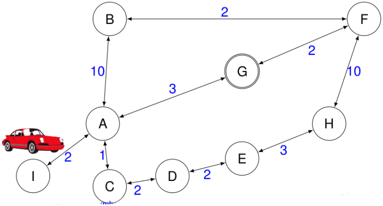
Definition. *A planning task is a 4-tuple* $\Pi = (\mathcal{V}, \mathcal{A}, \mathcal{I}, \mathcal{G})$ *where:*

- $\mathcal{V}$ *is a set of state variables, each* $v \in \mathcal{V}$ *with a finite domain* D_v.
- $\mathcal{A}$ *is a set of actions; each* $a \in \mathcal{A}$ *is a triple* (pre_a, eff_a, c_a), *of precondition and effect (partial assignments), and the action's cost* $c_a \in \mathbb{R}_0^+$.
- *Initial state* $\mathcal{I}$ *(complete assignment), goal* $\mathcal{G}$ *(partial assignment).*

$\rightarrow$ Solution ("Plan"): Action sequence mapping $\mathcal{I}$ into s s.t. $s \models \mathcal{G}$.

Running Example:

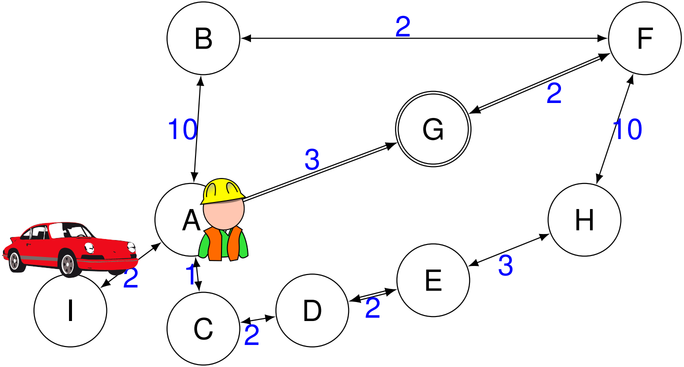
- $\mathcal{V} = \{c\}$
  with $D_c = \{A, B, C, D, E, F, G, H, I\}$.
- $\mathcal{A} = \{drive(x, x')\}$
- $\mathcal{I} = \{c \mapsto I\}$ and $\mathcal{G} = \{c \mapsto G\}$



Stackelberg Planning

Special case of two-player (leader and follower) game where:

- Leader acts first executing a sequence of actions
- Afterwards, the follower plans to achieve its goal



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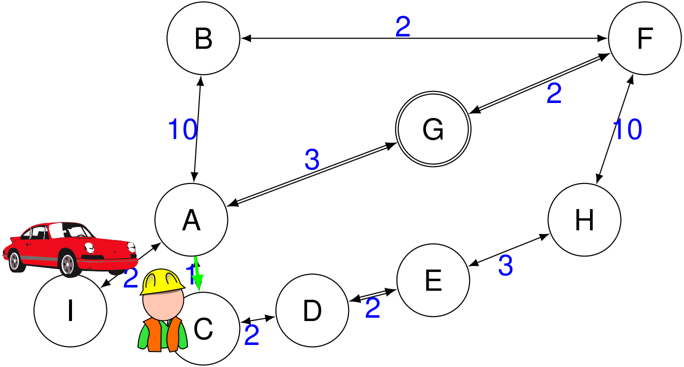

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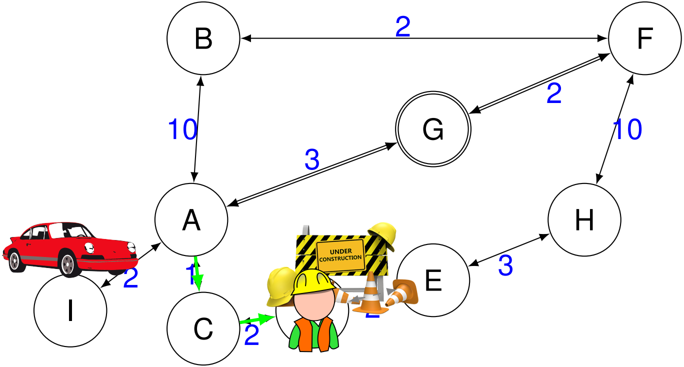

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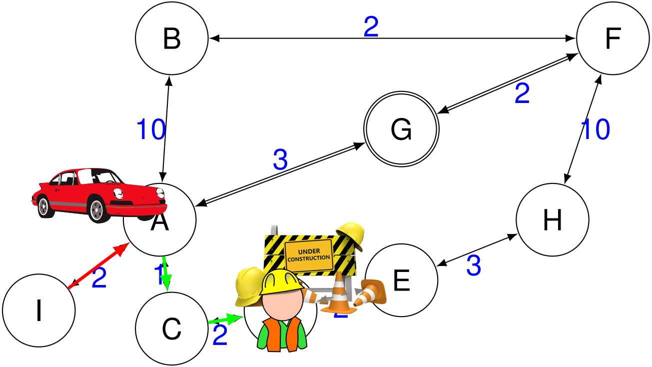

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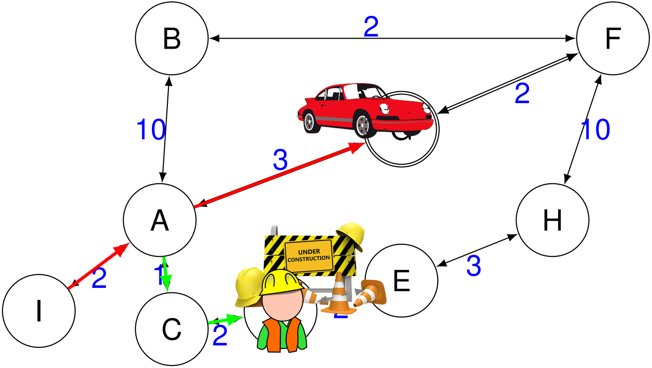

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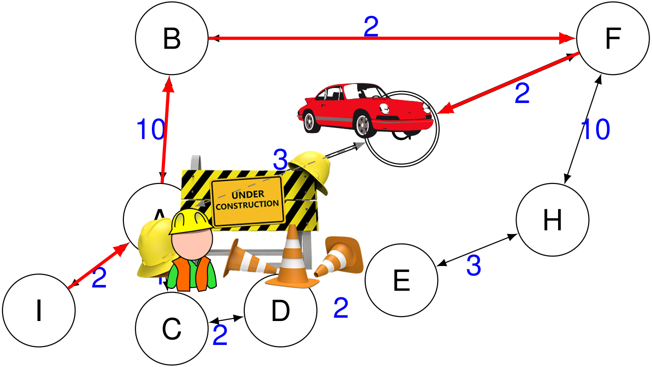
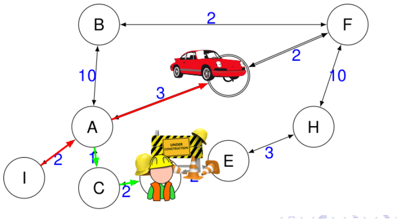
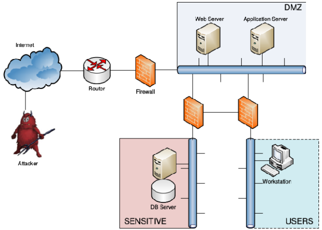


Applications

Robustness Analysis



Pentesting



- Follower: How to achieve the system's goal
- Leader: What could go wrong

- Follower: Attack the network to access sensible information
- Leader: How to defend the network

Stackelberg Planning

Definition. *A* Stackelberg planning task *is a 4-tuple* $\Pi = (\mathcal{V}, \mathcal{A}^L, \mathcal{A}^F, \mathcal{I}, \mathcal{G})$ *where:*

- V *is a set of state variables, each* $v \in V$ *with a finite domain* D_v.
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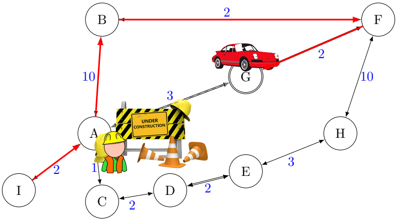
- *Initial state* I *(complete assignment), goal* G *(partial assignment).*

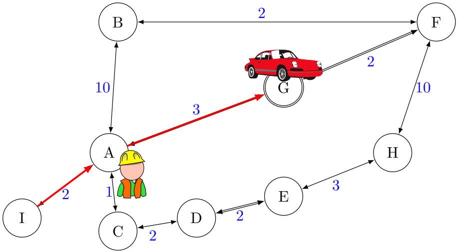
Running Example:

- $\mathcal{V} = \{c, w, r_{AG}, r_{FG}, r_{DE}\}$, $D_c = D_w = \{A, \ldots, I\}$, $D_{r_{ij}} = \{available, blocked\}$.
- $\mathcal{A}^F = \{drive_c(x, x')\}$, $\mathcal{A}^L = \{walk(x, x'), block(x, x')\}$
- $\mathcal{I} = \{c \mapsto I, w \mapsto A, r_{ij} \mapsto available\}$
- $\mathcal{G} = \{c \mapsto G\}$



Solution of a Stackelberg Planning Task

→Find a set of leader plans that minimize leader cost and maximize follower's cost



Leader Plan	c(L)	c(F)
⟨⟩	0	5

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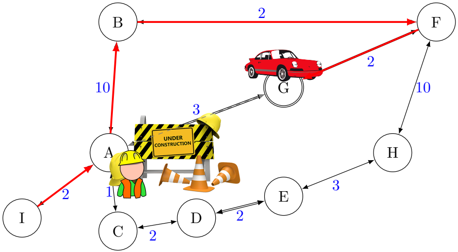

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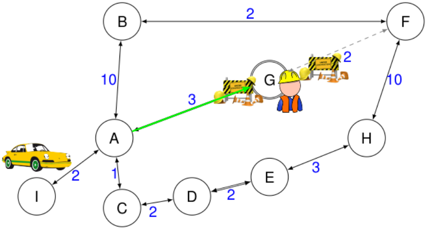

Leader Plan	c(L)	c(F)
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⟨block(A,G)⟩	1	16
⟨walk(A,C), walk(C,D), block(D,E)⟩	3	5

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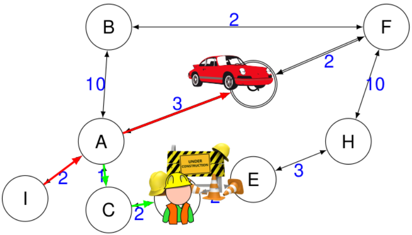

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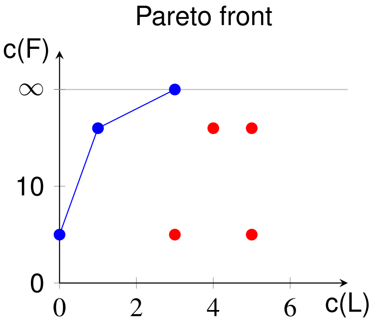

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Outline

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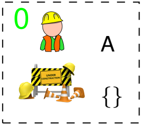

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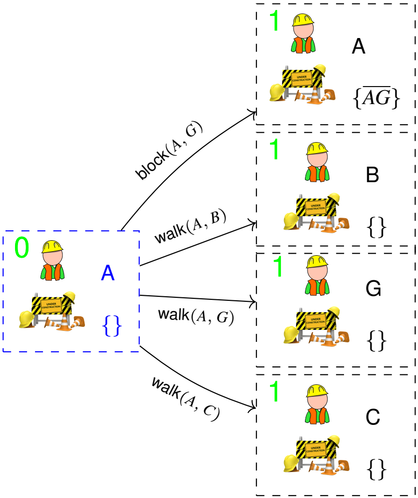

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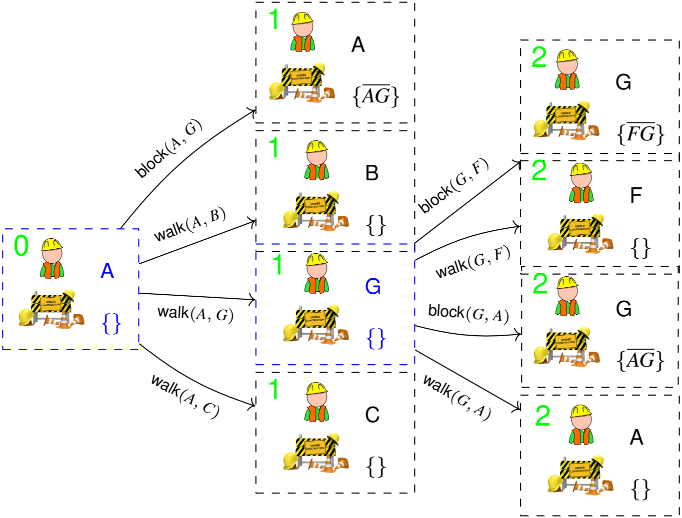

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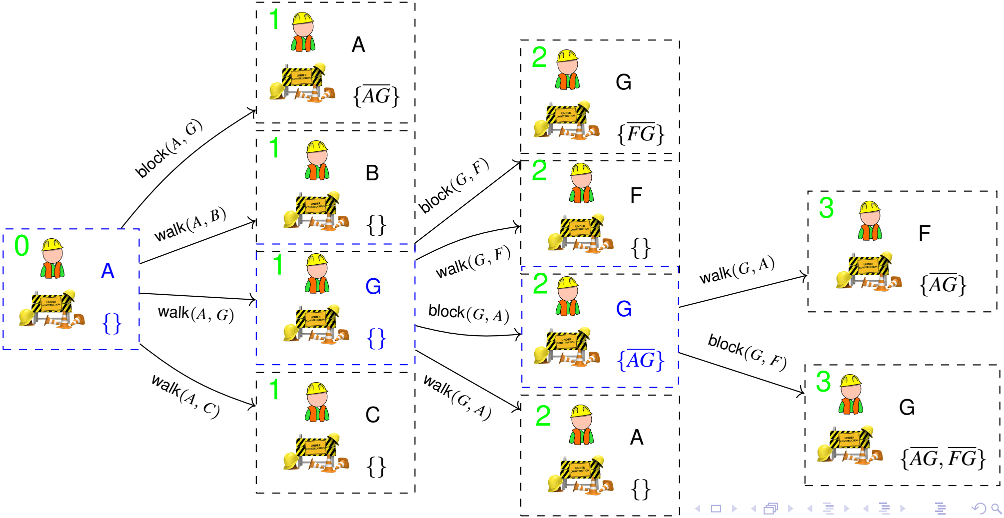
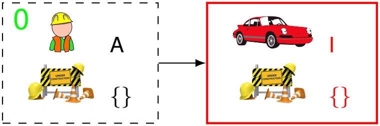


Follower Subtasks

Leader Search Node

Follower sub-task



Follower subtask:

- Classical Planning Task →set of actions depends on blocked roads
- Optimal planners:
  - A^* with LM-cut
  - Symbolic Bidirectional Search

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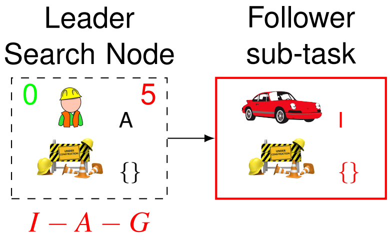

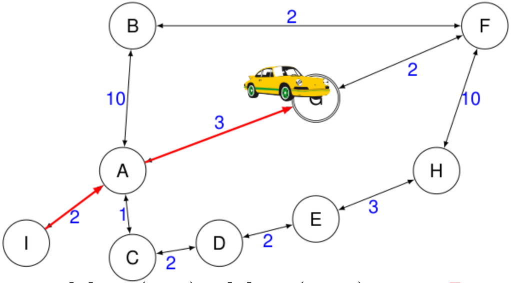

drive (I, A), **drive** (A, G) cost=5

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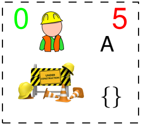
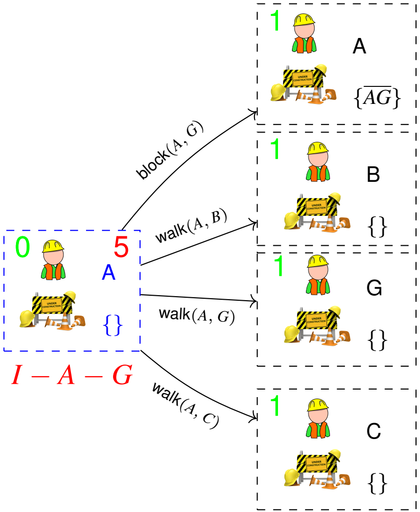
Leader Search (IDS)



$I - A - G$

Leader Search (IDS)



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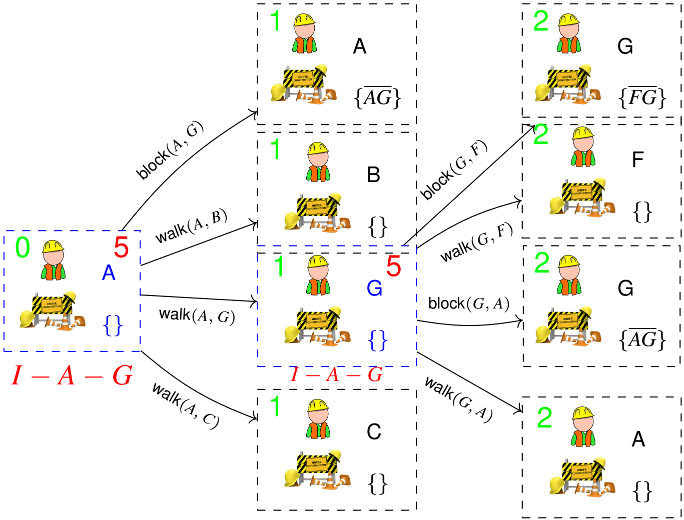

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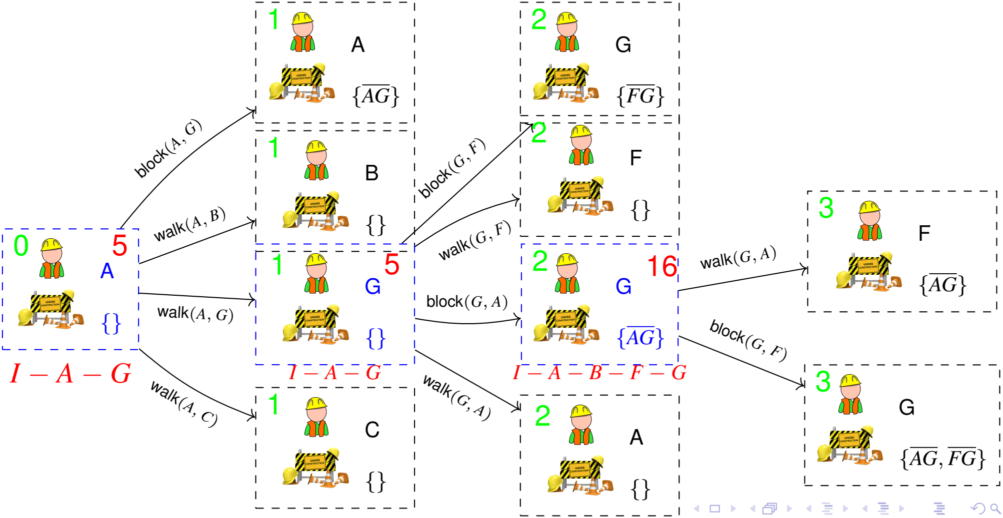
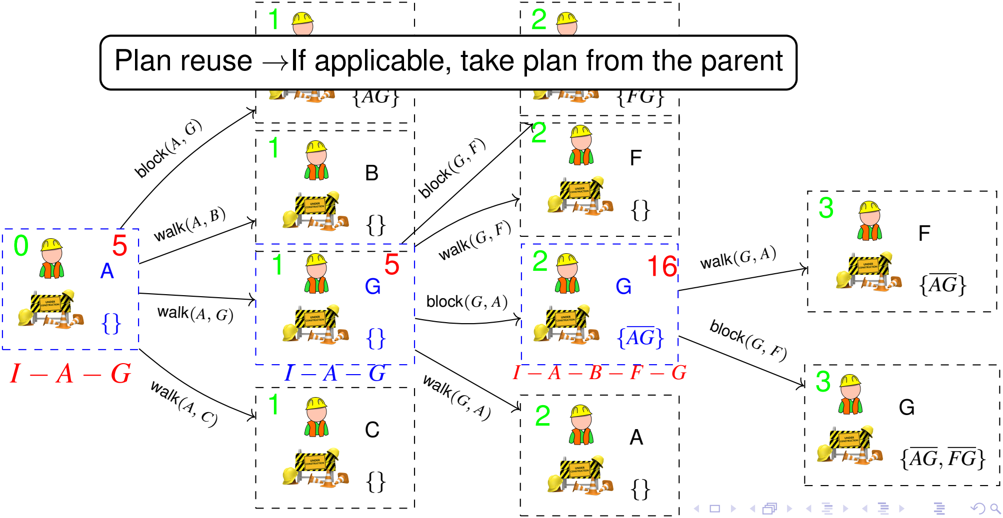


Leader Search (IDS)

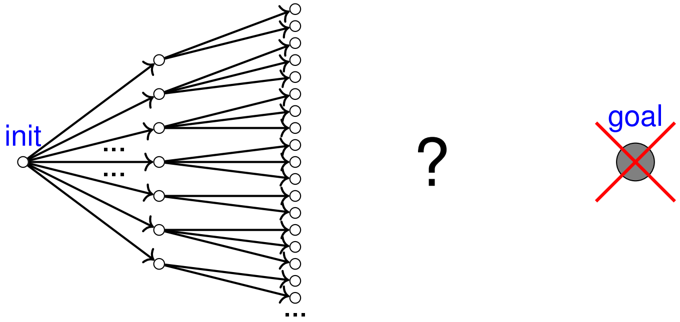
Plan reuse →If applicable, take plan from the parent



Outline

Symbolic Leader Search

Observation 1: Leader search is exhaustive → state space *explosion*



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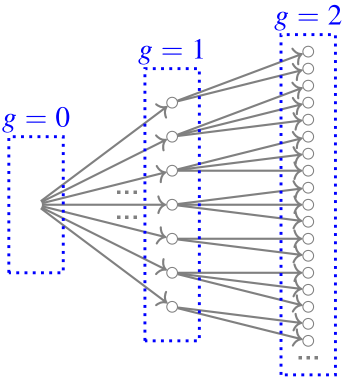
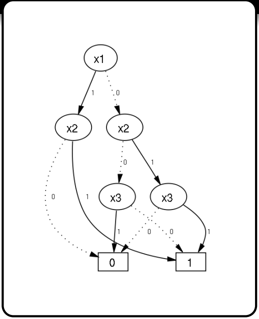


BDDs to the rescue!



Cost-bounded search

Observation 2: We do not always need to compute optimal solutions to follower sub-tasks

Global bounds:

- Leader cost L: explore leader space in ascending leader cost
- Follower cost F: the highest cost of any follower sub-problem so far

$\rightarrow$any new entry in the pareto front will have a cost greater than F

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Cost-bounded/Optimal Planning: Given a follower sub-task and a cost bound F, return any plan of cost $C \leq F$ if one exists, otherwise return an optimal plan if one exists, otherwise return “unsolvable”.

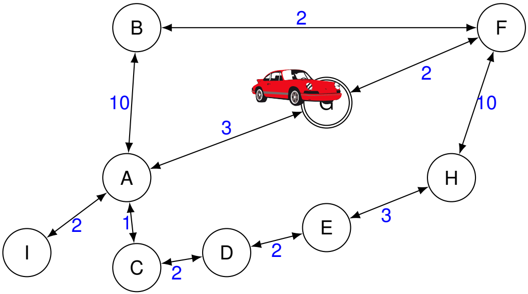
Information Sharing

Observation 3: We need to find optimal solutions for many follower sub-tasks, but they are very similar

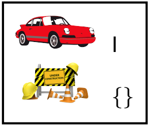
- Same set of variables $\mathcal{V}$ and goal $\mathcal{G}$
- Actions $A \subseteq \mathcal{A}^F$

$\rightarrow$How to re-use information among follower sub-searches?

Information Sharing: Goal Regression



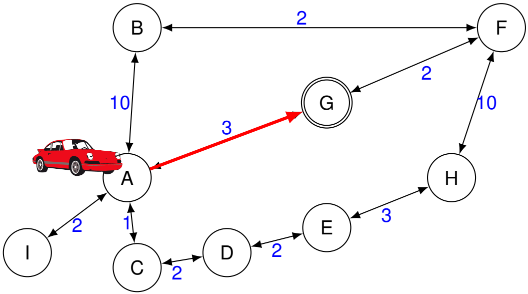
Whenever we compute a new plan for some follower sub-task, use regression from the goal:



drive (I, A), **drive** (A, G)

$\{c \mapsto G\}$ cost = 0

Information Sharing: Goal Regression



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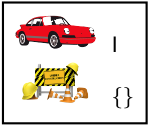

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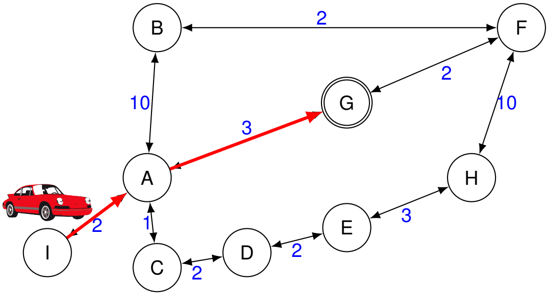
drive (I, A), **drive** (A, G) $\{c \mapsto A, r_{AG} \mapsto \mathit{available}\}$ cost = 3

Information Sharing: Goal Regression



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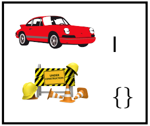


drive (I, A), **drive** (A, G)

$\{c \mapsto G\}$ cost = 0

$\{c \mapsto A, r_{AG} \mapsto available\}$ cost = 3

$\{c \mapsto I, r_{AG} \mapsto available\}$ cost = 5

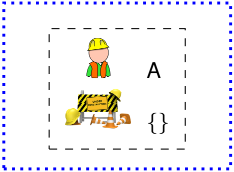
Symbolic Leader Search: Sharing at the Leader Level

L=0 F=0

Pareto Front

c(L)	c(F)

Solved Follower Tasks



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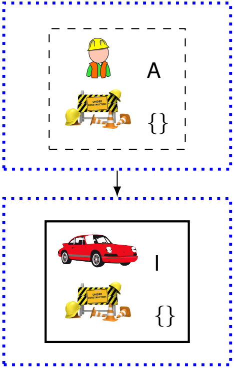

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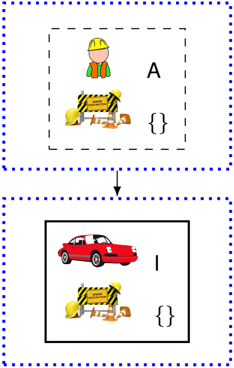

$I - A - G, c = 5$

Symbolic Leader Search: Sharing at the Leader Level

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Pareto Front	
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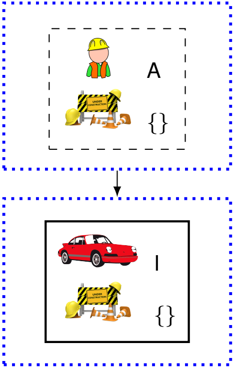
Solved Follower Tasks

$\{c \mapsto I, r_{AG} \mapsto available\}$



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c(L)	c(F)

Solved Follower Tasks

$\{c \mapsto I, r_{AG} \mapsto available\}$

A

{}

Symbolic Leader Search: Sharing at the Leader Level

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Pareto Front

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$$\{c \mapsto I, r_{AG} \mapsto available\}$$

A

{}

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Pareto Front	
c(L)	c(F)
0	5

Solved Follower Tasks

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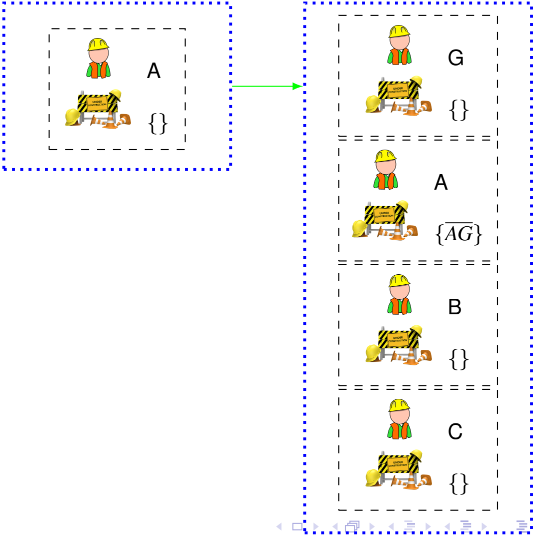

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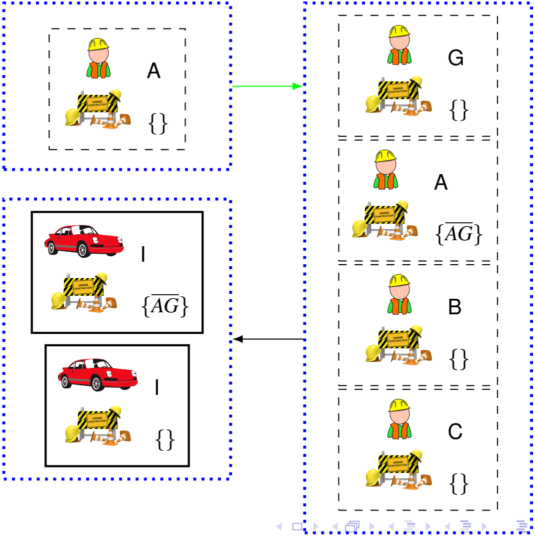

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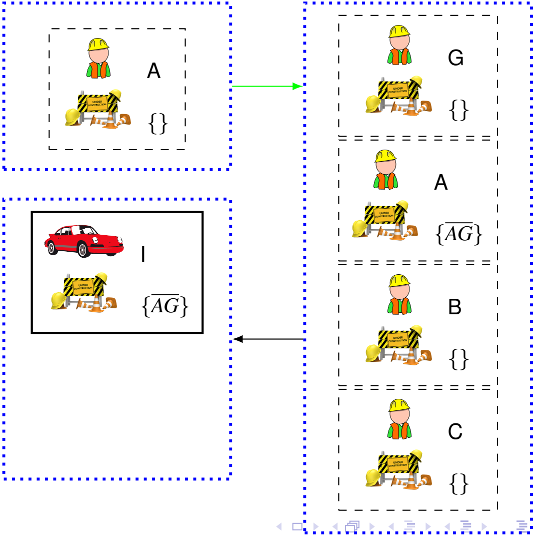

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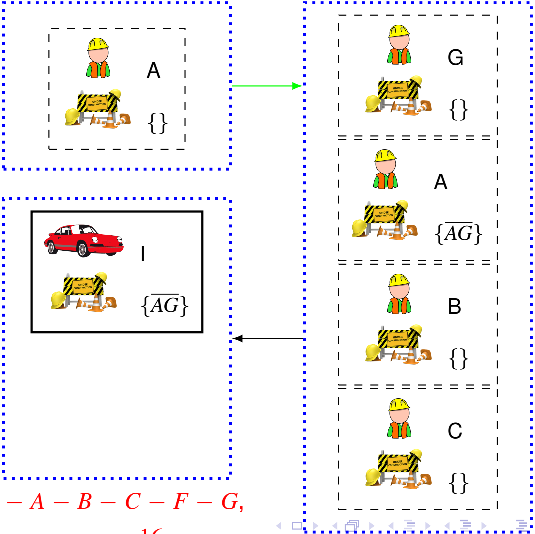
Solved Follower Tasks

$$\{c \mapsto I, r_{AG} \mapsto available\}$$



$I - A - B - C - F - G,$

$c = 16$

Symbolic Leader Search: Sharing at the Leader Level

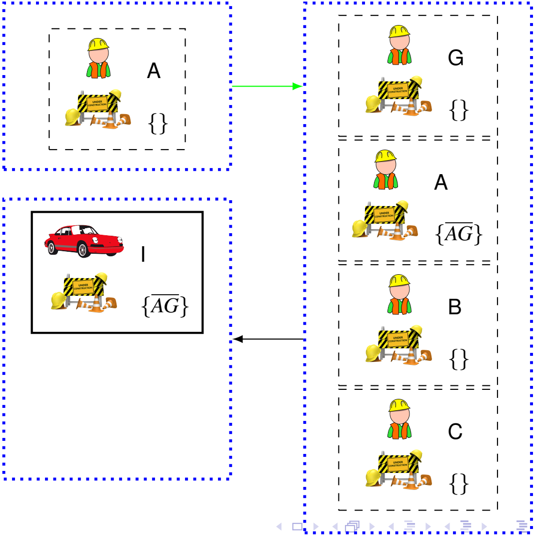
L=1 F=16

Pareto Front

c(L)	c(F)
0	5

Solved Follower Tasks

$$\{c \mapsto I, r_{AG} \mapsto available\}$$
$$\{c \mapsto I, r_{FG} \mapsto available\}$$



Symbolic Leader Search: Sharing at the Leader Level

L=1 F=16

Pareto Front	
c(L)	c(F)
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1	16

Solved Follower Tasks

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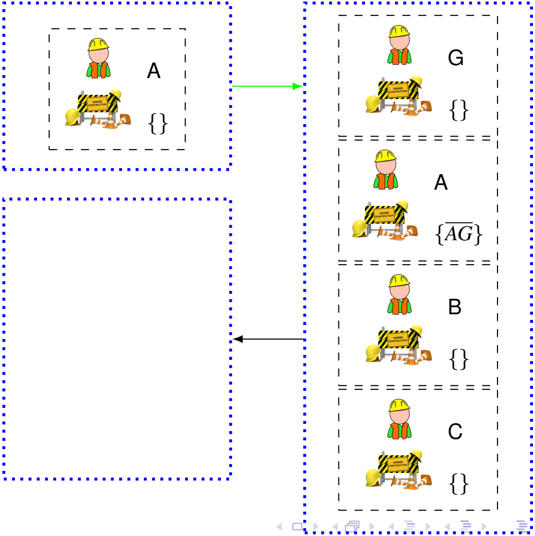

Sharing at the Follower Level

Upper Bound Function: pre-store plans for certain follower states

Sharing at the Follower Level

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How to obtain upper bound functions?

From plans:

$$\{c \mapsto G\} \quad \text{ub} = 0$$
$$\{c \mapsto A, r_{AG} \mapsto \mathit{available}\} \quad \text{ub} = 3$$
$$\{c \mapsto I, r_{AG} \mapsto \mathit{available}\} \quad \text{ub} = 5$$

Sharing at the Follower Level

Upper Bound Function: pre-store plans for certain follower states

How to obtain upper bound functions?

From plans:

$\{c \mapsto G\}$ ub = 0

$\{c \mapsto A, r_{AG} \mapsto \mathit{available}\}$ ub = 3

$\{c \mapsto I, r_{AG} \mapsto \mathit{available}\}$ ub = 5

How to use upper bound functions?

Transform any search-based optimal subsolver in a cost-bounded algorithm:
If some follower state s is seen such that $g(s) + ub(s) \leq F$ stop immediately!

Sharing at the Follower Level

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From backward search on the most constrained follower task (Π^{+}):

G

$\{\overline{AG}, \overline{FG}, \overline{DE}\}$

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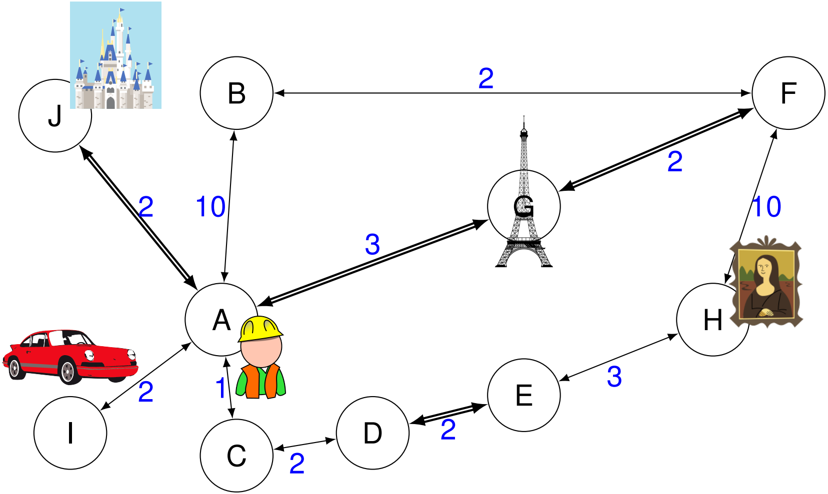
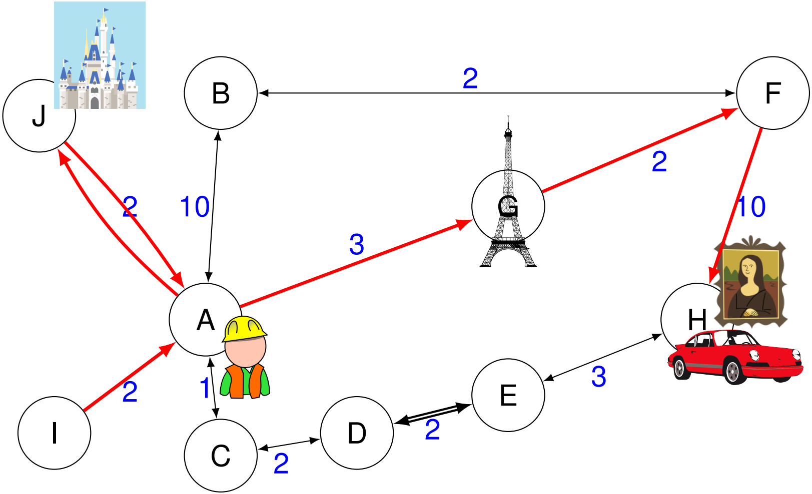
$\rightarrow$In the paper: General conditions under which lower- and upper-bounds can be shared accross follower sub-tasks

Outline

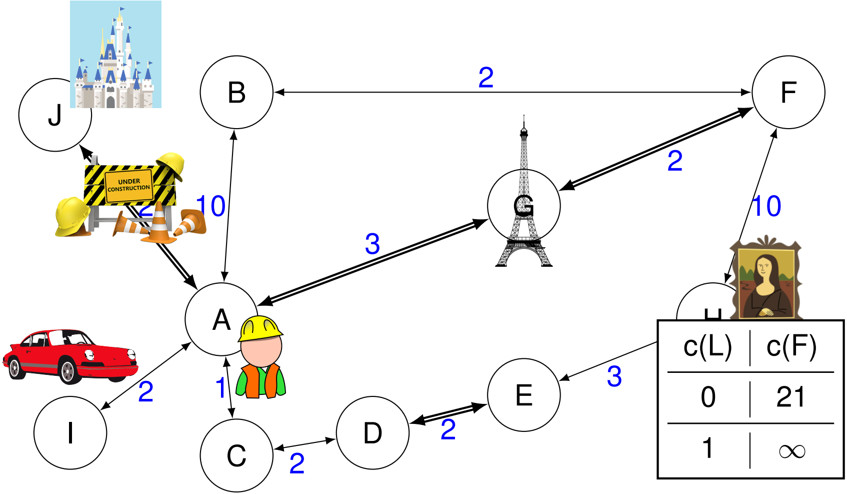
Net-Benefit Stackelberg Planning



Net-Benefit Stackelberg Planning



Net-Benefit Stackelberg Planning



c(L)	c(F)
0	21
1	∞

Net-Benefit Stackelberg Planning

- Soft goals: Each goal has a corresponding cost the follower must pay if it is not achieved

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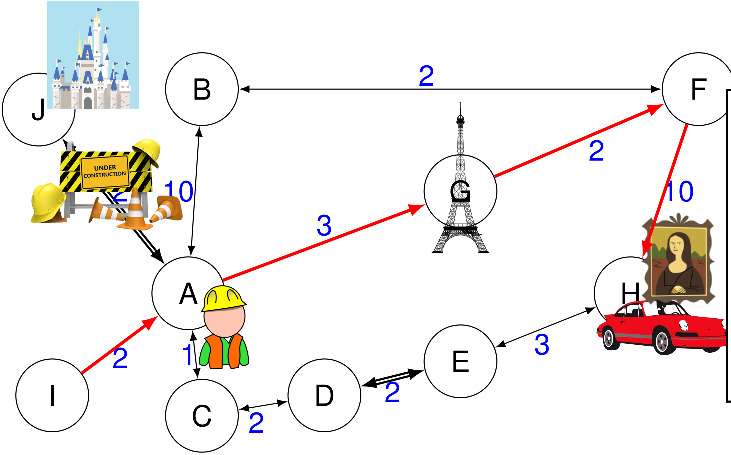
Net-Benefit Stackelberg Planning

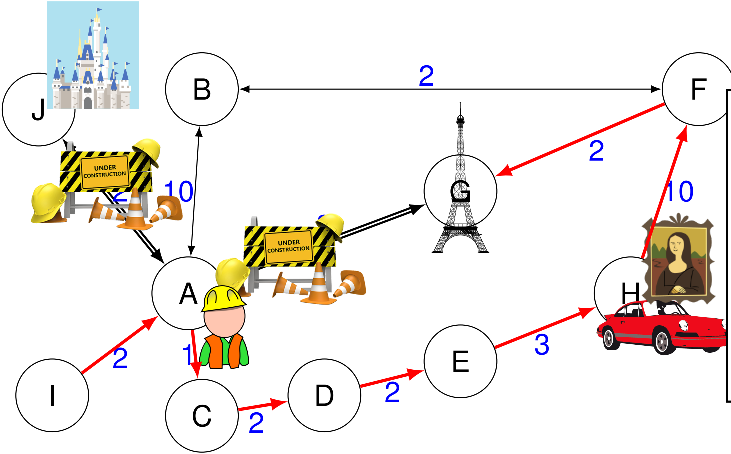
- Soft goals: Each goal has a corresponding cost the follower must pay if it is not achieved
- Compilation to classical planning by Keyder and Geffner (2009) →No specialized algorithms are required
- In our experiments we set a cost of 10000 for each individual sub-goal →follower chooses the cheapest plan among the ones that maximize the number of achieved goals

Net-Benefit Stackelberg Planning



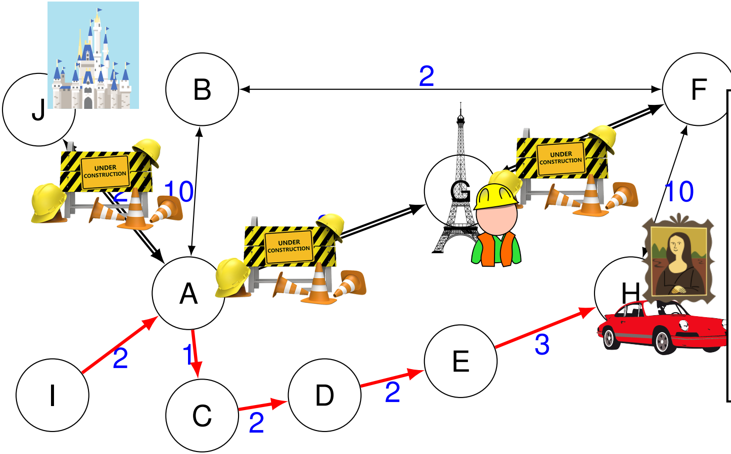
c(L)	c(F)
0	21
1	10017

Net-Benefit Stackelberg Planning



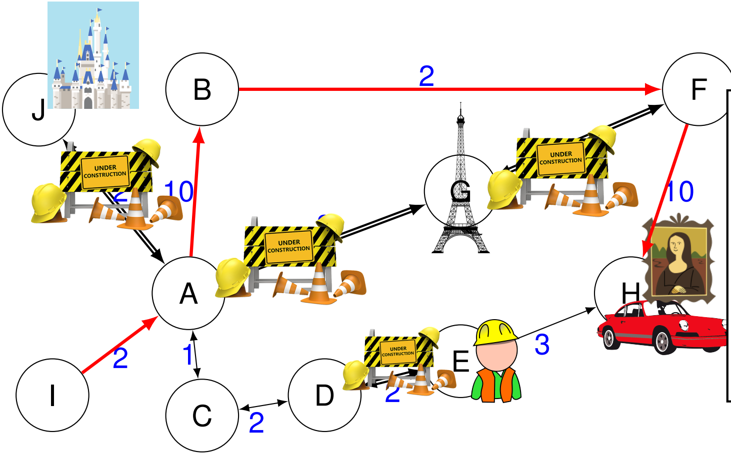
c(L)	c(F)
0	21
1	10017
2	10022

Net-Benefit Stackelberg Planning



c(L)	c(F)
0	21
1	10017
2	10022
4	20010

Net-Benefit Stackelberg Planning



c(L)	c(F)
0	21
1	10017
2	10022
4	20010
8	20024

Outline

Experimental Setup

Configurations:

- IDS (Speicher et al. 2018)
- Symbolic Leader Search
  - –
  - $+ ub$: use upper bound functions from plans
  - $+\Pi^{+}$: use upper bound functions from plans and backward search
  - +FF: use cost-bounded planner (GBFS with FF heuristic for $1s$)

$\rightarrow$Time limit of 30m and memory limit of 4GB.

Benchmarks

We use the benchmarks from previous work (Speicher et al. 2018)

Pareto frontier size ($|PF(\Pi^S)|$):

OLD (Speicher et al. 2018)

Domain	*avg*	*max*
Logistics	1.85	3
Mystery	1.59	3
Rovers	1.86	3
Sokoban	1.92	2
Tpp	2.00	2
Visitall	2.32	7
Pentesting	1.26	2

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We use the benchmarks from previous work (Speicher et al. 2018)

Pareto frontier size ($|PF(\Pi^S)|$):

OLD (Speicher et al. 2018)			NEW			NET	
Domain	*avg*	*max*	Domain	*avg*	*max*	*avg*	*max*
Logistics	1.85	3	Logistics	2.61	6	3.83	16
Mystery	1.59	3	Nomystery	2.58	7	3.25	13
Rovers	1.86	3	Rovers	1.85	4	3.21	13
Sokoban	1.92	2	Transport	4.24	17	3.71	9
Tpp	2.00	2	Tpp	2.15	7	2.35	13
Visitall	2.32	7	Visitall	2.97	7	5.45	34
Pentesting	1.26	2	Pentesting	1.71	4	4.06	13

→New benchmark set for standard and net-benefit planning

Overall Results

Coverage: Tasks solved under 30 minutes and 4GB

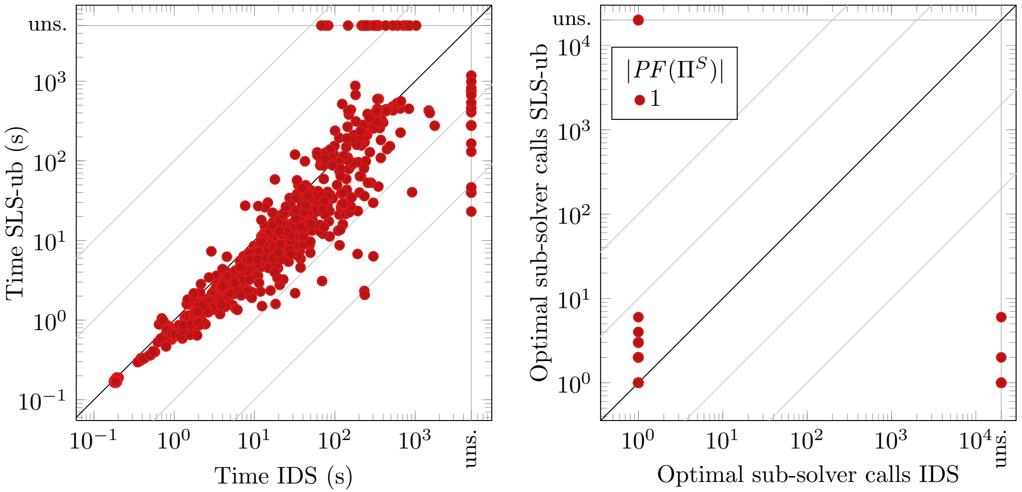
Follower sub-solver		IDS Π^+	**SLS** —
LMcut	OLD (1987)	**681**	630
	NEW (1059)	681	**708**
	NET (1064)	526	**536**
Symbolic Bidirectional	OLD (1987)	584	**621**
	NEW (1059)	632	**819**
	NET (1064)	540	**654**

Overall Results

Coverage: Tasks solved under 30 minutes and 4GB

Follower sub-solver		IDS Π^{+}	**SLS** —	**SLS** $+ub$
LMcut	OLD (1987)	**681**	630	634
	NEW (1059)	681	708	**743**
	NET (1064)	526	536	**574**
Symbolic Bidirectional	OLD (1987)	584	621	**628**
	NEW (1059)	632	819	**823**
	NET (1064)	540	654	**671**

Results: SLS-ub vs IDS



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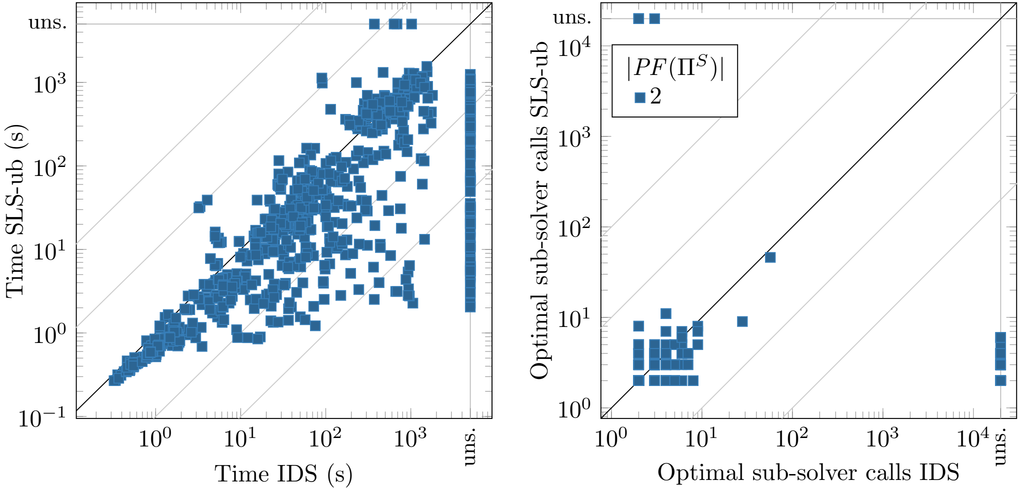

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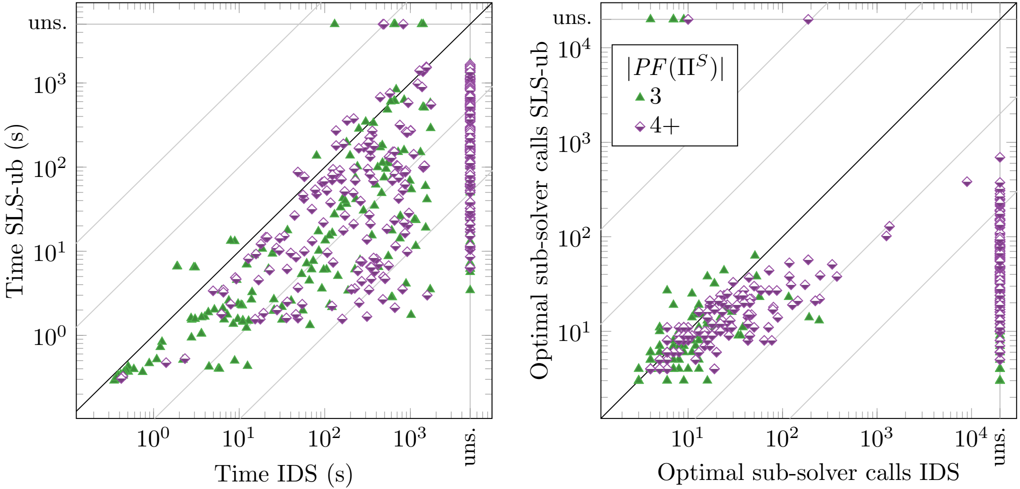

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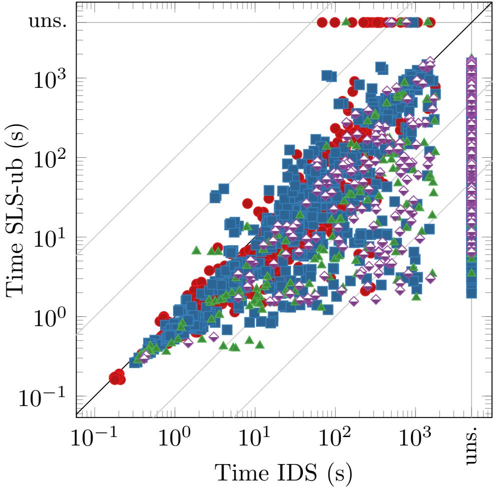

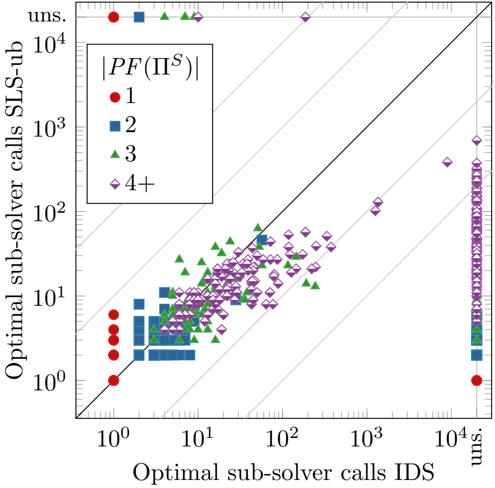

Overall Results

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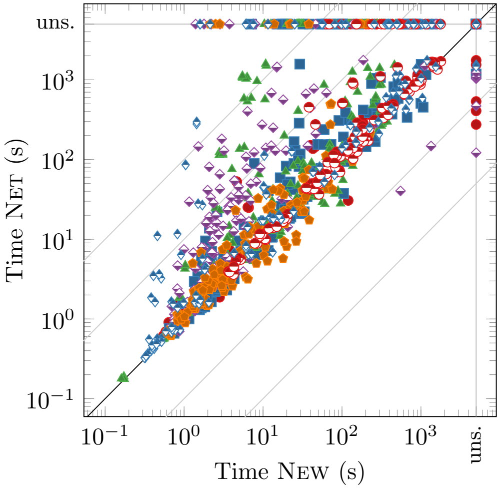
Follower sub-solver		IDS Π^+	SLS —	SLS $+ub$	SLS $+\Pi^+$
LMcut	OLD (1987)	**681**	630	634	652
	NEW (1059)	681	708	**743**	740
	NET (1064)	526	536	574	**613**
Symbolic Bidirectional	OLD (1987)	584	621	628	**652**
	NEW (1059)	632	819	823	**825**
	NET (1064)	540	654	671	**726**

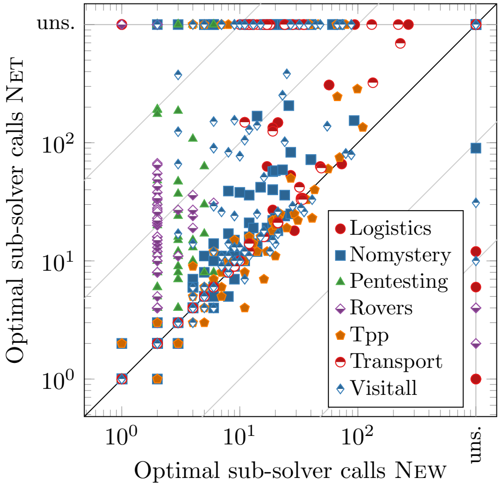
Overall Results

Coverage: Tasks solved under 30 minutes and 4GB

Follower sub-solver		IDS Π^+	SLS —	SLS $+ub$	SLS $+\Pi^+$	SLS + FF
LMcut	OLD (1987)	**681**	630	634	652	651
	NEW (1059)	681	708	**743**	740	736
	NET (1064)	526	536	574	613	**619**
Symbolic Bidirectional	OLD (1987)	584	621	628	**652**	**652**
	NEW (1059)	632	819	823	825	**826**
	NET (1064)	540	654	671	**726**	720

Results: Standard vs Net-Benefit Instances





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Conclusions

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    - More fine-grained analysis of how many goals can the follower achieve

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- Net-benefit Stackelberg planning
  - Soft goals
  - More fine-grained analysis of how many goals can the follower achieve

  $\rightarrow$increases usefulness of this framework
- Promising future work:
  - Apply information sharing ideas in other contexts
  - Improved cost-bounded planning algorithms