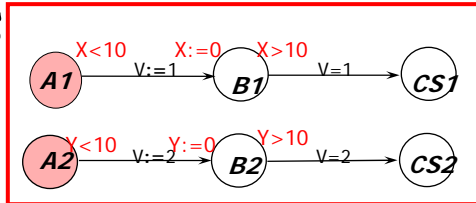
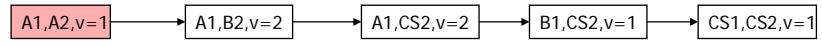


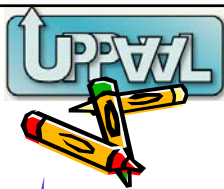
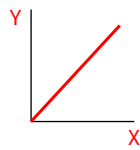
# Fischers cont.



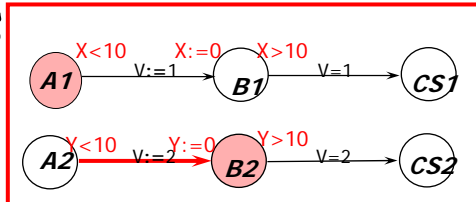
Untimed case



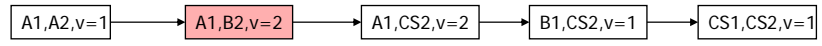
Taking time into account



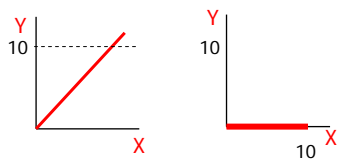
# Fischers cont.

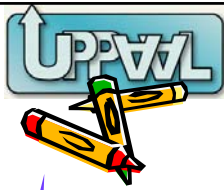


Untimed case

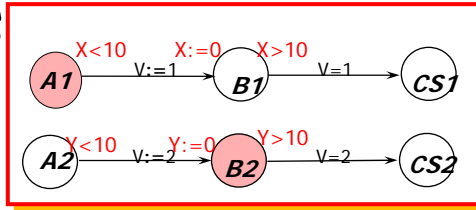


Taking time into account

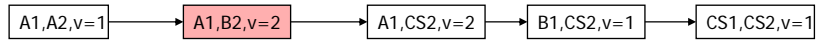




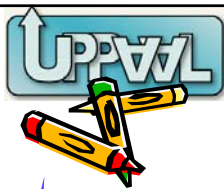
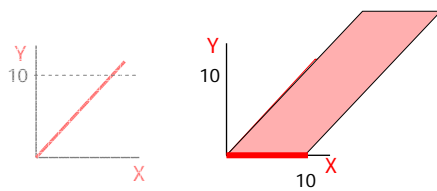
# Fischers cont.



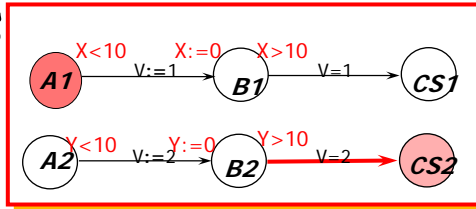
Untimed case



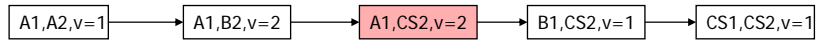
Taking time into account



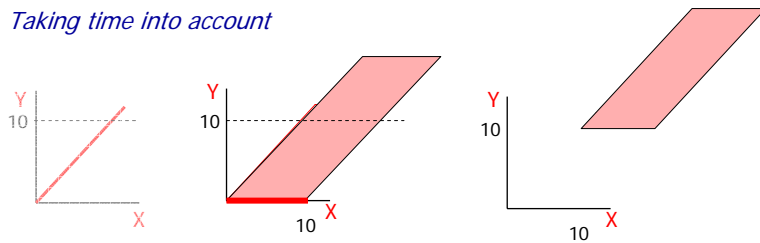
# Fischers cont.



Untimed case



Taking time into account



**UPPAAL** Fischers cont.

*Untimed case*

A1, A2, v=1 → A1, B2, v=2 → A1, CS2, v=2 → B1, CS2, v=1 → CS1, CS2, v=1

*Taking time into account*

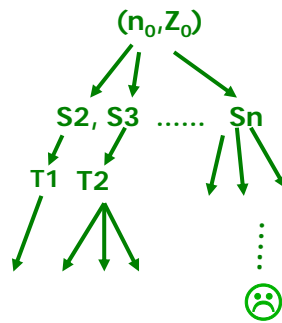
**UPPAAL**

## Inside the UPPAAL tool

- Example
- **Reachability Algorithm**
- Data structures
- Liveness Algorithm



# We have a search problem



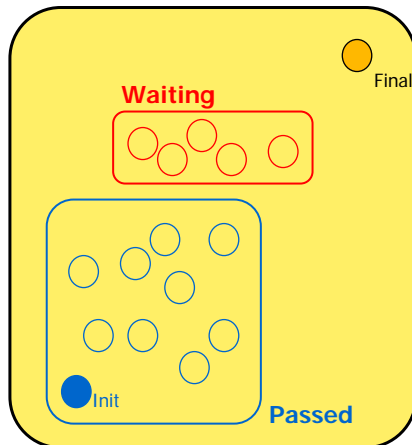
Symbolic state  
Symbolic transitions

Reachable?  
E ↔ ☹️ = : A[] : ☹️



# Forward Reachability

Init -> Final ?




**INITIAL** Passed := ∅;  
Waiting := {(n0, Z0)}

**REPEAT**  
 - pick (n, Z) in **Waiting**  
 - if for some Z' ⊇ Z (n, Z') in **Passed** then **STOP**  
 - else /explore/ add { (m, U) : (n, Z) => (m, U) } to **Waiting**;  
 Add (n, Z) to **Passed**

**UNTIL** **Waiting** = ∅  
 or  
 Final is in **Waiting**





# Forward Reachability


Init -> Final ?

**INITIAL** **Passed** :=  $\emptyset$ ;  
**Waiting** :=  $\{(n0,Z0)\}$

**REPEAT**

- pick  $(n,Z)$  in **Waiting**
- if for some  $Z' \supseteq Z$   $(n,Z')$  in **Passed** then **STOP**
- else (explore) add  $\{(m,U) : (n,Z) \Rightarrow (m,U)\}$  to **Waiting**;  
 Add  $(n,Z)$  to **Passed**

**UNTIL** **Waiting** =  $\emptyset$   
 or  
 Final is in **Waiting**



# Forward Reachability


Init -> Final ?

**INITIAL** **Passed** :=  $\emptyset$ ;  
**Waiting** :=  $\{(n0,Z0)\}$

**REPEAT**

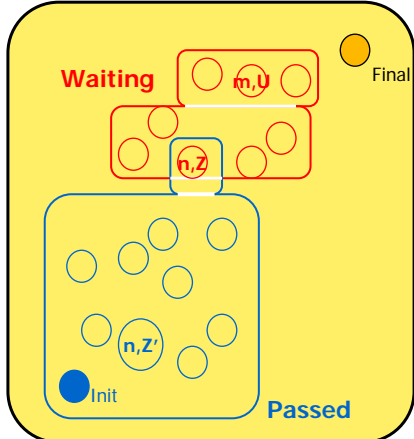
- pick  $(n,Z)$  in **Waiting**
- if for some  $Z' \supseteq Z$   $(n,Z')$  in **Passed** then **STOP**
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# Forward Reachability

Init -> Final ?




**INITIAL**  $Passed := \emptyset;$   
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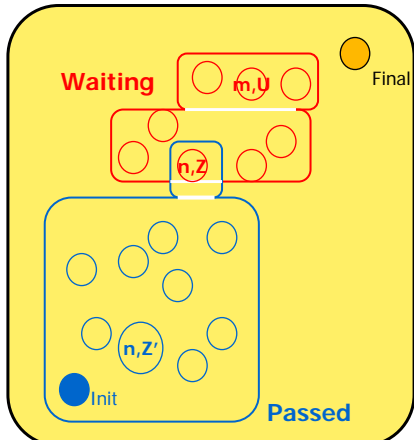
Add  $(n,Z)$  to **Passed**

**UNTIL** **Waiting** =  $\emptyset$   
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# Forward Reachability

Init -> Final ?



**INITIAL**  $Passed := \emptyset;$   
**Waiting**  $:= \{(n0,Z0)\}$

**REPEAT**

- pick  $(n,Z)$  in **Waiting**
- if for some  $Z' \supseteq Z$   $(n,Z')$  in **Passed** then **STOP**
- else /explore/ add  $\{ (m,U) : (n,Z) \Rightarrow (m,U) \}$  to **Waiting**;

Add  $(n,Z)$  to **Passed**

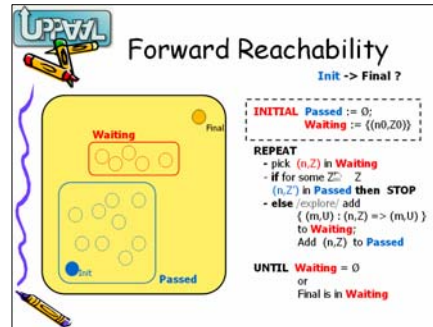
**UNTIL** **Waiting** =  $\emptyset$   
 or  
 Final is in **Waiting**





# Issues

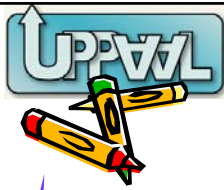
- Datastructures for **Passed** and **Waiting**
- Do we really need to *always* store in **Passed** ?
- Which symbolic state to select from **Waiting** ?
- Do we really need to add *all* successors ?



# Inside the UPPAAL tool

- Example
- Reachability Algorithm
- **Data structures**
- Liveness Algorithm





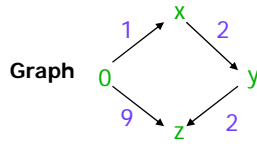
# Canonical Datastructures for Zones

## Difference Bounded Matrices

### Inclusion

**D1**

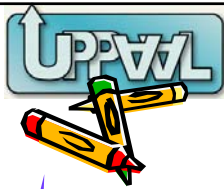
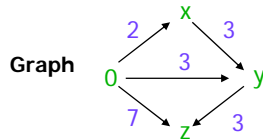
$$\begin{cases} x \leq 1 \\ y - x \leq 2 \\ z - y \leq 2 \\ z \leq 9 \end{cases}$$



? ⊆ ?

**D2**

$$\begin{cases} x \leq 2 \\ y - x \leq 3 \\ y \leq 3 \\ z - y \leq 3 \\ z \leq 7 \end{cases}$$



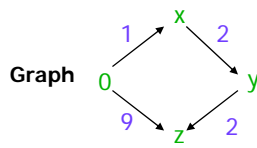
# Canonical Datastructures for Zones

## Difference Bounded Matrices

### Inclusion

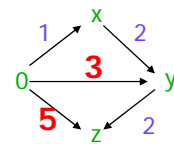
**D1**

$$\begin{cases} x \leq 1 \\ y - x \leq 2 \\ z - y \leq 2 \\ z \leq 9 \end{cases}$$



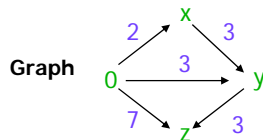
? ⊆ ?

Shortest Path Closure

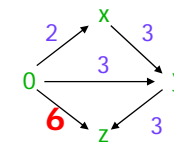


**D2**

$$\begin{cases} x \leq 2 \\ y - x \leq 3 \\ y \leq 3 \\ z - y \leq 3 \\ z \leq 7 \end{cases}$$



Shortest Path Closure



**UPPAAL**

## Canonical Datastructures for Zones

### Difference Bounded Matrices

**Emptiness**

**D**

$$\begin{cases} x \leq 1 \\ y \geq 5 \\ y - x \leq 3 \end{cases}$$

**Graph**

```

graph LR
    0((0)) -- 1 --> x((x))
    0 -- -5 --> y((y))
    x -- 3 --> y
  
```

**Negative Cycle iff empty solution set**

**UPPAAL**

## Canonical Datastructures for Zones

### Difference Bounded Matrices

**Future**

**D**

$$\begin{cases} 1 \leq x \leq 4 \\ 1 \leq y \leq 3 \end{cases}$$

**Future D**

$$\begin{cases} 1 \leq x, 1 \leq y \\ -2 \leq x - y \leq 3 \end{cases}$$

**Shortest Path Closure**

```

graph LR
    0((0)) -- 4 --> x((x))
    0 -- -1 --> y((y))
    x -- -1 --> y
    y -- 3 --> x
  
```

**Remove upper bounds on clocks**

```

graph LR
    0((0)) -- -1 --> x((x))
    0 -- -1 --> y((y))
    x -- 3 --> y
    y -- 2 --> x
  
```

**UPPAAL**

## Canonical Datastructures for Zones

### *Difference Bounded Matrices*

**Reset / Project**

$D$

$$\begin{matrix} y \\ | \\ 1 \leq x, 1 \leq y \\ -2 \leq x-y \leq 3 \\ | \\ x \end{matrix}$$

$\{y\}D$

$$\begin{matrix} y \\ | \\ y=0, 1 \leq x \\ | \\ x \end{matrix}$$

Remove all bounds involving  $y$  and set  $y$  to 0

**UPPAAL**

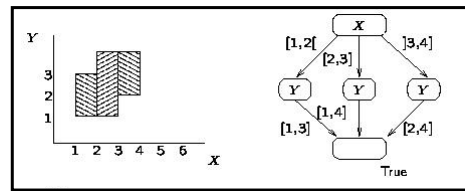
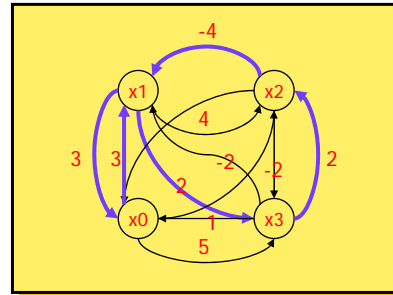
## Complexity

- Computing the shortest path closure, the canonical form of a zone:  $O(n^3)$  [Floyd's alg.]
- Run-time complexity, mostly in  $O(n)$  (if we keep all zones in canonical form)



# Datastructures for Zones

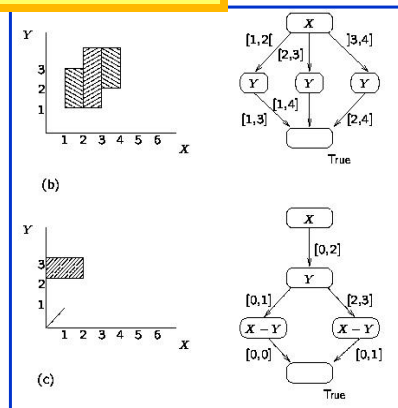
- Difference Bounded Matrices [Bellman58, Dill89]
- Minimal Constraint Form [RTSS97]
- Clock Difference Diagrams [CAV99]




# Other Symbolic Datastructures

- Regions Alur, Dill
- Minimal Constraint Form
- $n^2$  [RTSS97]
- NDD's Maler et. al.
- CDD's UPPAAL/CAV99
- DDD's Möller, Lichtenberg
- Polyhedra HyTech
- .....



### CDD-representations





## Inside the UPPAAL tool

- Example
- Reachability Algorithm
- Data structures
- **Liveness Algorithm**

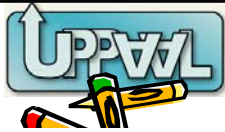



## Liveness Properties in UPPAAL

$F ::= E \Box P$		—	<b>Possibly always P</b>
$A\} P$		—	<b>Eventually P</b> is equivalent to $(\Box E \Box P)$
$P \rightarrow Q$		—	<b>P leads to Q</b> is equivalent to $A \Box (P \ A\} Q)$

Bouajjani, Tripakis, Yovine'97  
 On-the-fly symbolic model checking of TCTL

28

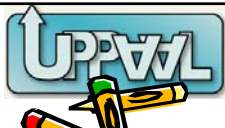
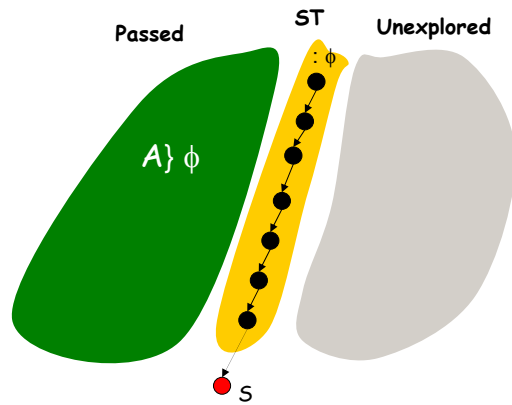


# Liveness Algorithm

Bouajjani, Tripakis, Yovine, 97

```

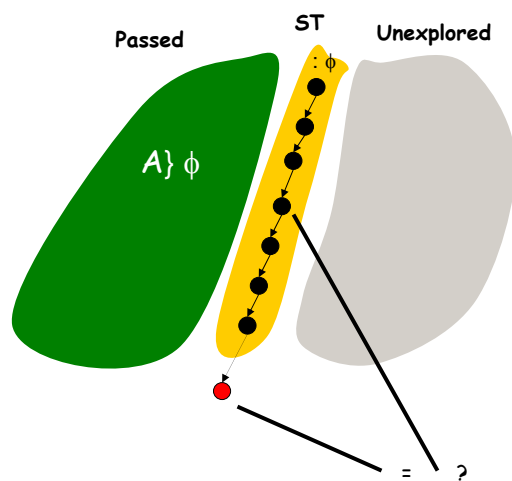
proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end
proc Search( $S$ )  $\equiv$ 
  if loop( $S, ST$ ) then exit(false) fi
   $S := S \wedge \neg\varphi$ 
  push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
    exit(false) fi
  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
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    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end
  
```

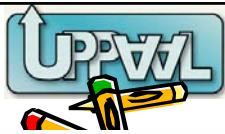


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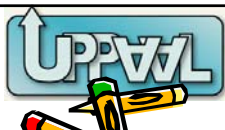
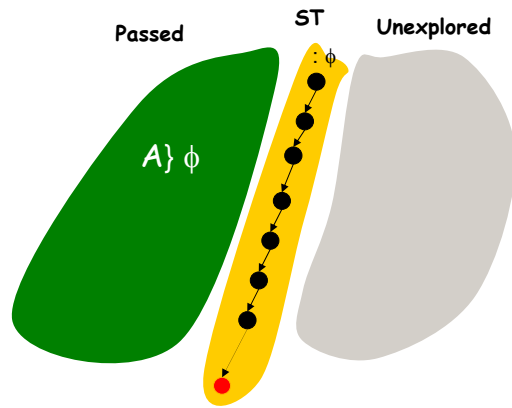




# Liveness Algorithm

```

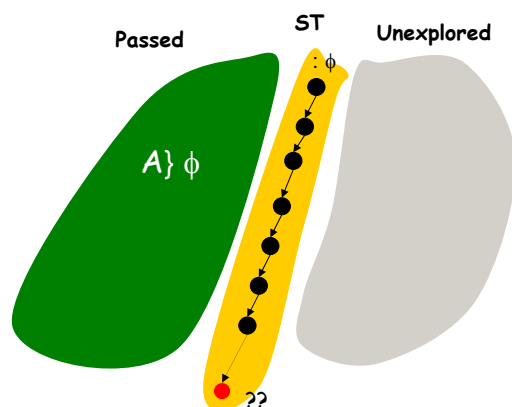
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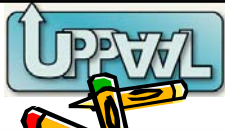
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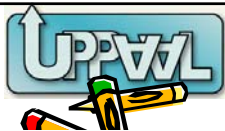
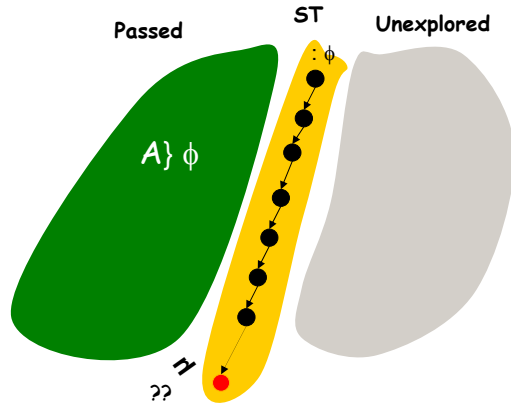




# Liveness Algorithm

```

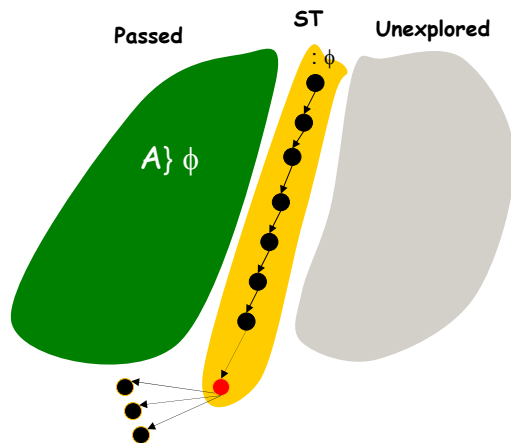
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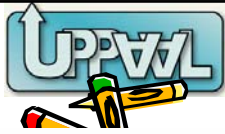


# Liveness Algorithm

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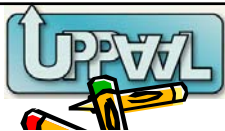
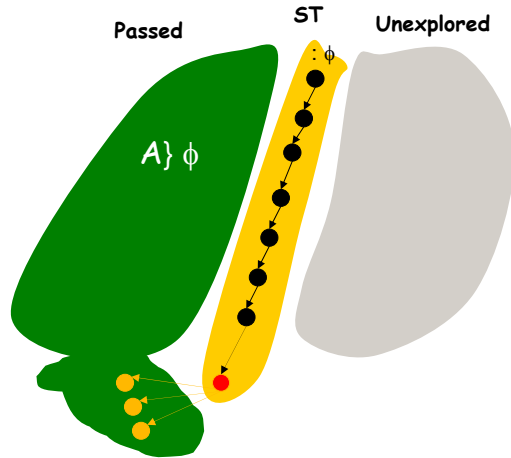


# Liveness Algorithm

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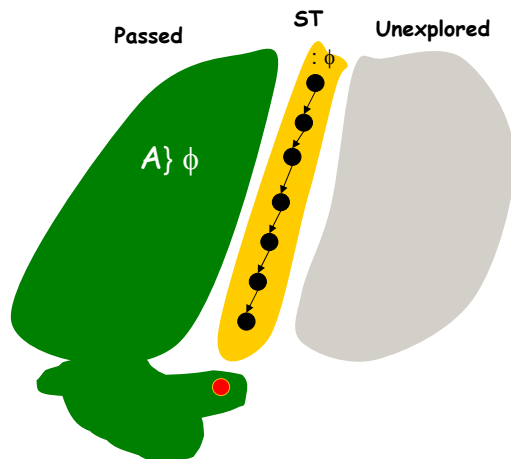


# Liveness Algorithm

```

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   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end

proc Search( $S$ )  $\equiv$ 
  if loop( $S, ST$ ) then exit(false) fi
   $S := S \wedge \neg\varphi$ 
  push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
    exit(false) fi
  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end
  
```





## Inside the UPPAAL tool

- Example
- Reachability Algorithm
- Data structures
- Liveness Algorithm

