Semantics and Verification 2008

Lecture 3

- value passing CCS
- behavioural equivalences
- strong bisimilarity and bisimulation games
- properties of strong bisimilarity

Value Passing CCS

Main Idea
Handshake synchronization is extended with the possibility to exchange integer values.

$$\overline{pay(6)}.Nil \mid pay(x).\overline{save(x/2)}.Nil \mid Bank(100)$$
$$\downarrow \tau$$
$$Nil \mid \overline{save(3)}.Nil \mid Bank(100)$$
$$\downarrow \tau$$
$$Nil \mid Nil \mid Bank(103)$$

Parametrized Process Constants
For example: $Bank(total) \stackrel{\mathrm{def}}{=} save(x).Bank(total + x)$.

Translation of Value Passing CCS to Standard CCS

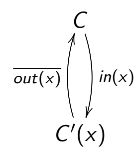
Value Passing CCS

$$C \stackrel{\mathrm{def}}{=} in(x).C'(x)$$
$$C'(x) \stackrel{\mathrm{def}}{=} \overline{out(x)}.C$$

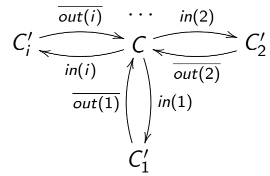
$\longrightarrow$

Standard CCS

$$C \stackrel{\mathrm{def}}{=} \sum_{i \in \mathbb{N}} in(i).C'_i$$
$$C'_i \stackrel{\mathrm{def}}{=} \overline{out(i)}.C$$



symbolic LTS



infinite LTS

CCS Has Full Turing Power

Fact
CCS can simulate a computation of any Turing machine.

Remark
Hence CCS is as expressive as any other programming language but its use is to rather **describe** the behaviour of reactive systems than to perform specific calculations.

Behavioural Equivalence

Implementation

$$CM \stackrel{\mathrm{def}}{=} coin.\overline{coffee}.CM$$
$$CS \stackrel{\mathrm{def}}{=} \overline{pub}.\overline{coin}.coffee.CS$$
$$Uni \stackrel{\mathrm{def}}{=} (CM \mid CS) \smallsetminus \{coin, coffee\}$$

Specification

$$Spec \stackrel{\mathrm{def}}{=} \overline{pub}.Spec$$

Question
Are the processes Uni and $Spec$ behaviorally equivalent?

$$Uni \equiv Spec$$

Goals

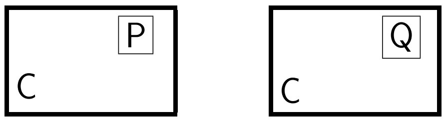
What should a reasonable behavioural equivalence satisfy?

- abstract from states (consider only the behaviour – actions)
- abstract from nondeterminism
- abstract from internal behaviour

What else should a reasonable behavioural equivalence satisfy?

- **reflexivity** $P \equiv P$ for any process P
- **transitivity** $Spec_0 \equiv Spec_1 \equiv Spec_2 \equiv \cdots \equiv Impl$ gives that $Spec_0 \equiv Impl$
- **symmetry** $P \equiv Q$ iff $Q \equiv P$

Congruence



$C(P)$ $C(Q)$

Congruence Property

$P \equiv Q$ implies that $C(P) \equiv C(Q)$

Trace Equivalence

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

Trace Set for $s \in Proc$

$$Traces(s) = \{w \in Act^* \mid \exists s' \in Proc.\ s \xrightarrow{w} s'\}$$

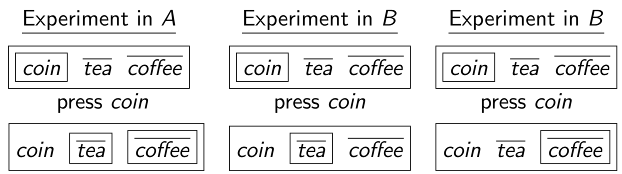
Let $s \in Proc$ and $t \in Proc$.

Trace Equivalence

We say that s and t are **trace equivalent** ($s \equiv_t t$) if and only if

$$Traces(s) = Traces(t)$$

Black-Box Experiments



Main Idea

Two processes are behaviorally equivalent if and only if an **external observer** cannot tell them apart.

Strong Bisimilarity

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS.

Strong Bisimulation

A binary relation $R \subseteq Proc \times Proc$ is a **strong bisimulation** iff whenever $(s, t) \in R$ then for each $a \in Act$:

- if $s \xrightarrow{a} s'$ then $t \xrightarrow{a} t'$ for some t' such that $(s', t') \in R$
- if $t \xrightarrow{a} t'$ then $s \xrightarrow{a} s'$ for some s' such that $(s', t') \in R$.

Strong Bisimilarity

Two processes $p_1, p_2 \in Proc$ are **strongly bisimilar** ($p_1 \sim p_2$) if and only if there exists a strong bisimulation R such that $(p_1, p_2) \in R$.

$$\sim\ = \cup\{R \mid R \text{ is a strong bisimulation}\}$$

Basic Properties of Strong Bisimilarity

Theorem

$\sim$ is an equivalence (reflexive, symmetric and transitive)

Theorem

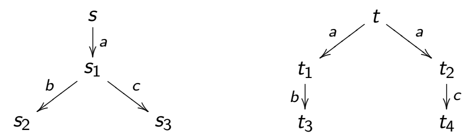
$\sim$ is the largest strong bisimulation

Theorem

$s \sim t$ if and only if for each $a \in Act$:

- *if $s \xrightarrow{a} s'$ then $t \xrightarrow{a} t'$ for some t' such that $s' \sim t'$*
- *if $t \xrightarrow{a} t'$ then $s \xrightarrow{a} s'$ for some s' such that $s' \sim t'$.*

How to Show Nonbisimilarity?



To prove that $s \not\sim t$:

- Enumerate **all binary relations** and show that none of them at the same time contains (s, t) and is a strong bisimulation. (Expensive: $2^{|Proc|^2}$ relations.)
- Make certain **observations** which will enable to disqualify many bisimulation candidates in one step.
- Use **game characterization** of strong bisimilarity.

Strong Bisimulation Game

Let $(Proc, Act, \{\xrightarrow{a} \mid a \in Act\})$ be an LTS and $s, t \in Proc$.

We define a two-player game of an **'attacker'** and a **'defender'** starting from s and t.

- The game is played in **rounds** and configurations of the game are pairs of states from $Proc \times Proc$.
- In every round exactly one configuration is called **current**. Initially the configuration (s, t) is the current one.

Intuition

The defender wants the show that s and t are strongly bisimilar while the attacker aims to prove the opposite.

Rules of the Bisimulation Games

Game Rules

In each round the players change the current configuration as follows:

1. the attacker chooses one of the processes in the current configuration and makes an $\xrightarrow{a}$-move for some $a \in Act$, and
2. the defender must respond by making an $\xrightarrow{a}$-move in the other process under the same action a.

The newly reached pair of processes becomes the current configuration. The game then continues by another round.

Result of the Game

- If one player cannot move, the other player wins.
- If the game is infinite, the defender wins.

Game Characterization of Strong Bisimilarity

Theorem

- States s and t are strongly bisimilar if and only if the defender has a **universal** winning strategy starting from the configuration (s, t).
- States s and t are not strongly bisimilar if and only if the attacker has a **universal** winning strategy starting from the configuration (s, t).

Remark

Bisimulation game can be used to prove both bisimilarity and nonbisimilarity of two processes. It very often provides elegant arguments for the negative case.

Strong Bisimilarity is a Congruence for CCS Operations

Theorem

Let P and Q be CCS processes such that $P \sim Q$. Then

- $\alpha.P \sim \alpha.Q$ *for each action* $\alpha \in Act$
- $P + R \sim Q + R$ *and* $R + P \sim R + Q$ *for each CCS process* R
- $P \mid R \sim Q \mid R$ *and* $R \mid P \sim R \mid Q$ *for each CCS process* R
- $P[f] \sim Q[f]$ *for each relabelling function* f
- $P \setminus L \sim Q \setminus L$ *for each set of labels* L.

Other Properties of Strong Bisimilarity

Following Properties Hold for any CCS Processes P, Q and R

- $P + Q \sim Q + P$
- $P \mid Q \sim Q \mid P$
- $P + Nil \sim P$
- $P \mid Nil \sim P$
- $(P + Q) + R \sim P + (Q + R)$
- $(P \mid Q) \mid R \sim P \mid (Q \mid R)$