## Semantics and Verification 2008

Lecture 2

- informal introduction to CCS
- syntax of CCS
- semantics of CCS

# CCS Basics (Sequential Fragment)

- Nil (or 0) process (the only atomic process)
- action prefixing (a.P)
- names and recursive definitions  $(\stackrel{\text{def}}{=})$
- nondeterministic choice (+)

#### This is Enough to Describe Sequential Processes

Any finite LTS can be (up to isomorphism) described by using the operations above.

Sequential Fragment Parallelism and Renaming

## CCS Basics (Parallelism and Renaming)

- parallel composition (|) (synchronous communication between two components = handshake synchronization)
- restriction  $(P \smallsetminus L)$
- relabelling (*P*[*f*])

Introduction to CCS Notation Syntax of CCS CCS Process Expressions Semantics of CCS CCS Defining Equations

Definition of CCS (channels, actions, process names)

Let

- A be a set of channel names (e.g. *tea*, *coffee* are channel names)
- L = A ∪ A be a set of labels where
   A = {ā | a ∈ A} (A are called names and A are called co-names)
   by convention ā = a
- Act = L ∪ {τ} is the set of actions where

   τ is the internal or silent action
   (e.g. τ, tea, coffee are actions)
- $\mathcal{K}$  is a set of process names (constants) (e.g. CM).

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## Definition of CCS (expressions)

$$P := K$$

$$\alpha.P$$

$$\sum_{i \in I} P_i$$

$$P_1 | P_2$$

$$P \smallsetminus L$$

$$P[f]$$

process constants ( $K \in K$ ) prefixing ( $\alpha \in Act$ ) summation (I is an arbitrary index set) parallel composition restriction ( $L \subseteq A$ ) relabelling ( $f : Act \rightarrow Act$ ) such that •  $f(\tau) = \tau$ •  $f(\overline{a}) = \overline{f(a)}$ 

The set of all terms generated by the abstract syntax is called CCS process expressions (and denoted by  $\mathcal{P}$ ).

#### Notation

$$P_1 + P_2 = \sum_{i \in \{1,2\}} P_i$$

 $Nil = 0 = \sum_{i \in \emptyset} P_i$ 

Notation CCS Process Expressions CCS Defining Equations

### Precedence

#### Precedence

- restriction and relabelling (tightest binding)
- 2 action prefixing
- oparallel composition

summation

Example:  $R + a.P|b.Q \setminus L$  means  $R + ((a.P)|(b.(Q \setminus L)))$ .

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# Definition of CCS (defining equations)

#### CCS program

A collection of defining equations of the form

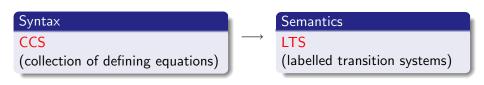
$$K\stackrel{\mathrm{def}}{=} P$$

where  $K \in \mathcal{K}$  is a process constant and  $P \in \mathcal{P}$  is a CCS process expression.

- Only one defining equation per process constant.
- Recursion is allowed: e.g.  $A \stackrel{\text{def}}{=} \overline{a}.A \mid A$ .

Motivation SOS Rules for CCS Examples

### Semantics of CCS



HOW?

Motivation SOS Rules for CCS Examples

## Structural Operational Semantics for CCS

#### Structural Operational Semantics (SOS) – G. Plotkin 1981

Small-step operational semantics where the behaviour of a system is inferred using syntax driven rules.

Given a collection of CCS defining equations, we define the following LTS (*Proc*, Act,  $\{\stackrel{a}{\longrightarrow} | a \in Act\}$ ):

- $Proc = \mathcal{P}$  (the set of all CCS process expressions)
- $Act = \mathcal{L} \cup \{\tau\}$  (the set of all CCS actions including  $\tau$ )
- transition relation is given by SOS rules of the form:

Introduction to CCS Motivation Syntax of CCS Sos Rules for CCS Semantics of CCS Examples

## SOS rules for CCS ( $\alpha \in Act$ , $a \in \mathcal{L}$ )

ACT 
$$\alpha . P \xrightarrow{\alpha} P$$
 SUM<sub>j</sub>  $\frac{P_j \xrightarrow{\alpha} P'_j}{\sum_{i \in I} P_i \xrightarrow{\alpha} P'_j} j \in I$ 

COM1	$\underline{P \xrightarrow{\alpha} P'}$	COM2	$\underline{\qquad Q \xrightarrow{\alpha} Q'}$
	$P Q \xrightarrow{lpha} P' Q$	00012	$\overline{P Q \stackrel{\alpha}{\longrightarrow} P Q'}$

COM3 
$$\xrightarrow{P \xrightarrow{a} P' \quad Q \xrightarrow{\overline{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

$$\operatorname{RES} \quad \frac{P \xrightarrow{\alpha} P'}{P \smallsetminus L \xrightarrow{\alpha} P' \smallsetminus L} \quad \alpha, \overline{\alpha} \notin L \qquad \qquad \operatorname{REL} \quad \frac{P \xrightarrow{\alpha} P'}{P[f] \xrightarrow{f(\alpha)} P'[f]}$$

 $\begin{array}{ccc} \text{CON} & \frac{P \stackrel{\alpha}{\longrightarrow} P'}{K \stackrel{\alpha}{\longrightarrow} P'} & K \stackrel{\text{def}}{=} P \\ \\ \text{Lecture 2} & \text{Semantics and Verification 2008} \end{array}$ 

Motivation SOS Rules for CCS Examples

### Deriving Transitions in CCS

Let  $A \stackrel{\text{def}}{=} a.A$ . Then

 $((A | \overline{a}.Nil) | b.Nil)[c/a] \xrightarrow{c} ((A | \overline{a}.Nil) | b.Nil)[c/a].$ 

$$\operatorname{REL} \frac{\operatorname{COM1} \frac{A\operatorname{CT}}{A \xrightarrow{a} A} A \stackrel{\text{def}}{=} a.A}{(A \mid \overline{a}.Nil \xrightarrow{a} A) A} A \stackrel{\text{def}}{=} a.A}{(A \mid \overline{a}.Nil \xrightarrow{a} A \mid \overline{a}.Nil)}$$



