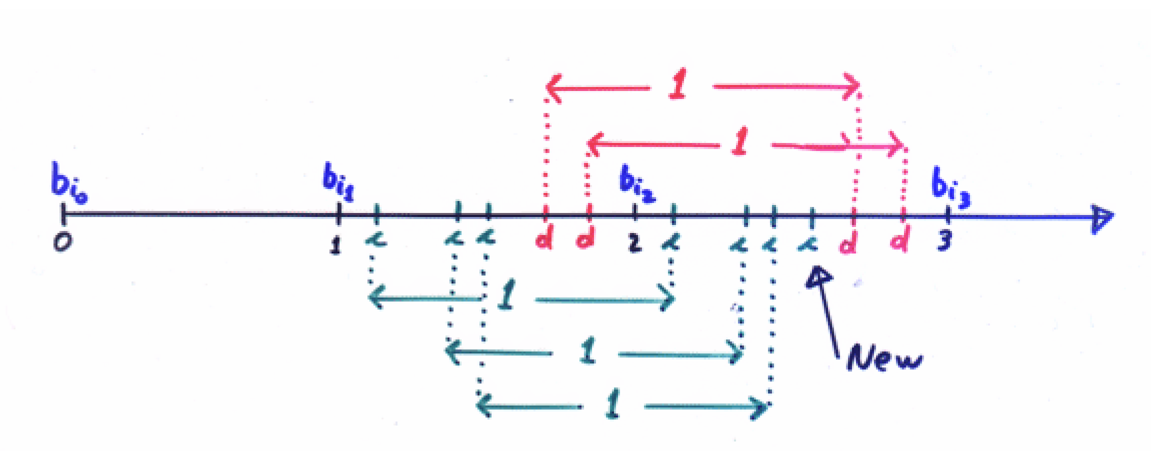
EXERCISE
Undecidability of Timed Language Inclusion

This exercise has the purpose of completing Theorem 5.2 in Alur&Dill: "A Theory of Timed Automata" (AD)

We are considering timed words over the alphabet $\Sigma=\{b_1,..,b_n,c,d\}$ where $\{b_1,.,b_n\}$ are the Instructions of the Two-Counter Machine M and c and d are used to count the value of the counters x and y.

The figure below illustrates what is required in order that a timed word over Σ represents the computation of M. A formal definition can be found on p 26 AD.

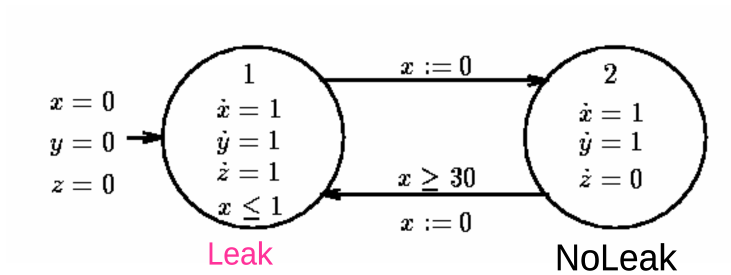


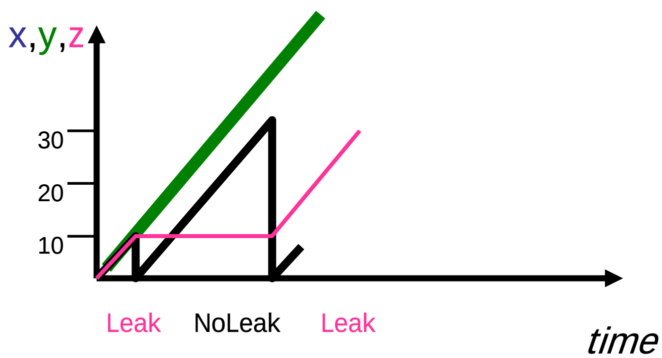
Consider the instruction of M:

I: (p) x:=x+1 ; goto (q)

Give a collection of timed automata, whose union captures all timed word that (somewhere) violates the requirement to an encoding of the instruction **I**.

EXERCISE
Undecidability of
Reachability for Stopwatch Automata





In this exercise we are considering a “small” extension of timed automata, namely so-called Stopwatch Automata. As indicated by the name, in a stopwatch automata clocks may be stopped when entering a new location. Formally, for each location l there is a set of clocks $C_l \subseteq C$ defining the clocks that are stopped.

Ex 1: Define formally the semantics of Stopwatch Automata.

In the figure above you see a Stopwatch Automata modeling the so-called *Gasburner.* Here z describes the accumulated time spent in location Leak.

Ex 2: Prove that whenever $y \geq 60$ then $20 \cdot z \leq y$

Ex 3: Prove that location-reachability for Stopwatch Automata is undecidable.
Hint: reduce the halting problem for two-counter machines to location-reachability for Stopwatch Automata.