Take-home Assignment 3

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The solutions are to be written **individually**, though a group discussion about the general strategy how to solve the problems is allowed. The students are expected to write down the solution in latex (as this will be a part of the semester report) and deliver it to Jiri latest by **November 26th**, 2007. You will then receive the corrected assignments within a week or so.

Let Σ be a finite alphabet. A *rewrite system* (RS) is a finite set Δ of rules of the form $u \to v$ where $u \in \Sigma^+$ and $v \in \Sigma^*$. For two words $x, y \in \Sigma^*$ we write $x \Rightarrow y$ iff there is a rule $(u \to v) \in \Delta$ such that $x = z_1 u z_2$ and $y = z_1 v z_2$ for some $z_1, z_2 \in \Sigma^*$.

By $\Rightarrow^*$ we denote the reflexive and transitive closure of $\Rightarrow$.

- Show the undecidability of the word problem for RS, i.e., given a set of rules Δ and two strings $x, y \in \Sigma^*$ that it is undecidable whether $x \Rightarrow^* y$.
  Hint: provide a mapping reduction either from the halting problem of 2-counter Minsky machine or directly from the halting problem of Turing machines.

- Try to find the most restrictive (but still undecidable) variant of the word problem by restricting e.g. the size of Σ and/or the lenghts of the words used in the rules. Prove that your restricted variant of the problem is still undecidable.