Take-home Assignment 1

by Jiri Srba

The solutions are to be written **individually**, though a group discussion about the general strategy how to solve the problems is allowed and in fact very welcome. The students are expected to write down the solution in latex (as this will be a part of the semester report) and deliver it to Jiri latest by **October 8th**, 2007. You will then receive the corrected assignments within a week or so.

1. A one-counter automaton is a pushdown automaton with only two stack symbols X and Z (X representing the counter value and Z is the bottom of the stack) and the allowed rules are restricted: they can be either of the form $pZ \xrightarrow{a} qX^nZ$ or $pX \xrightarrow{a} qX^n$ for any $n \geq 0$.

   a) Prove that there is a one-counter automaton that is not strongly bisimilar to any finite-state process.

   b) Prove that there is a pushdown automaton that is not strongly bisimilar to any one-counter automaton.

2. Assume the following PDA rules: $pX \rightarrow qY$, $pX \rightarrow pXY$, $qY \rightarrow p$.

   a) Construct a multi-automaton that recognizes the set $pre*(pX^*Y)$.

   b) Construct a multi-automaton that recognizes the set $post*(\{pX\})$.

3. Construct a PDA system of size n and two configurations $p\alpha$ and $q\beta$ such that:

   a) $|\alpha| = O(n)$ and $|\beta| = O(n)$, and

   b) $q\beta$ is reachable from $p\alpha$, and

   c) the minimum number of steps for reaching $q\beta$ from $p\alpha$ is $\Omega(2^n)$.

   Hint: simulate a binary addition on a stack.

4. Let Δ be a PDA system, let pX be its initial configuration, and let q be a control state. The control-state reachability problem is to decide whether a configuration $q\alpha$ can be reached from pX for some sequence of stack symbols α. Prove the following lemma.

   Lemma: The control-state reachability problem is decidable in (deterministic) polynomial time. Moreover, if the configuration $q\alpha$ is reachable from pX, then we can assume without loss of generality that the length of α is linear in the size of the pushdown automaton.

   Hint: use the *post∗* multi-automaton to argue for the claim.