Priced Timed Automata and Timed Games

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Scheduling and Synthesis

Priced Timed Automata

Timed Games

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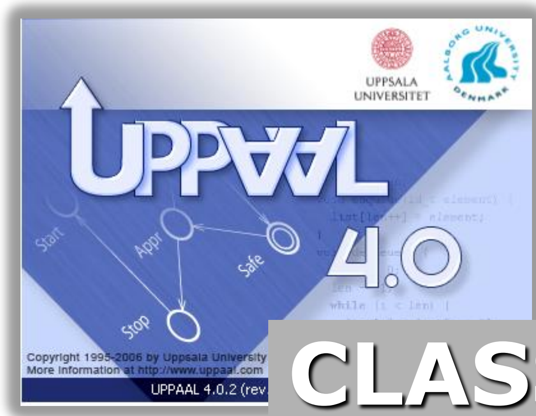
Overview

- **Timed** Automata & UPPAAL
- **Symbolic** Verification & UPPAAL Engine, Options
- **Priced** Timed Automata and Timed **Games**
- **Stochastic** Timed Automata **Statistical** Model Checking

(Lecture+Exercise)⁴



CLASSIC

CORA

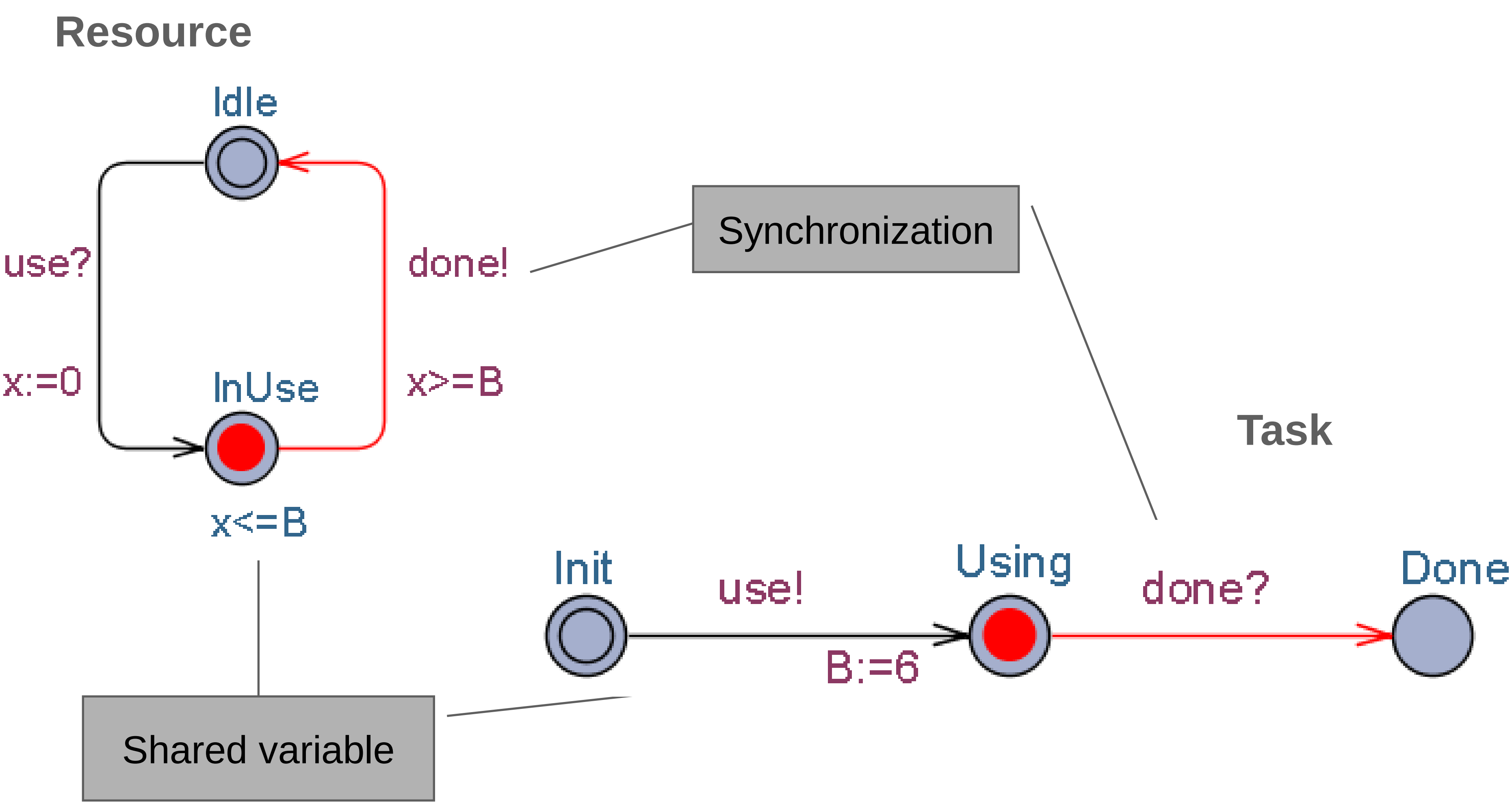
TIGA

ECDAR

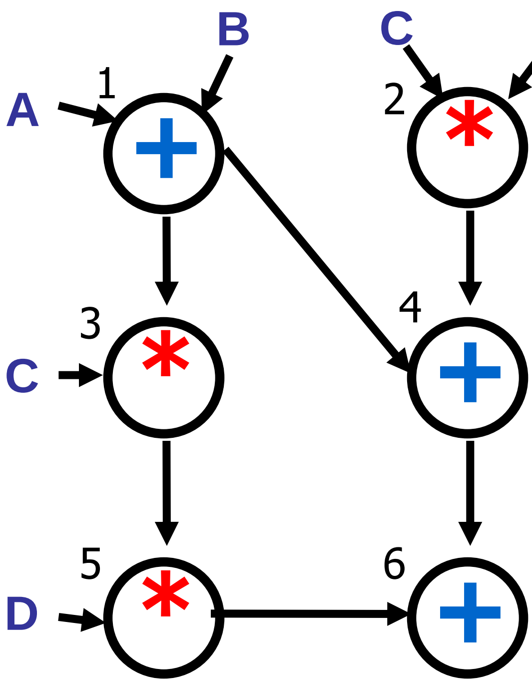
SMC

TRON

Resources & Tasks



Task Graph Scheduling – Example



Compute :

(D * (C * (A + B)) + ((A + B) + (C * D))

4

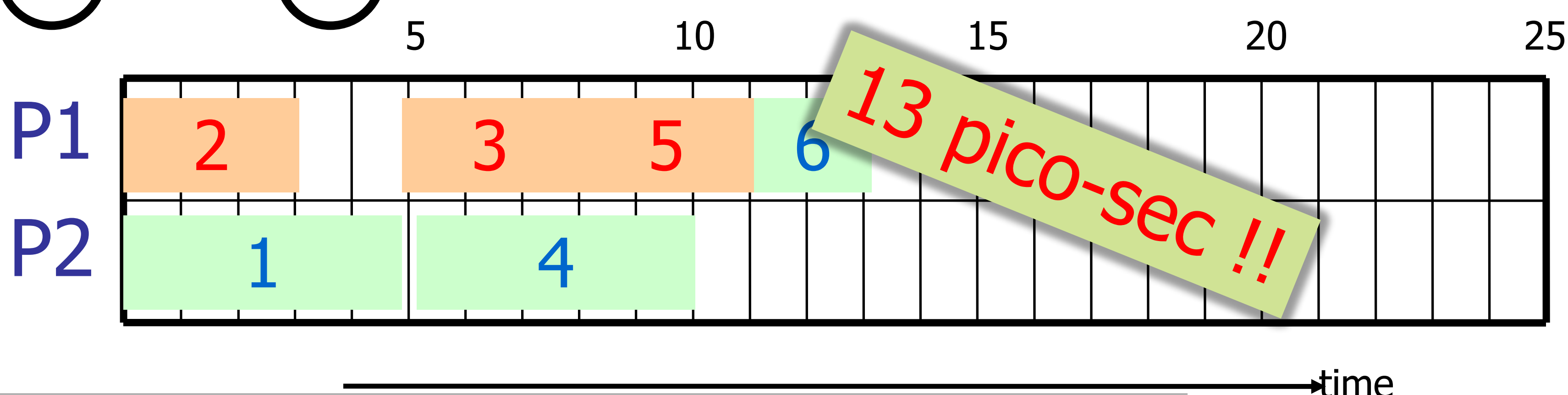
using 2 processors

P1 (fast)

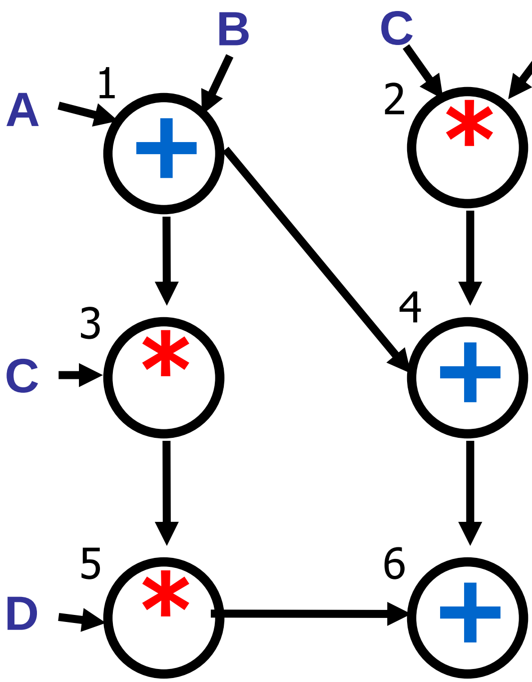
+	2ps
*	3ps

P2 (slow)

+	5ps
*	7ps



Task Graph Scheduling – Example



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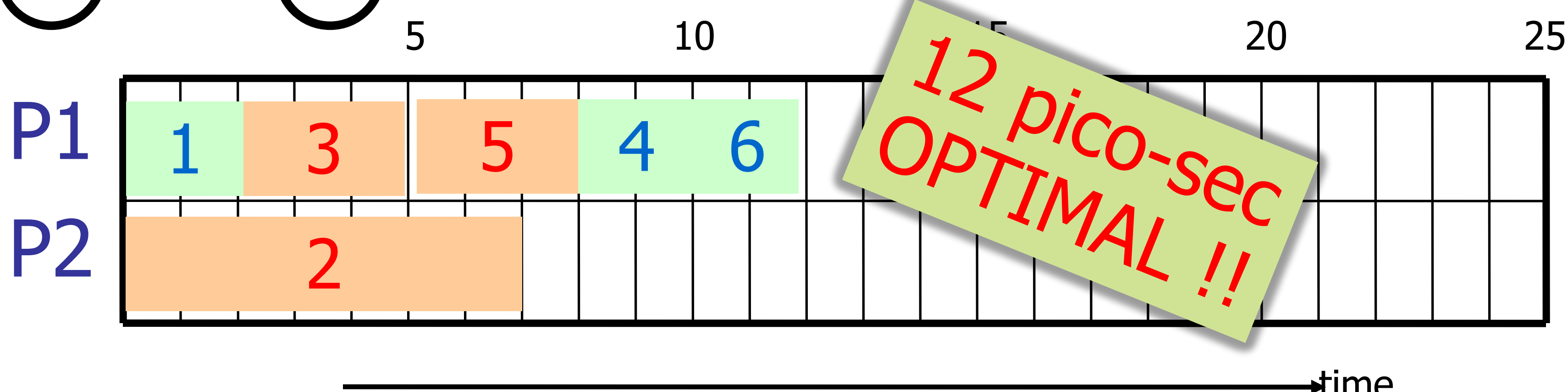
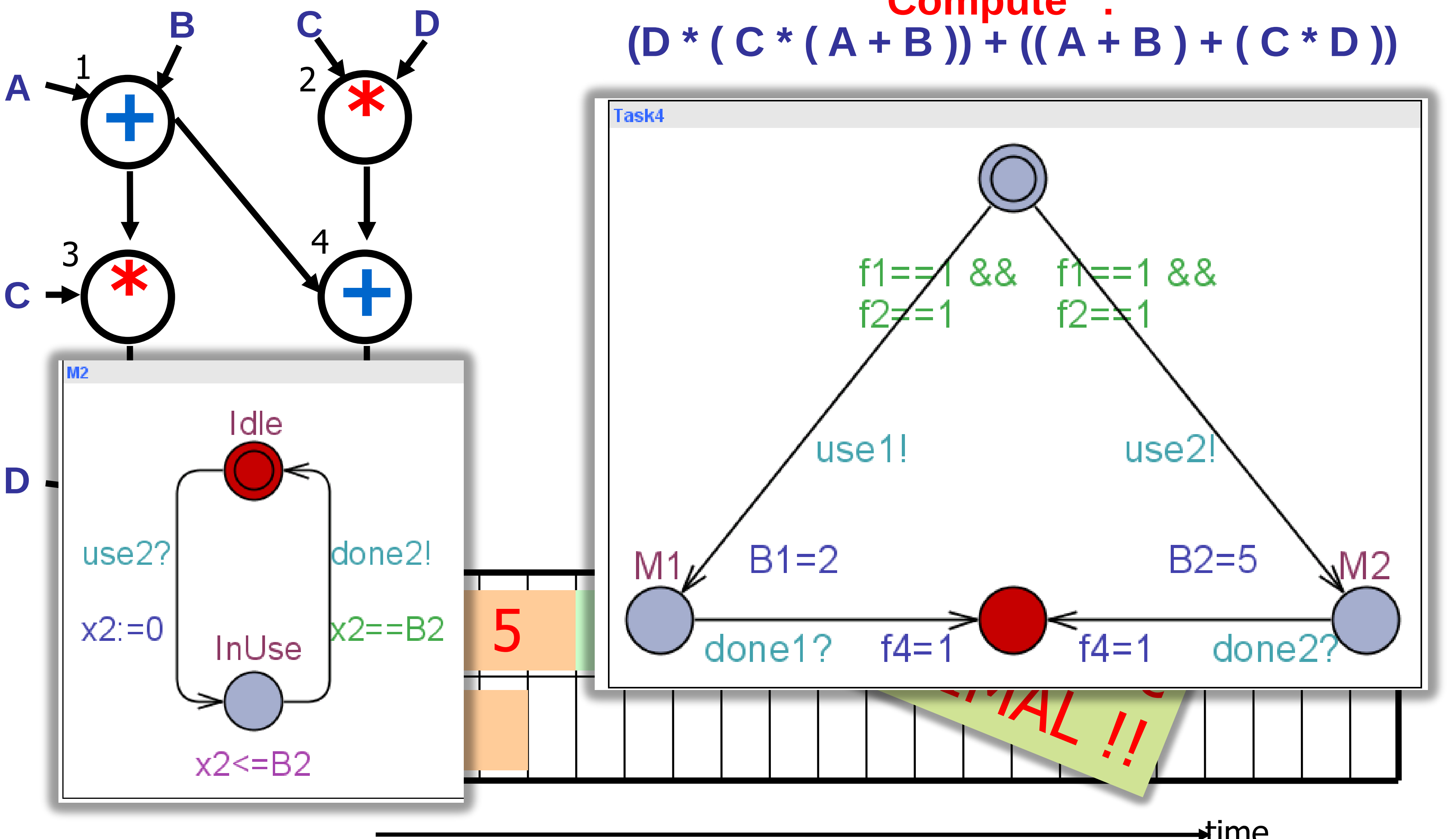
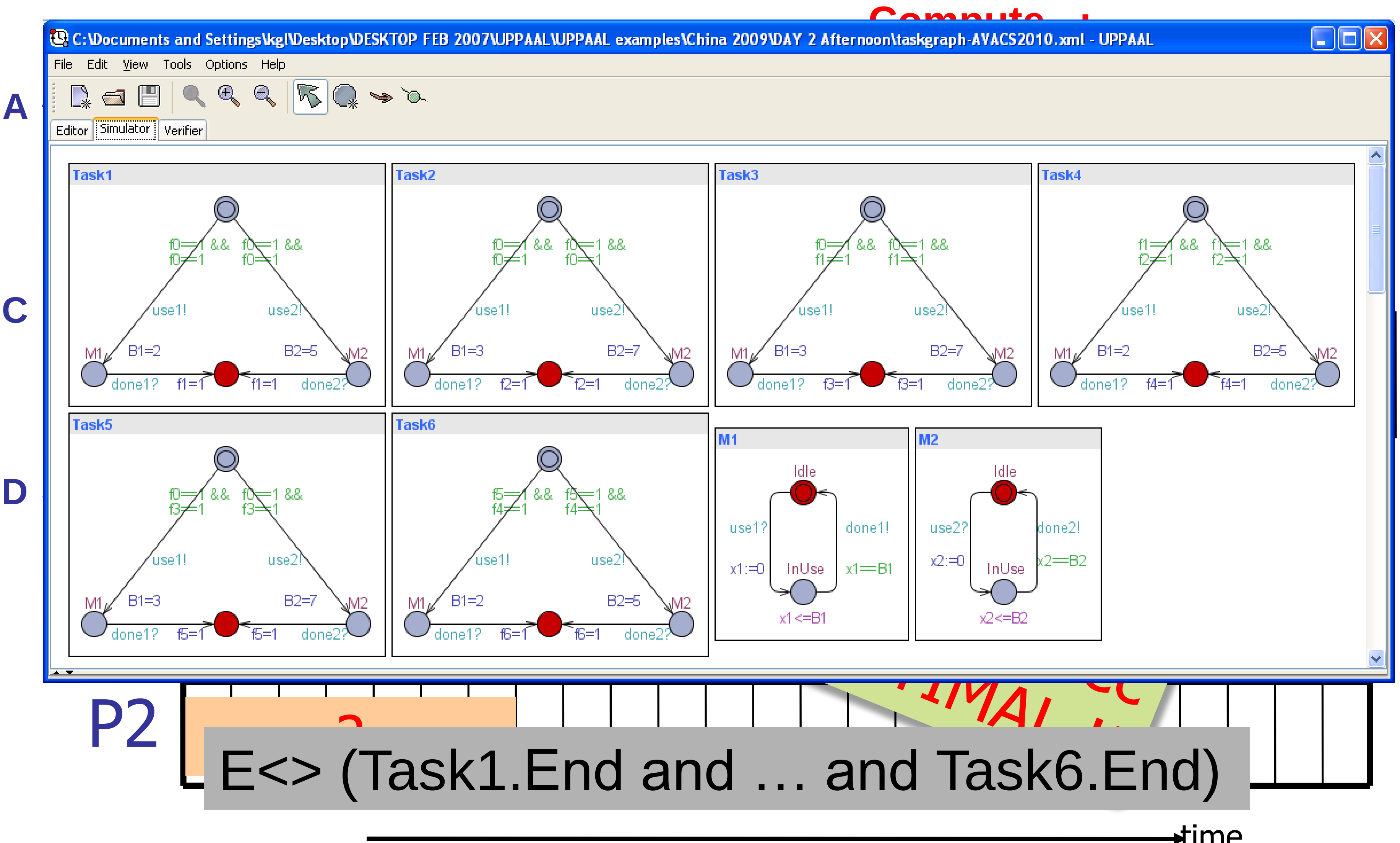


Task Graph Scheduling – Example

Compute :
(D * (C * (A + B)) + ((A + B) + (C * D))



Task Graph Scheduling – Example



E<> (Task1.End and … and Task6.End)

Experimental Results

name	#tasks	#chains	# machines	optimal	TA
001	437	125	4	1178	1182
000	452	43	20	537	537
018	730	175	10	700	704
074	1007	66	12	891	894
021	1145	88	20	605	612
228	1187	293	8	1570	1574
071	1193	124	20	629	634
271	1348	127	12	1163	1164
237	1566	152	12	1340	1342
231	1664	101	16	t.o.	1137
235	1782	218	16	t.o.	1150
233	1980	207	19	1118	1121
294	2014	141	17	1257	1261
295	2168	965	18	1318	1322
292	2333	318	3	8009	8009
298	2399	303	10	2471	2473

AMETIST
advanced methods for timed systems

Symbolic A*
Branch-&-Bound
60 sec

Abdeddaïm, Kerbaa, Maler

Jobshop Scheduling

[TACAS'2001]

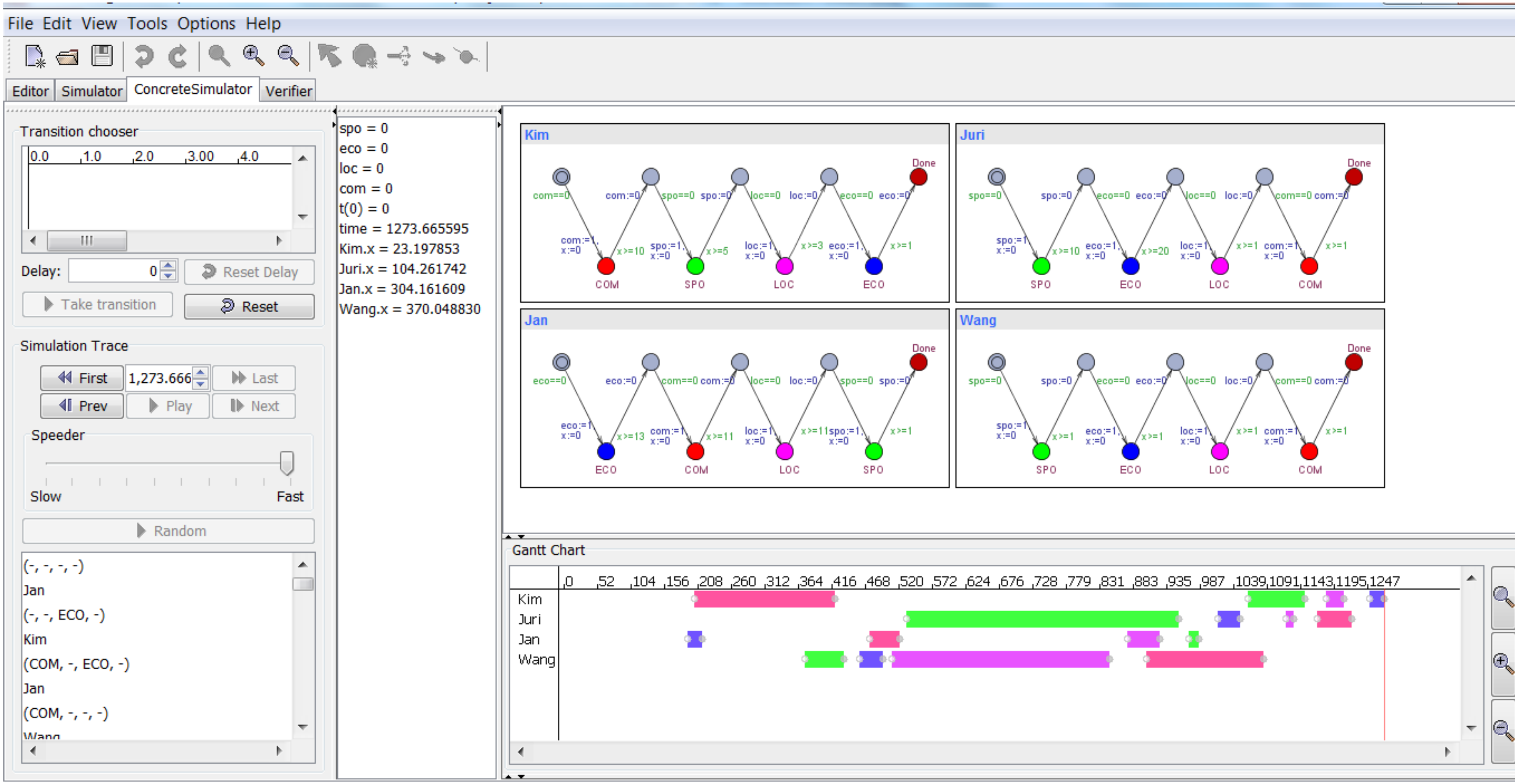
	Sport	Economy	Local News	Comic Stip
Kim	2. 5 min	4. 1 min	3. 3 min	1. 10 min
Jüri	1. 10 min	2. 20 min	3. 1 min	4. 1 min
Jan	4. 1 min	1. 13 min	3. 11 min	2. 11 min
Wang	1. 1 min	2. 1 min	3. 1 min	4. 1 min

Problem: compute the minimal **MAKESPAN**

NP-hard

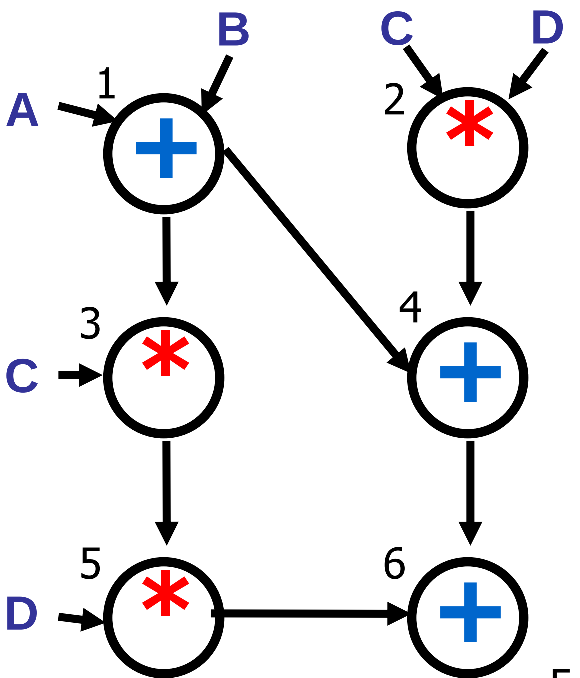
Simulated annealing
Shiffted bottleneck
Branch-and-Bound
Gentic Algorithms

Jobshop Scheduling in UPPAAL



Priced Timed Automata

Task Graph Scheduling – Revisited



Compute :

(D * (C * (A + B)) + ((A + B) + (C * D))

using 2 processors

P1 (fast)

+	2ps
*	3ps

Idle	10W
In use	90W

P2 (slow)

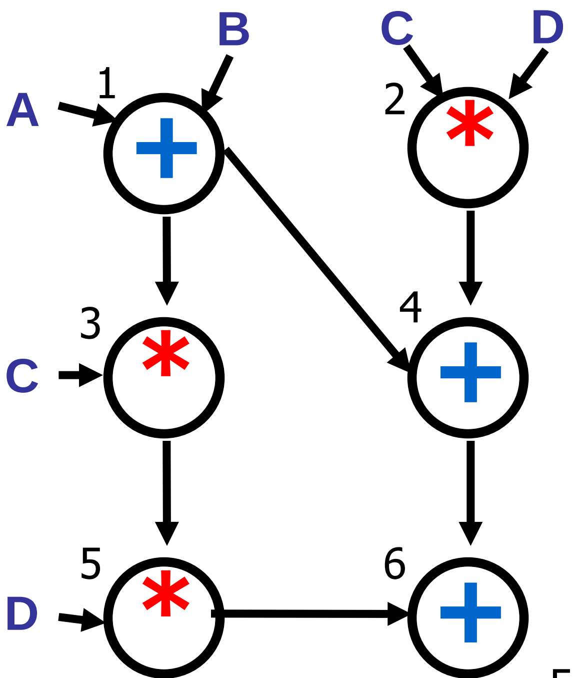
+	5ps
*	7ps

Idle	20W
In use	30W

ENERGY:

Energy:
1.39 nano-joule !!

Task Graph Scheduling – Revisited



Compute :

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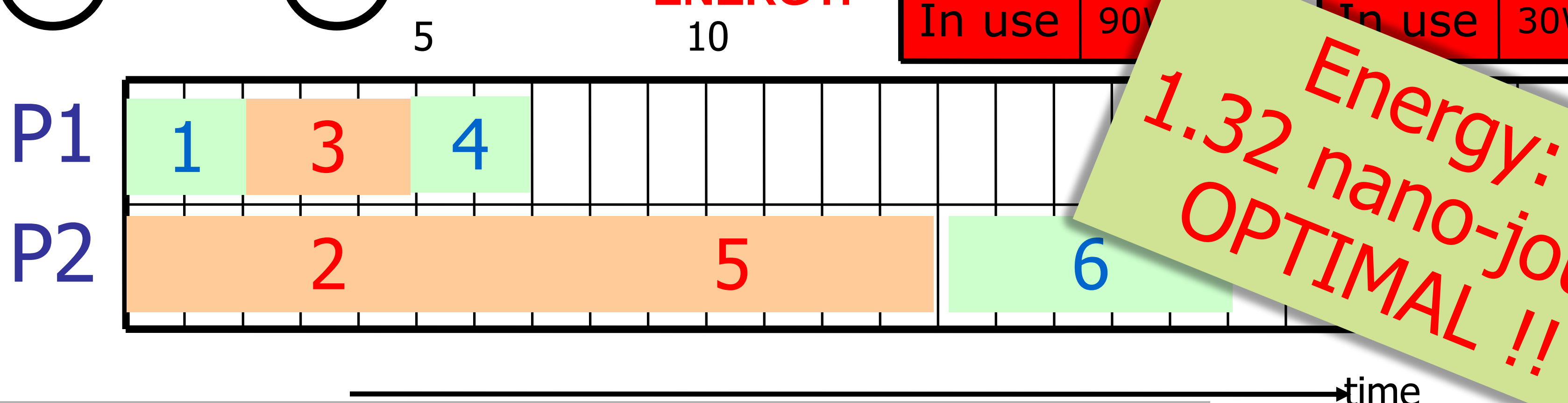
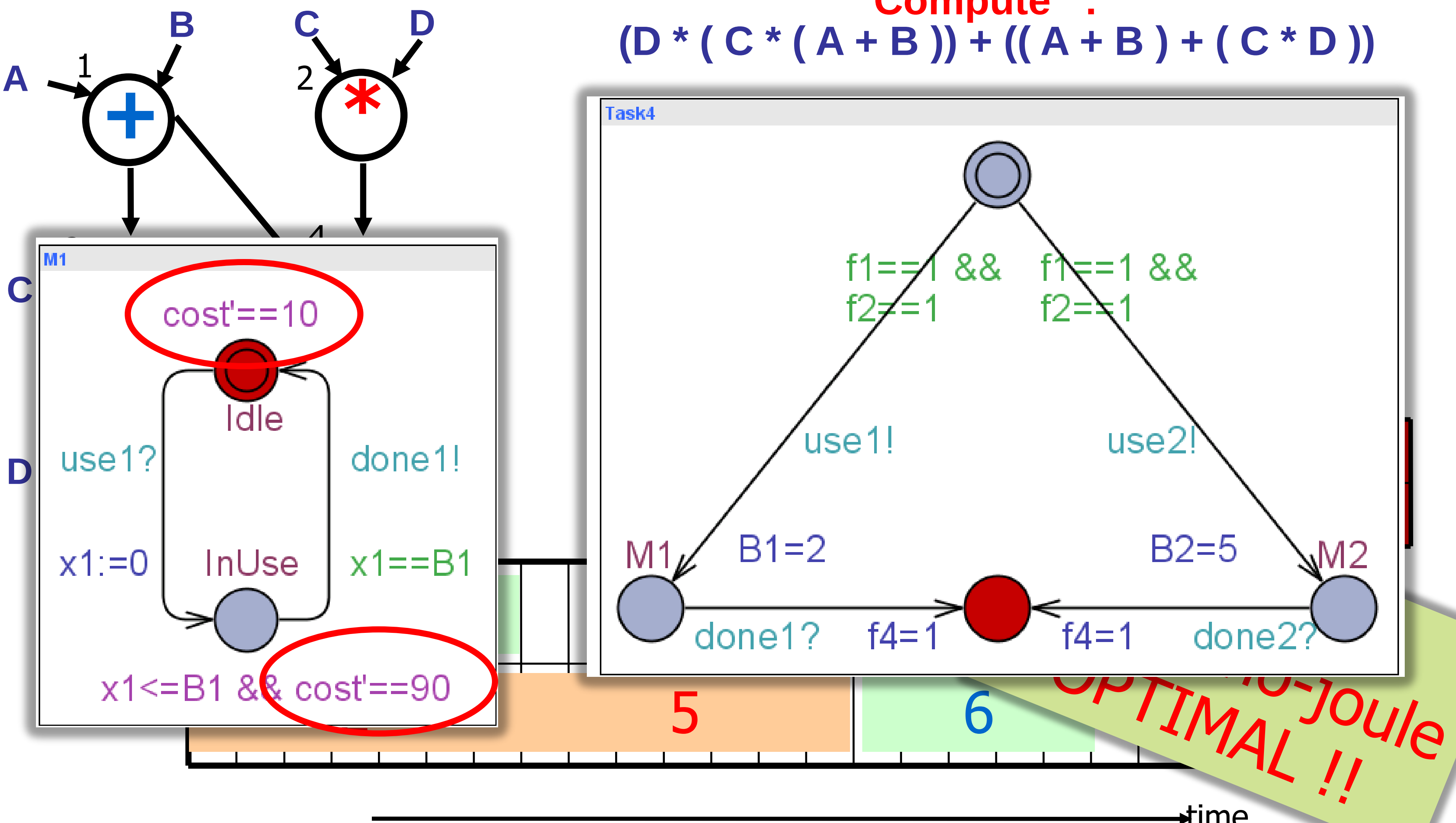
ENERGY:



Energy: 1.32 nano-joule OPTIMAL !!

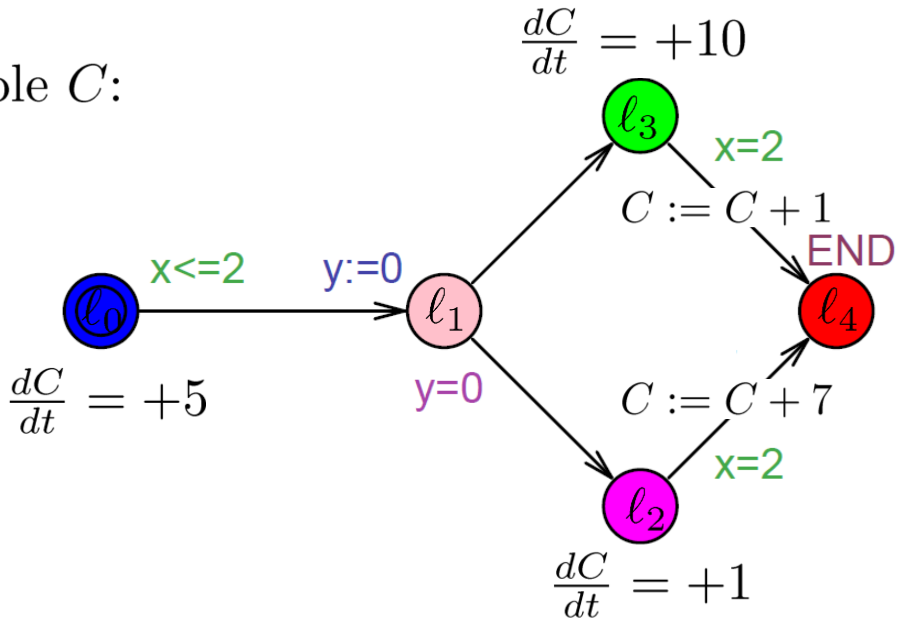
Task Graph Scheduling – Revisited

Compute :
(D * (C * (A + B)) + ((A + B) + (C * D))



A simple example

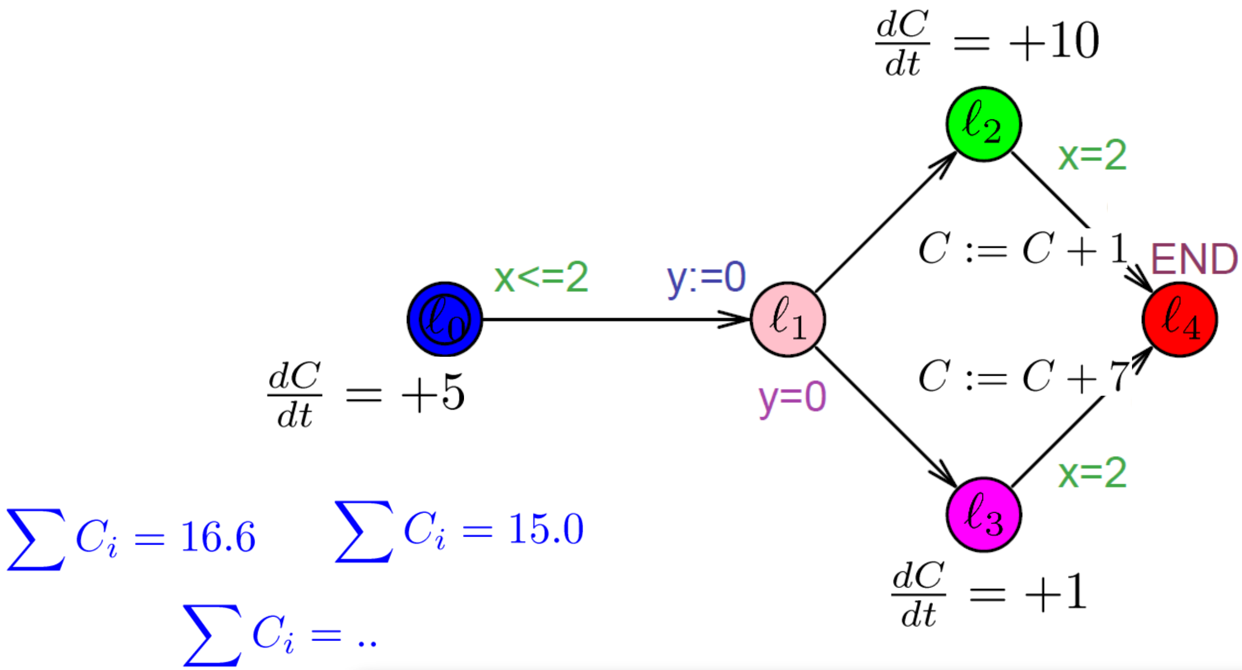
Observer variable C:



$$(\ell_0, [0,0]) \xrightarrow{1.9}_{9.5} (\ell_0, [1.9, 1.9]) \rightarrow_0 (\ell_1, [1.9, 0]) \rightarrow_0 (\ell_2, [1.9, 0]) \xrightarrow{0.1}_{0.1} (\ell_2, [2, 0.1]) \rightarrow_7 (\ell_4, [2, 0.1]) \qquad \sum C_i = 16.6$$

$$(\ell_0, [0,0]) \xrightarrow{1.2}_{6.0} (\ell_0, [1.2, 1.2]) \rightarrow_0 (\ell_1, [1.2, 0]) \rightarrow_0 (\ell_3, [1.2, 0]) \xrightarrow{0.8}_{8.0} (\ell_3, [2, 0.8]) \rightarrow_1 (\ell_4, [2, 0.8]) \qquad \sum C_i = 15.0$$

A simple example



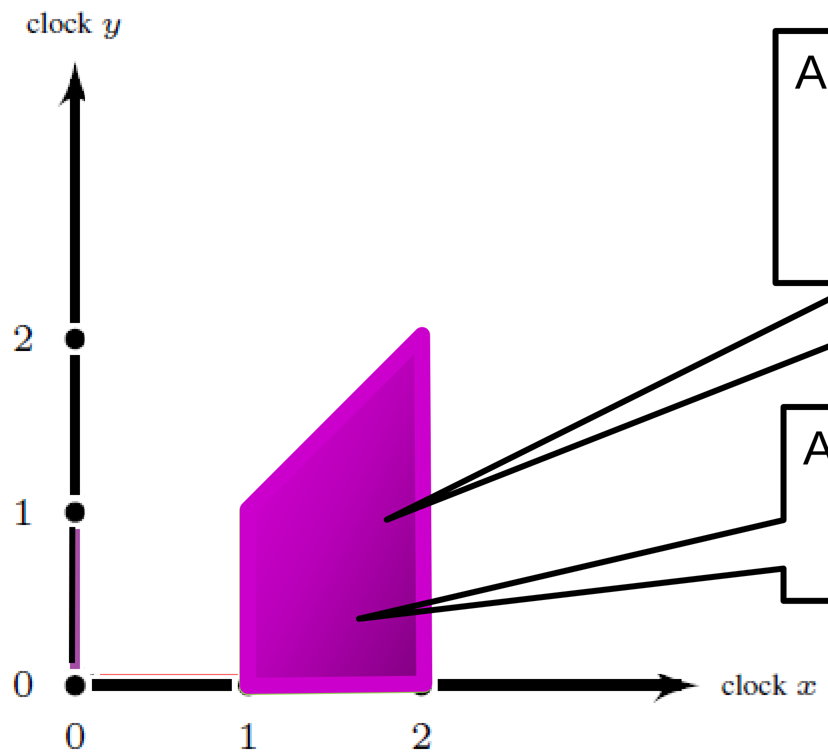
Q: What is cheapest cost for reaching ℓ_4 ?

$$\inf_{0 \leq t \leq 2} \min\{5t + 10(2 - t) + 1, 5t + (2 - t) + 4\} = 9$$

➜ **strategy:** leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

Priced Zones

[CAV01]



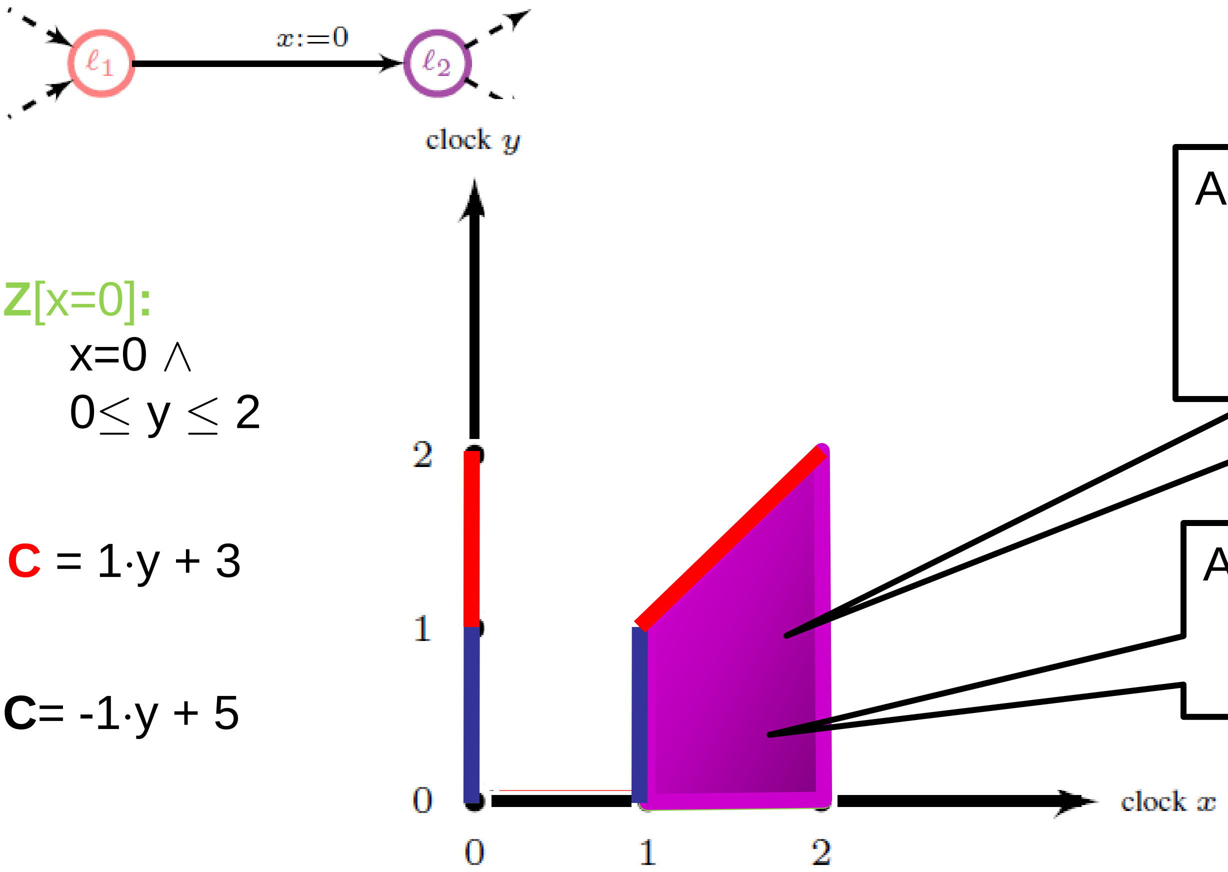
A zone **Z**:

$1 \leq x \leq 2 \quad \wedge$
$0 \leq y \leq 2 \quad \wedge$
$x - y \geq 0$

A cost function **C**

$C(x,y) =$
$2 \cdot x - 1 \cdot y + 3$

Priced Zones – Reset [CAV01]



Z[x=0]:
x=0 $\wedge$
0$\leq$ y $\leq$ 2

C = 1·y + 3

C= -1·y + 5

A zone **Z**:
1$\leq$ x $\leq$ 2 $\wedge$
0$\leq$ y $\leq$ 2 $\wedge$
x - y $\geq$ 0

A cost function **C**
C(x,y) =
2·x - 1·y + 3

Symbolic Branch & Bound Algorithm

THM [Behrmann, Fehnker ..01] [Alur,Torre,Pappas 01]
Optimal reachability is decidable for PTA

THM [Bouyer, Brojaue, Briuere, Raskin 07]
Optimal reachability is PSPACE-complete for PTA

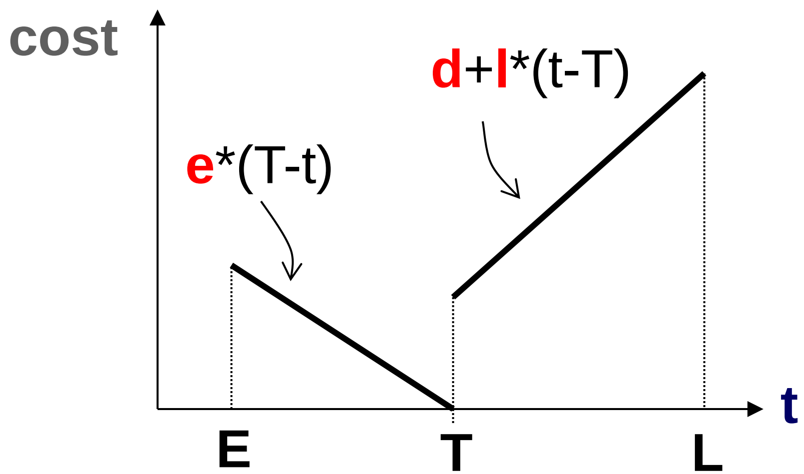
Cost := ∞
Passed := $\emptyset$
Waiting := $\{(l_0, Z_0)\}$
while Waiting $\neq \emptyset$ **do**
  select (l, Z) from Waiting
  if $l = l_g$ and minCost(Z) < Cost **then**
    Cost := minCost(Z)
  if minCost(Z) + Rem$_{(l,Z)} \geq$ Cost **then** break
  if for all (l, Z') in Passed: $Z' \not\leq Z$ **then**
    add (l, Z) to Passed
    add all (l', Z') with $(l, Z) \rightarrow (l', Z')$ to
return Cost

$Z' \leq Z$

Z′* is bigger & cheaper than *Z

$\leq$ **is a well-quasi ordering which guarantees termination!**

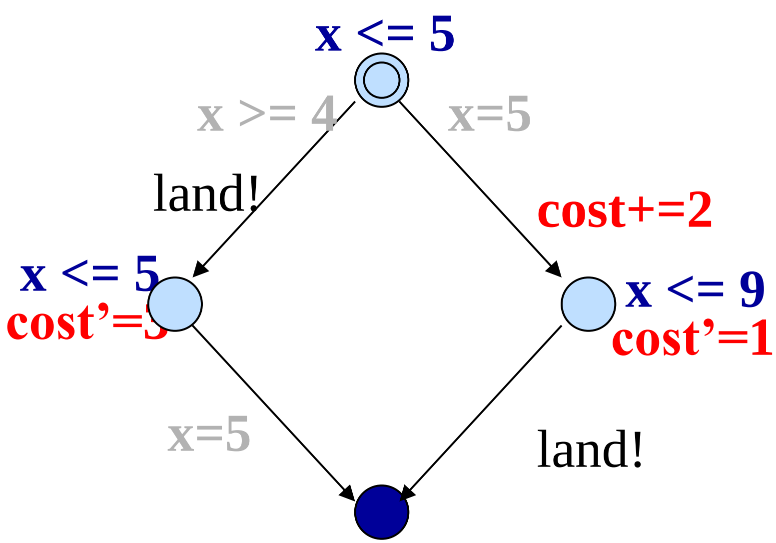
Example: Aircraft Landing



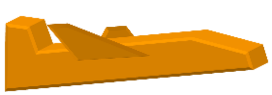
E earliest landing time
T target time
L latest time
e cost rate for being early
l cost rate for being late
d fixed cost for being late

Planes have to keep separation distance to avoid turbulences caused by preceding planes

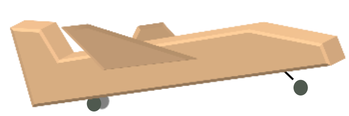
Example: Aircraft Landing

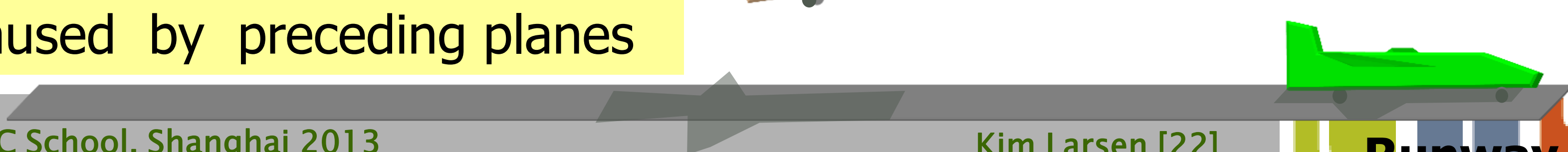


4 earliest landing time
5 target time
9 latest time
3 cost rate for being early
1 cost rate for being late
2 fixed cost for being late



Planes have to keep separation distance to avoid turbulences caused by preceding planes





Aircraft Landing

Source of examples:
Baesley et al'2000

	problem instance	1	2	3	4	5	6	7
	number of planes	10	15	20	20	20	30	44
	number of types	2	2	2	2	2	4	2
1	optimal value	700	1480	820	2520	3100	24442	1550
	explored states	481	2149	920	5693	15069	122	662
	cputime (secs)	4.19	25.30	11.05	87.67	220.22	0.60	4.27
2	optimal value	90	210	60	640	650	554	0
	explored states	1218	1797	669	28821	47993	9035	92
	cputime (secs)	17.87	39.92	11.02	755.84	1085.08	123.72	1.06
3	optimal value	0	0	0	130	170	0	N/A
	explored states	24	46	84	207715	189602	62	
	cputime (secs)	0.36	0.70	1.71	14786.19	12461.47	0.68	
4	optimal value	N/A	N/A	N/A	0	0	N/A	N/A
	explored states				65	64		
	cputime (secs)				1.97	1.53		

Timed Games

TIGA

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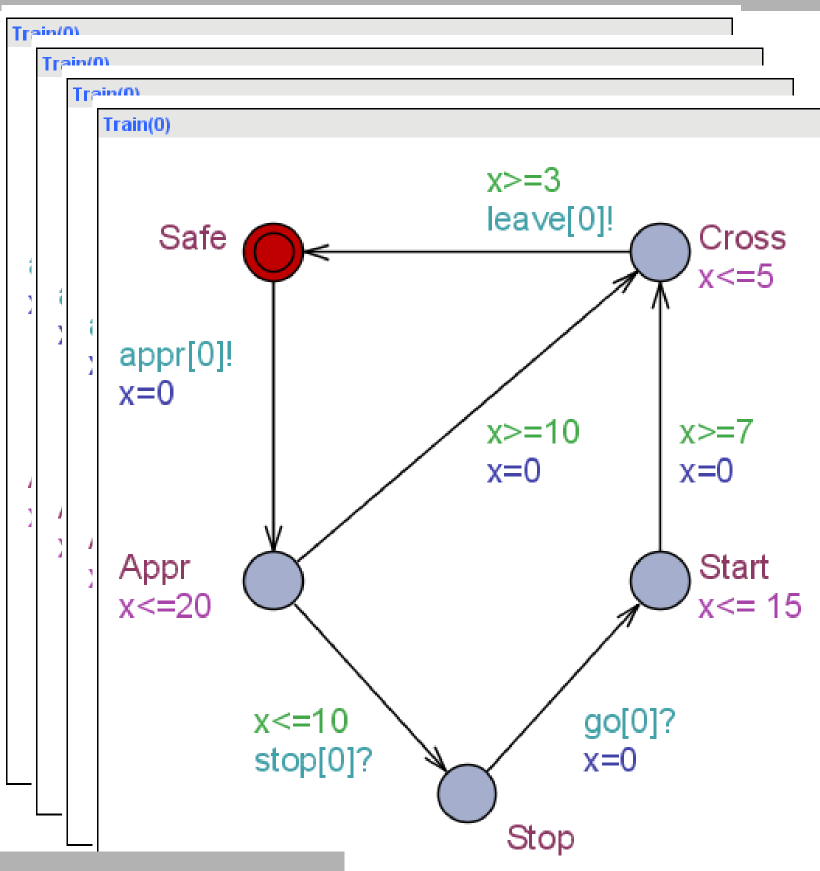
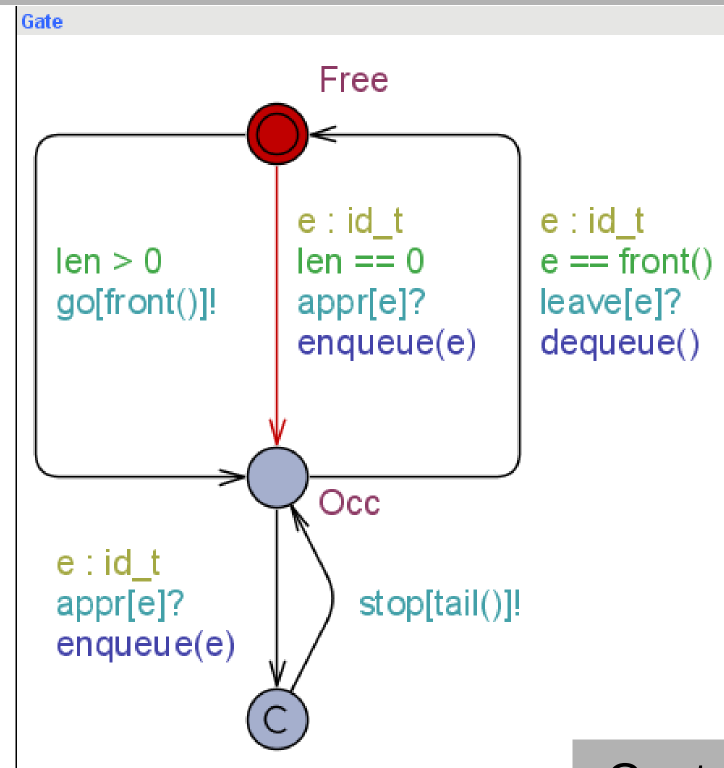
Model Checking



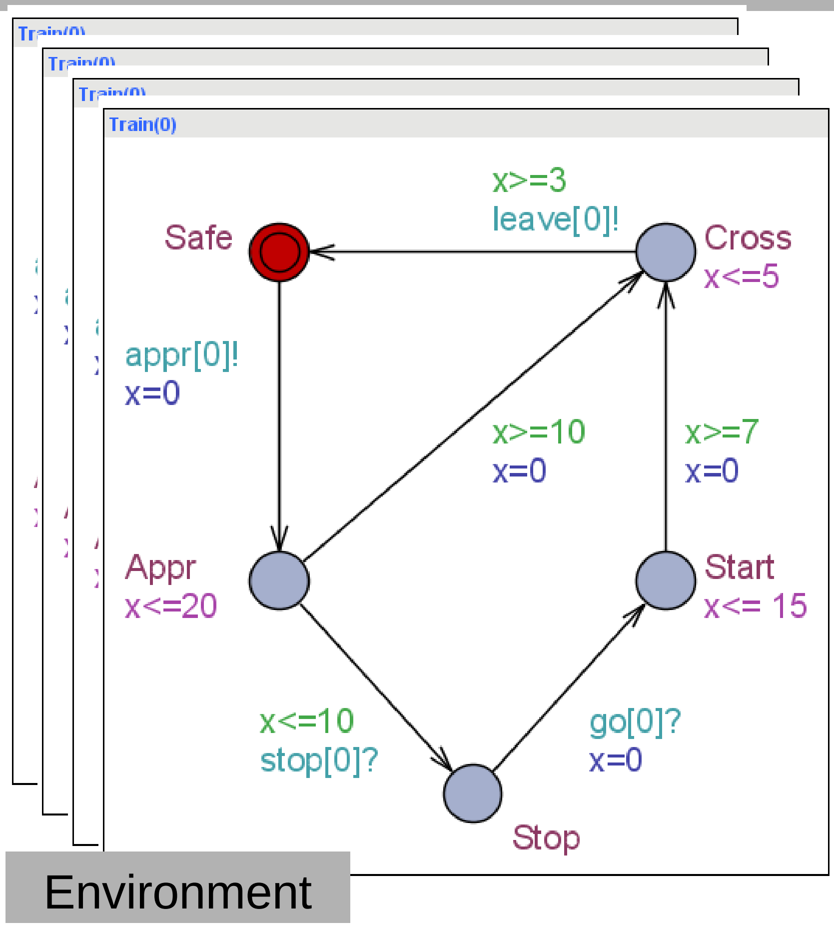
Environment

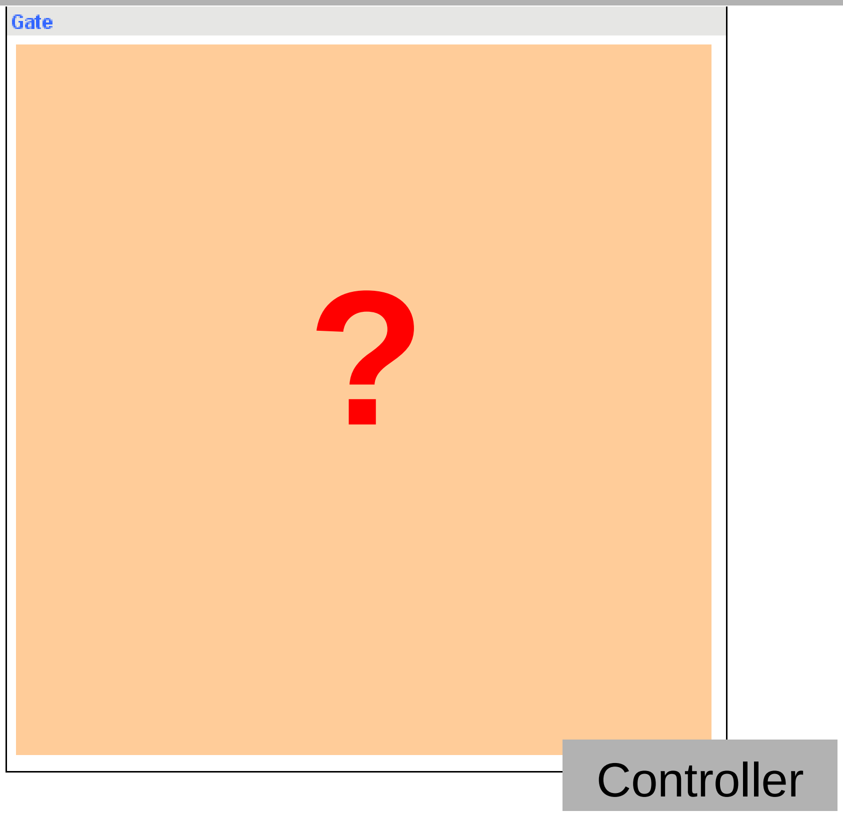


Controller

ϕ: Never two trains at the crossing at the same time

Synthesis



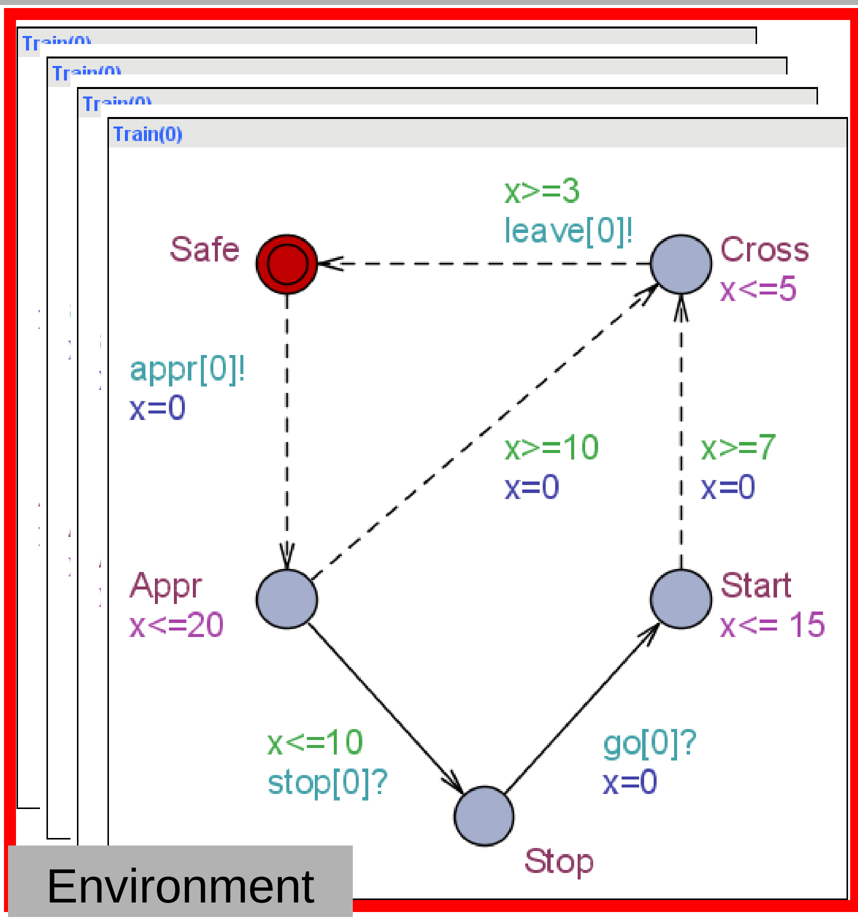


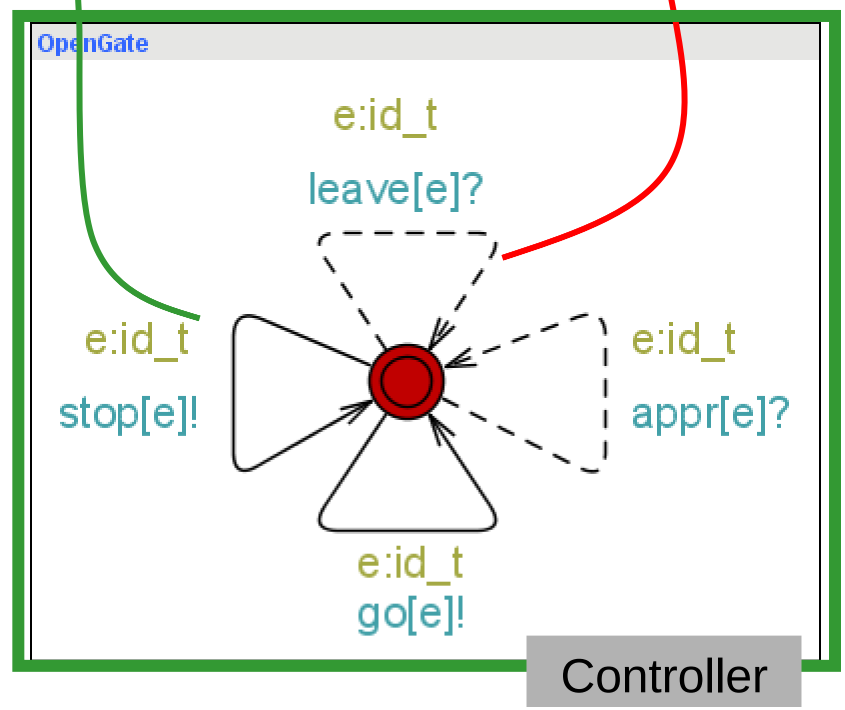
ϕ: Never two trains at the crossing at the same time

Synthesis

Two Player Game

Controllable Uncontrollable

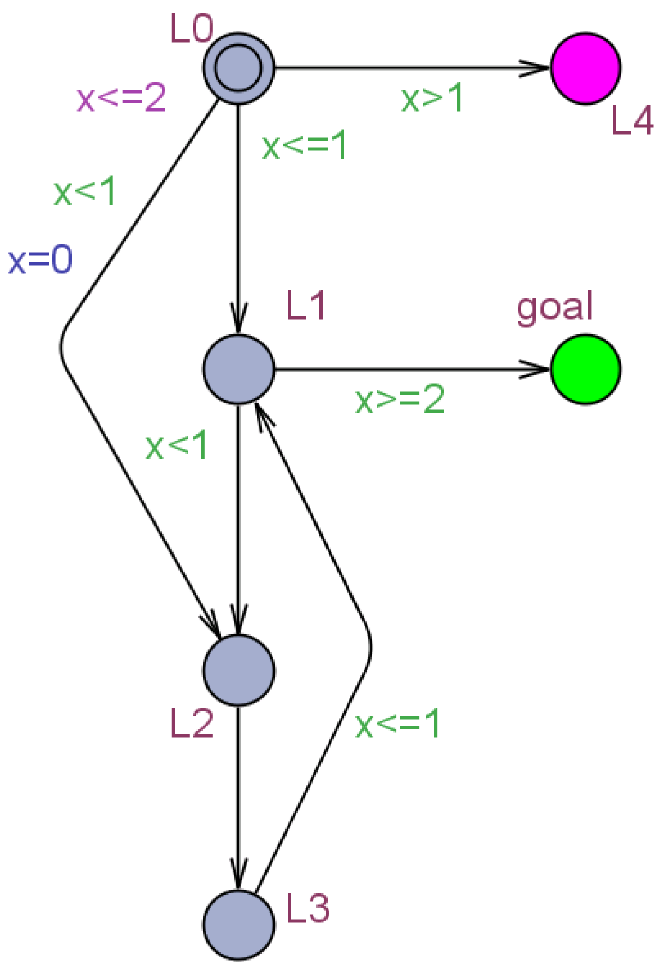




Find strategy for controllable actions st behaviour satisfies ϕ

ϕ: Never two trains at the crossing at the same time

Timed Automata & Model Checking



State (L1, x=0.81)

Transitions

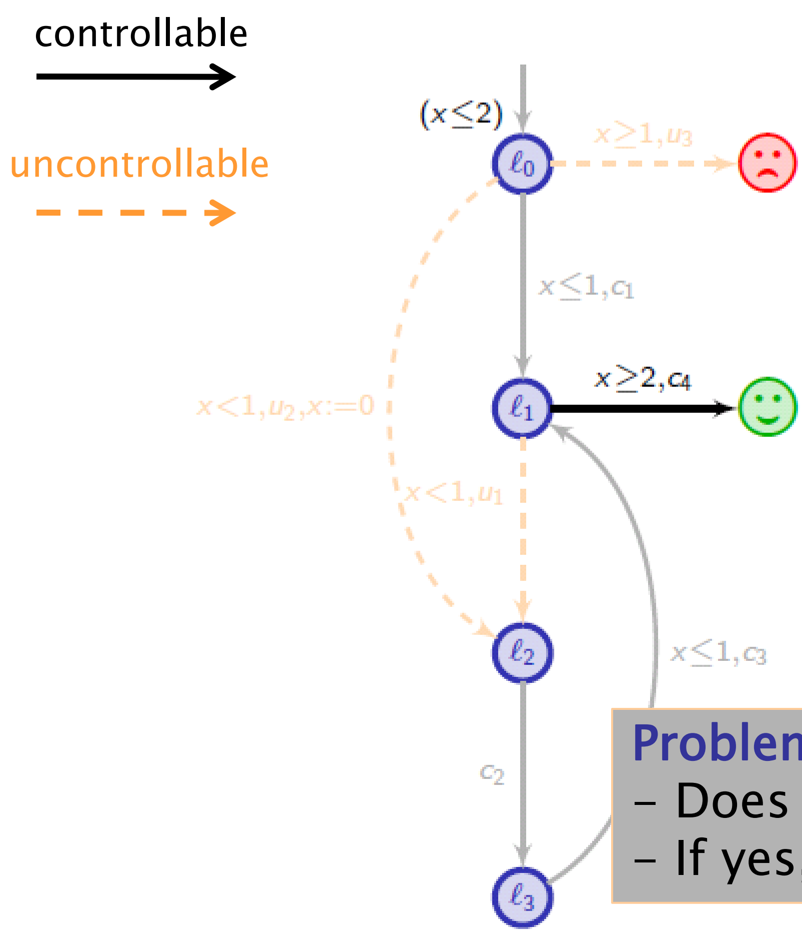
(L1 , x=0.81)
– 2.1 –>
(L1 , x=2.91)
–>
(goal , x=2.91)

E⟨⟩ goal ?

A⟨⟩ goal ?

A[ ] ¬ L4 ?

Timed Game Automata & Synthesis



A (memoryless) winning strategy

- from $(\ell_0, 0)$, play $(0.5, c_1)$

  ⇝ can be preempted by u_2

Problems to be considered:

- Does there exist a winning strategy?
- If yes, compute one (as simple as possible)

Decidability of Timed Games

Theorem [AMPS98,HK99]

Reachability and safety timed games are decidable and EXPTIME-complete. Furthermore memoryless and "region-based" strategies are sufficient.

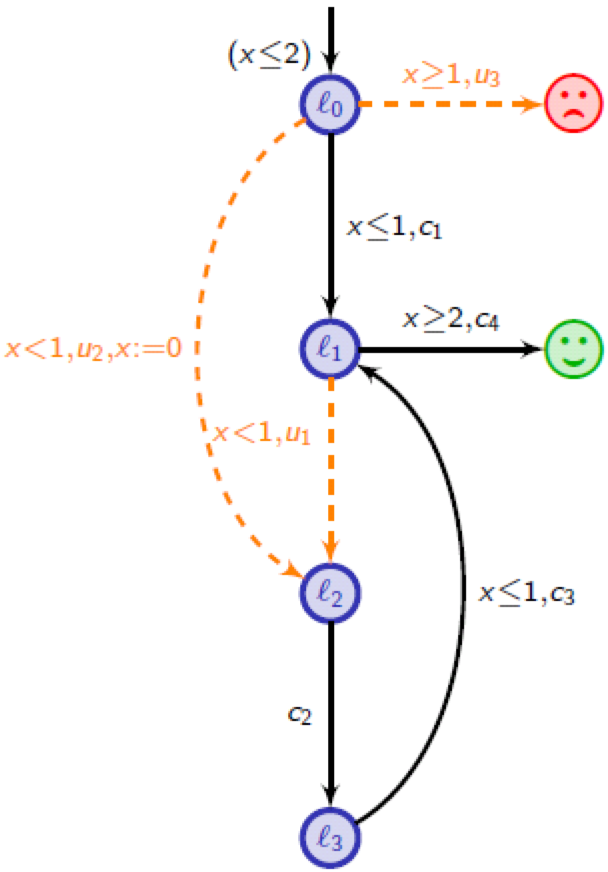
$\rightsquigarrow$ classical regions are sufficient for solving such problems

Theorem [AM99,BHPR07,JT07]

Optimal-time reachability timed games are decidable and EXPTIME-complete.

[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdziński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

Computing Winning States



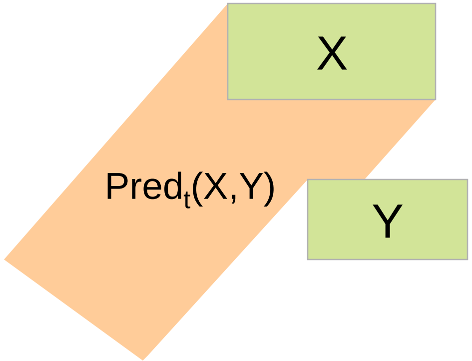
Reachability Games

Backwards Fixed-Point Computation

Definitions

$$\text{cPred}(X) = \{ q \in Q \mid \exists q' \in X.\ q \rightarrow_c q' \}$$
$$\text{uPred}(X) = \{ q \in Q \mid \exists q' \in X.\ q \rightarrow_u q' \}$$
$$\text{Pred}_t(X,Y) = \{ q \in Q \mid \exists t.\ q^t \in X \ \text{ and } \ \forall s \leq t.\ q^s \in Y^C \}$$

$$\pi(X) = \text{Pred}_t[\, X \cup \text{cPred}(X)\ ,\ \text{uPred}(X^C)\,]$$



Theorem:

The set of winning states is obtained as the least fixpoint of the function: $X \mapsto \pi(X) \cup \text{Goal}$

Symbolic On-the-fly Algorithms for Timed Games [CDF+05, BCD+07]

- $S, S', \ldots$
  are *symbolic* states, i.e. sets of concrete states;
- G
  is the set of (concrete) goal states;
- $E = \{S \xrightarrow{c} S', S \xrightarrow{u} S'\}$
  the (finite) set of symbolic transitions (controlla
- $Waiting \subseteq E$
  is the list of symbolic transitions waiting to be p
- $Passed$
  is the list of the passed symbolic states;
- $Win[S] \subseteq S$
  is the subset of S currently known to be winnin
- $Depend[S] \subseteq E$
  indicates the edges (predecessors) of S which mu
  information about S is obtained.

symbolic version of on-the-fly MC algorithm
for modal mu-calculus
Liu & Smolka 98

Initialization:

$Passed \leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, \vec{0})\}^{\nearrow}$;
$Waiting \leftarrow \{(S_0, \alpha, S') \mid S' = \mathsf{Post}_\alpha(S_0)^{\nearrow}\}$;
$Win[S_0] \leftarrow S_0 \cap (\{\mathsf{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
$Depend[S_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge (s_0 \notin Win[S_0]))$ **do**
  $e = (S, \alpha, S') \leftarrow pop(Waiting)$;
  if $S' \notin Passed$ **then**
    $Passed \leftarrow Passed \cup \{S'\}$;
    $Depend[S'] \leftarrow \{(S, \alpha, S')\}$;
    $Win[S'] \leftarrow S' \cap (\{\mathsf{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
    $Waiting \leftarrow Waiting \cup \{(S', \alpha, S'') \mid S'' = \mathsf{Post}_\alpha(S')^{\nearrow}\}$;
    if $Win[S'] \neq \emptyset$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
  e (* reevaluate *)[a]
    $Win^* \leftarrow \mathsf{Pred}_t(Win[S] \cup \bigcup_{S \xrightarrow{c} T} \mathsf{Pred}_c(Win[T]),$
      $\bigcup_{S \xrightarrow{u} T} \mathsf{Pred}_u(T \setminus Win[T])) \cap S$;
    if $(Win[S] \subsetneq Win^*)$ **then**
      $Waiting \leftarrow Waiting \cup Depend[S]$; $Win[S] \leftarrow Win^*$;
    $Depend[S'] \leftarrow Depend[S'] \cup \{e\}$;
  endif
endwhile

[CDF+05] Cassez, David, Fleury, Larsen, Lime. Efficient on-the-fly algorithms for the analysis of timed games *(CONCUR'05)*.
[BCD+07] Berhmann, Cougnard, David, Fleury, Larsen, Lime. Uppaal-Tiga: Time for playing games! *(CAV'07)*.

UPPAAL Tiga [CDF+05, BCD+07]

- Reachability properties:
  - control: A[p U q] *until*
  - control: A⟨⟩ q ⇔ control: A[true U q]
- Safety properties:
  - control: A[p W q] *weak until*
  - control: A[] p ⇔ control: A[p W false]
- Time-optimality :
  - control_t*(u,g): A[p U q]
    - u is an upper-bound to prune the search
    - g is the time to the goal from the current state

[CDF+05] Cassez, David, Fleury, Larsen, Lime. Efficient on-the-fly algorithms for the analysis of timed games *(CONCUR'05)*.
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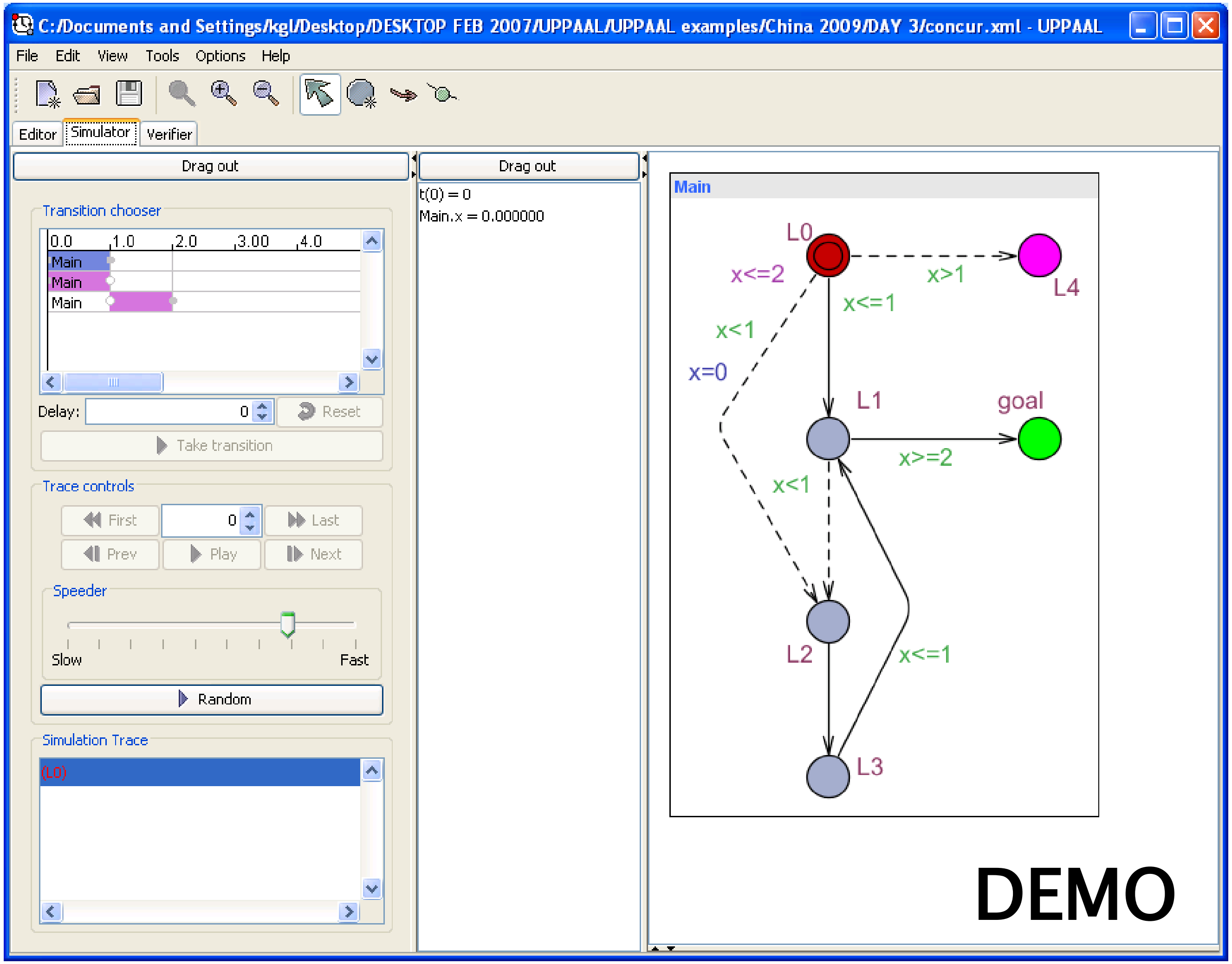
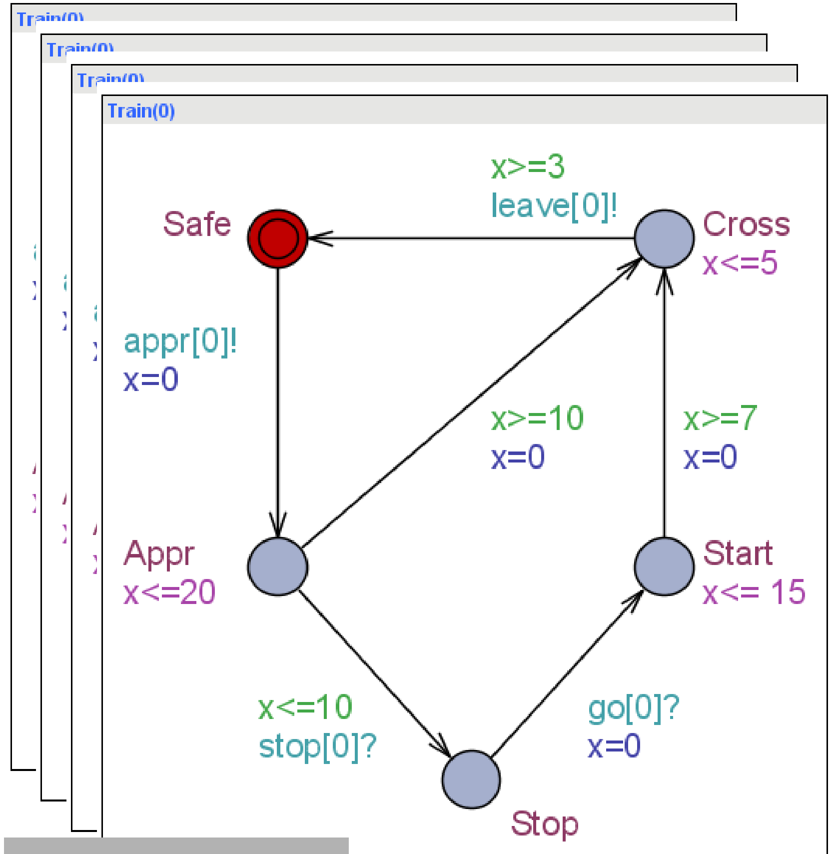
UPPAAL Tiga

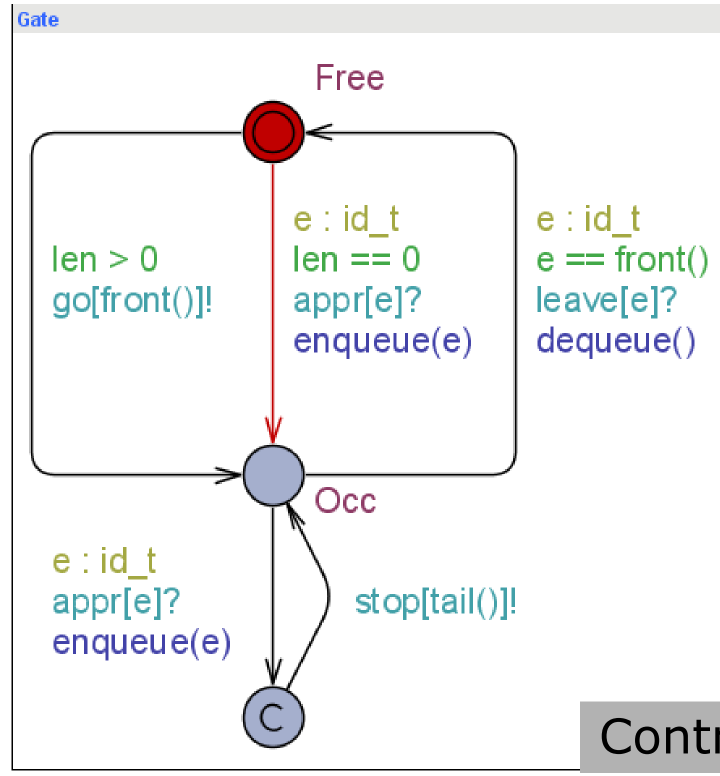


C:/Documents and Settings/kgl/Desktop/DESKTOP FEB 2007/UPPAAL/UPPAAL examples/China 2009/DAY 3/concur.xml - UPPAAL
File Edit View Tools Options Help
Editor Simulator Verifier
Drag out
Transition chooser
Delay:
Reset
Take transition
Trace controls
First Last Prev Play Next
Speeder
Slow Fast
Random
Simulation Trace
(L0)
Drag out
t(0) = 0
Main.x = 0.000000
Main
L0
x<=2
x>1
L4
x<=1
x<1
x=0
L1
goal
x>=2
x<1
L2
x<=1
L3
DEMO

Model Checking (ex Train Gate)



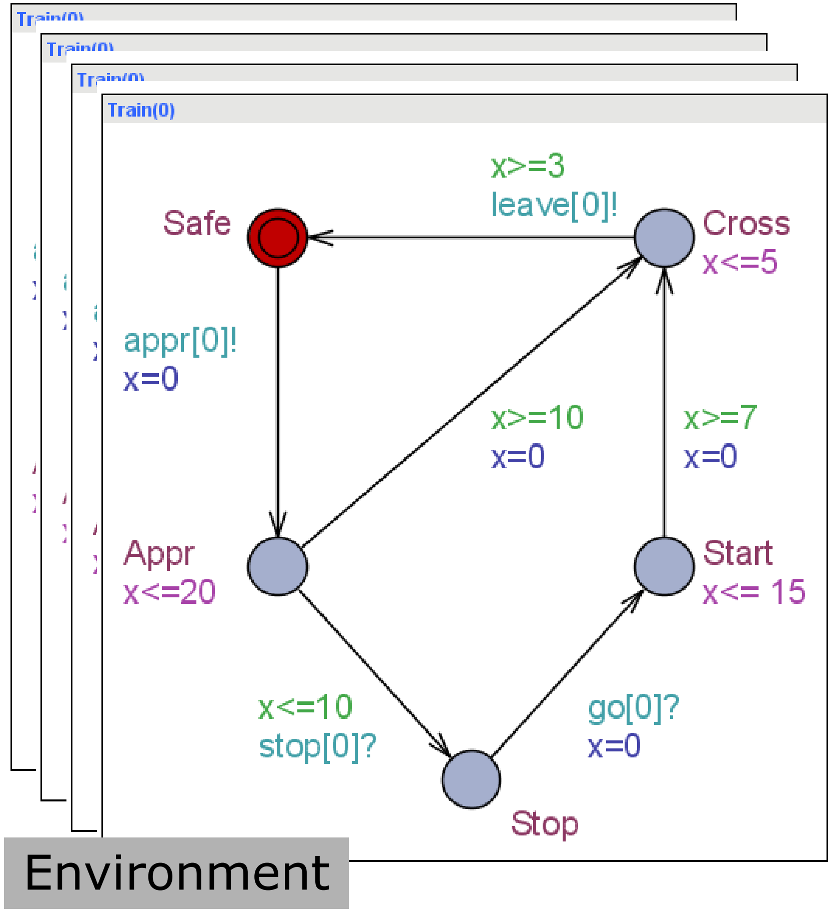
Environment

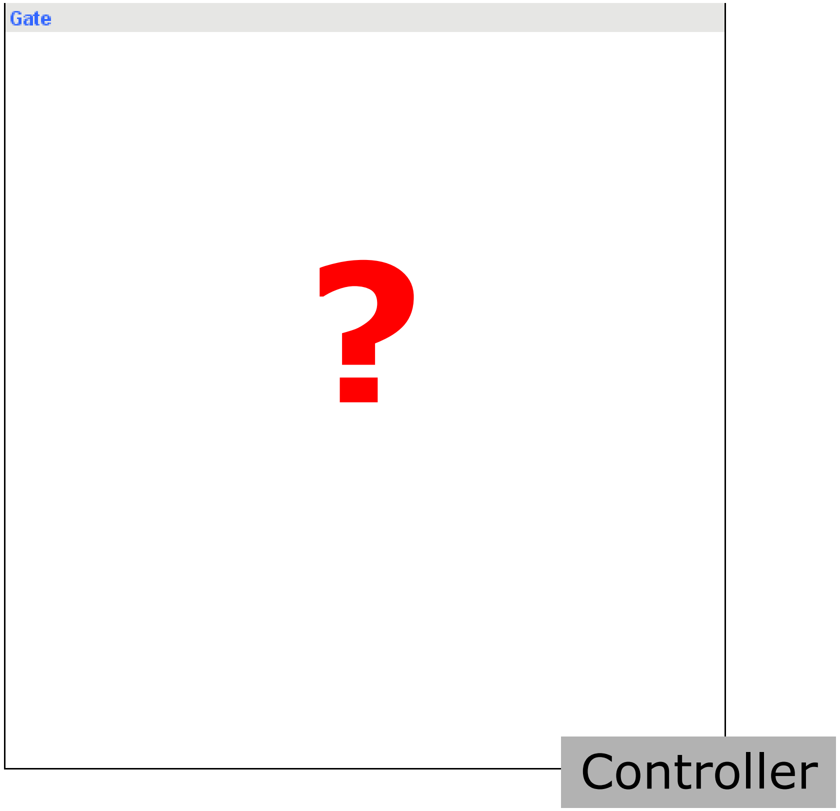


Controller

ϕ: Never two trains at the crossing at the same time

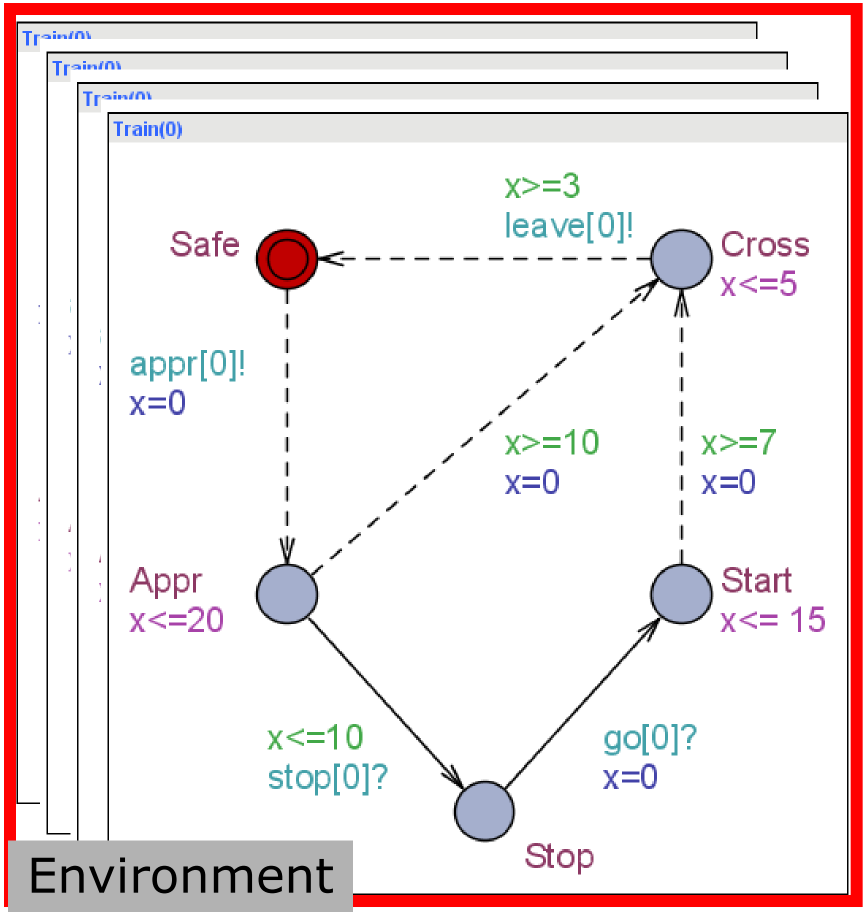
Synthesis (ex Train Gate)

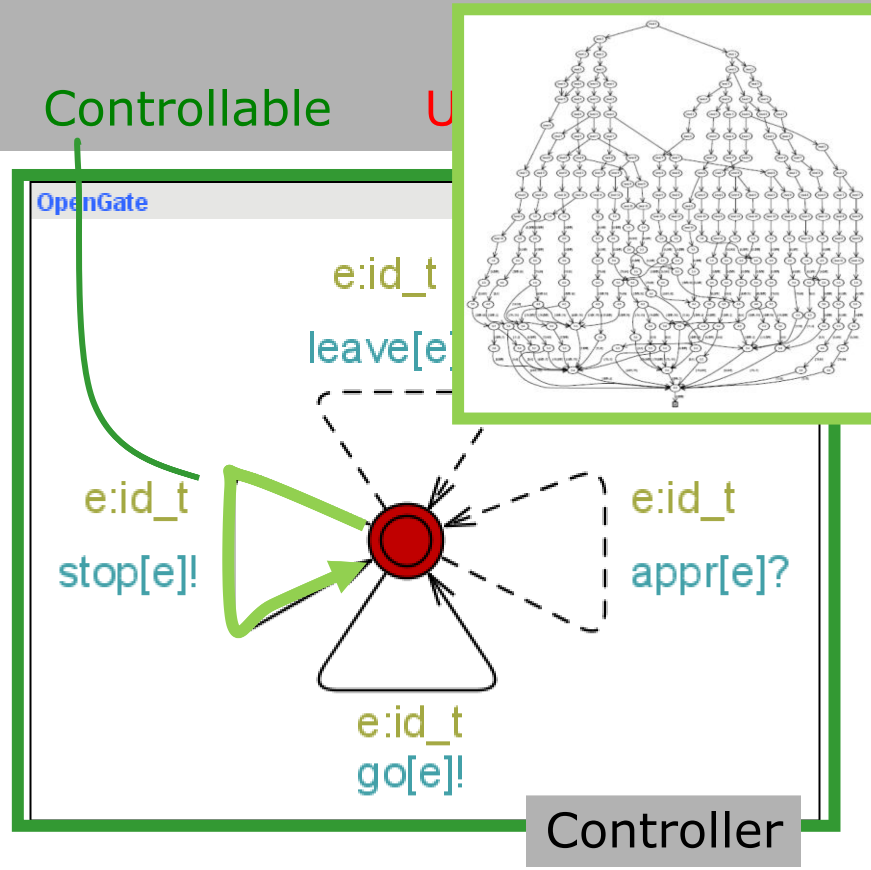




ϕ: Never two trains at the crossing at the same time

Timed Games





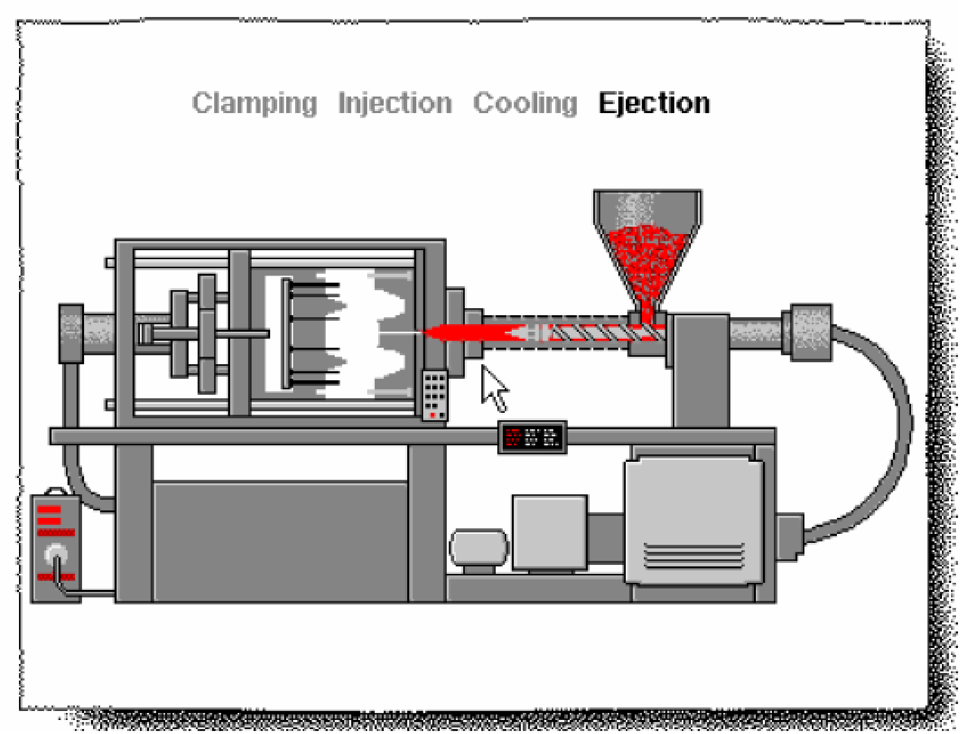
Find strategy for controllable actions st behaviour satisfies ϕ

ϕ: Never two trains at the crossing at the same time

Plastic Injection Molding Machine

Quasimodo

[CJL+09]

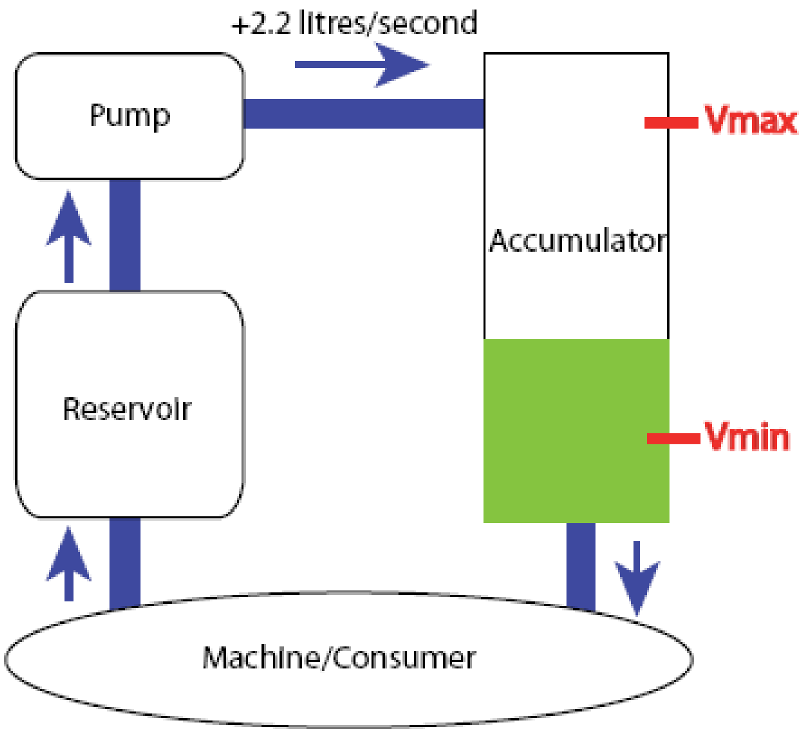


HYDAC

- Robust and optimal control
- Tool Chain
  - Synthesis: UPPAAL TIGA
  - Verification: PHAVer
  - Performance: SIMULINK
- 40% improvement of existing solutions..

[CJL+09] Cassez, Jessen, Larsen, Raskin, Reynier.Automatic Synthesis of Robust and Optimal Controllers – An Industrial Case Study *(HSCC'09)*.

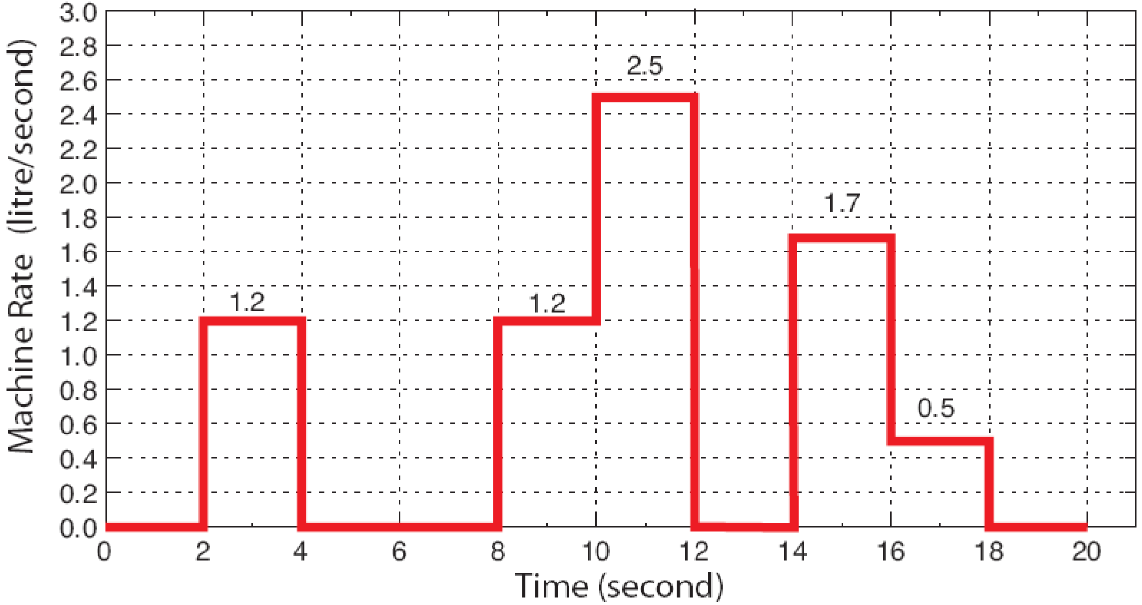
Oil Pump Control Problem



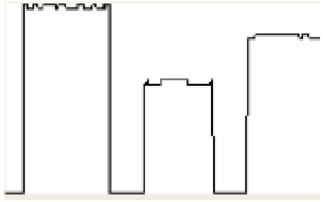
- R1: stay within safe interval [4.9,25.1]
- R2: minimize average/overall oil volume

$$\int_{t=0}^{t=T} v(t)dt \, / \, T$$

The Machine (consumption)



- Infinite cyclic demand to be satisfied by our control strategy.
- P: latency 2 s between state change of pump
- F: noise 0.1 l/s



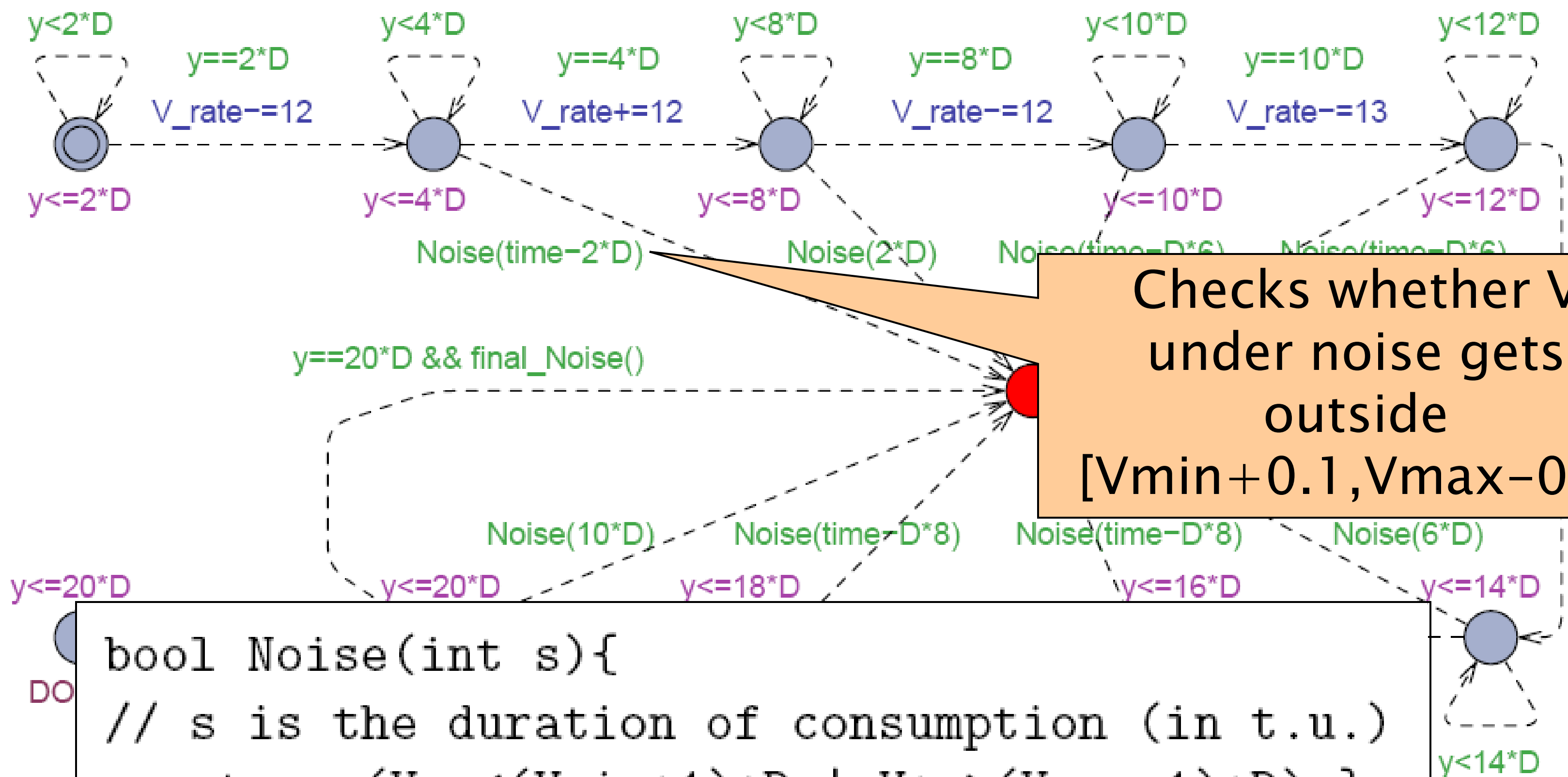
Abstract Game Model

- UPPAAL Tiga
  offers games of perfect information

- Abstract game model such that states only contain information about:
  - Volume of oil at the beginning of cycle
  - The ideal volume as predicted by the consumption cycle
  - Current time within the cycle
  - State of the Pump (on/off)
  - Discrete model

D
V, V_rate
V_acc
time

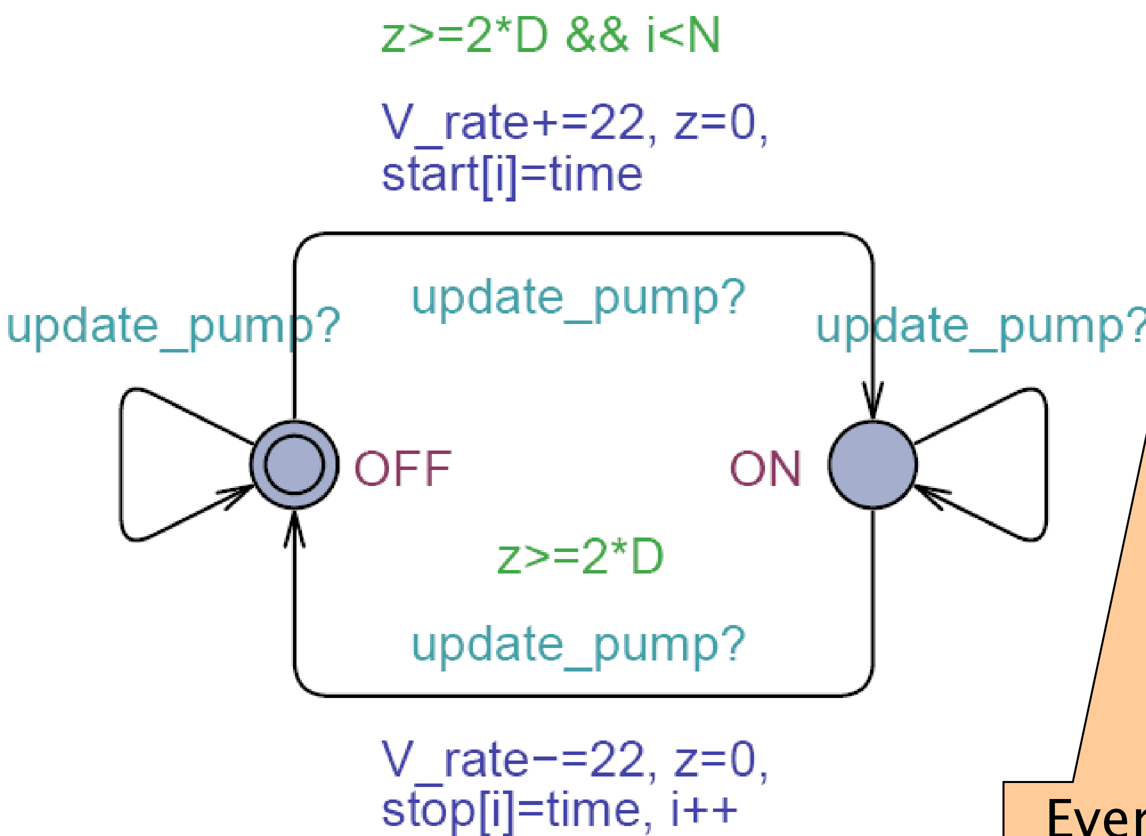
Machine (uncontrollable)



Checks whether V under noise gets outside [Vmin+0.1,Vmax-0.1]

```
bool Noise(int s){
// s is the duration of consumption (in t.u.)
  return (V-s<(Vmin+1)*D | V+s>(Vmax-1)*D);}
```

Pump (controllable)

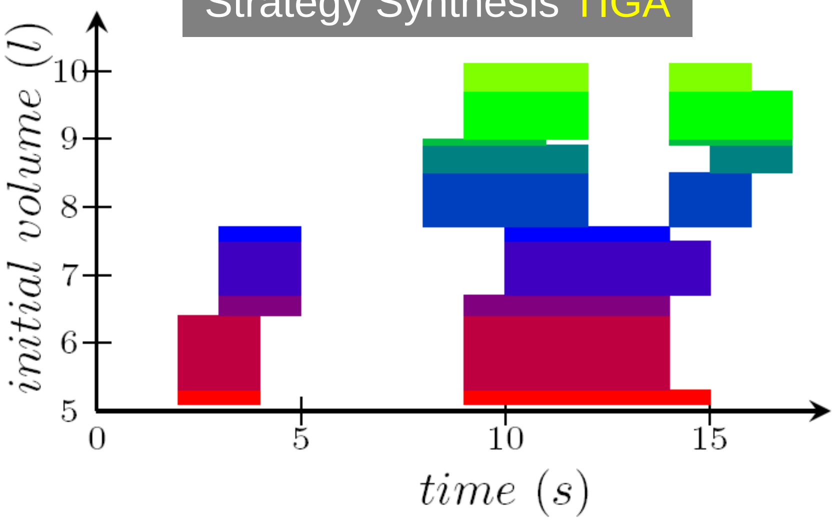


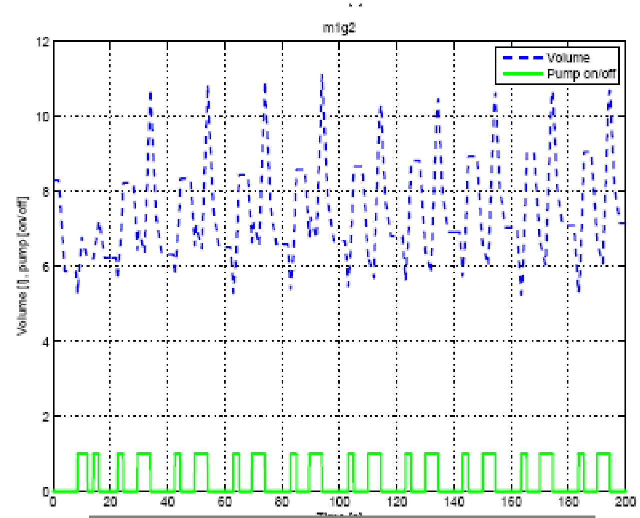

```
void update_val(){
   int V_pred = V;
   time++;
   V+=V_rate;
   V_acc+=V+V_pred;
}
```

Every 1 (one) seconds

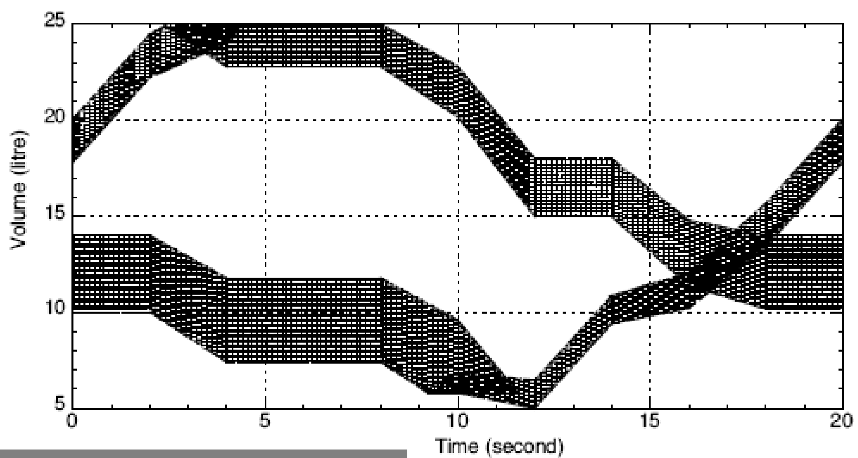
Tool Chain

Strategy Synthesis TIGA





Performance Evaluation SIMULINK



Verification PHAVER

Guaranteed
Correctness
Robustness

with
40% Improvement

LAB Exercises

www.cs.aau.dk/~kgl/Shanghai2013

Exercise 28 (Jobshop Scheduling Part 1)
Exercise 19 (Train Gate Part 1)