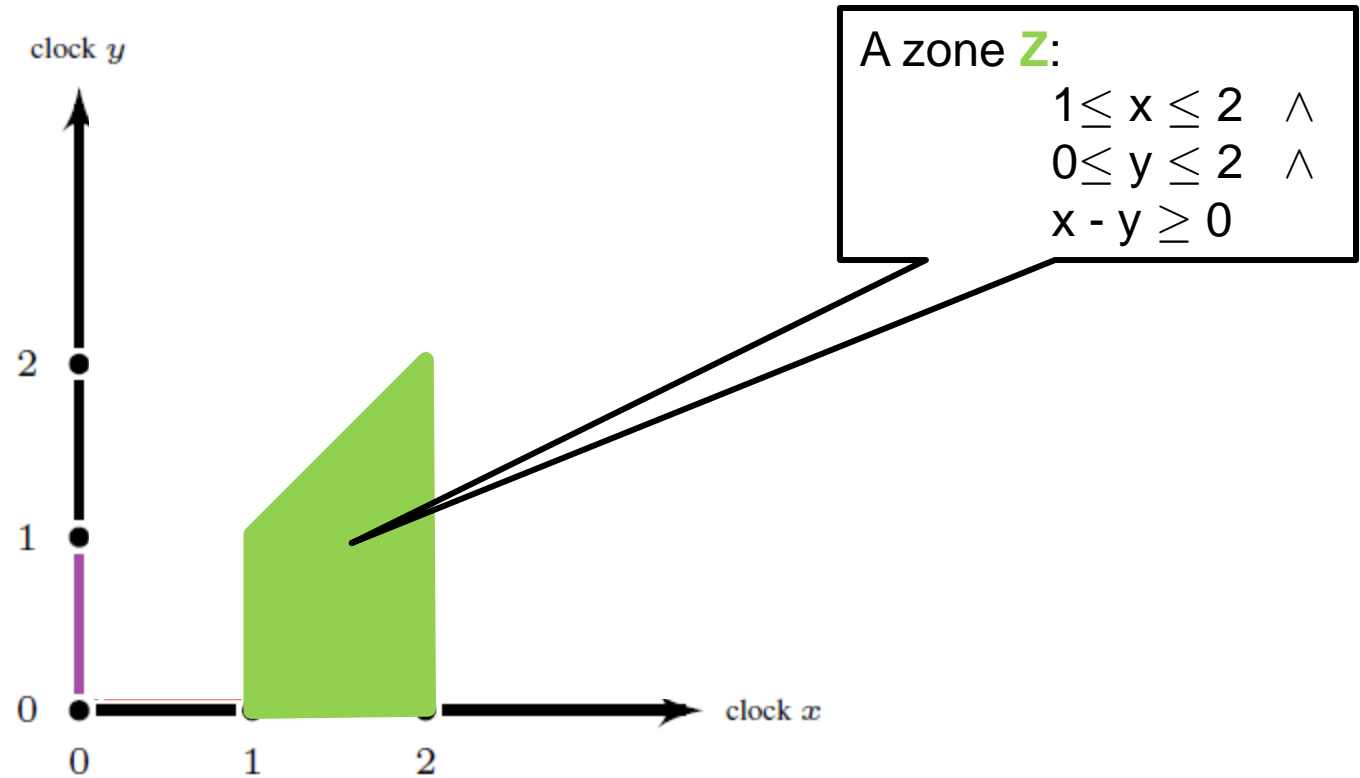


# Symbolic Verification

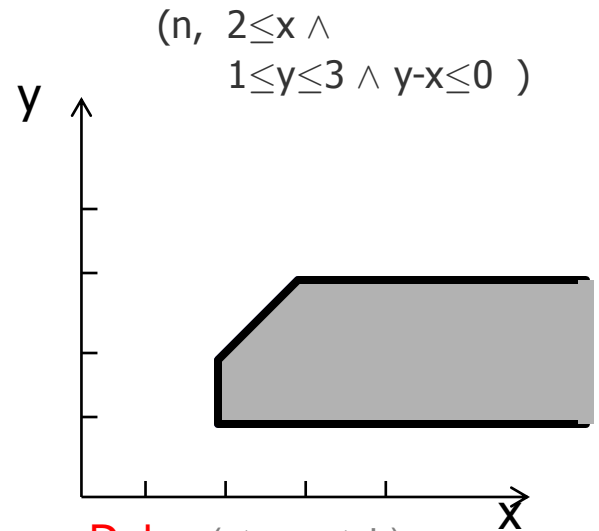
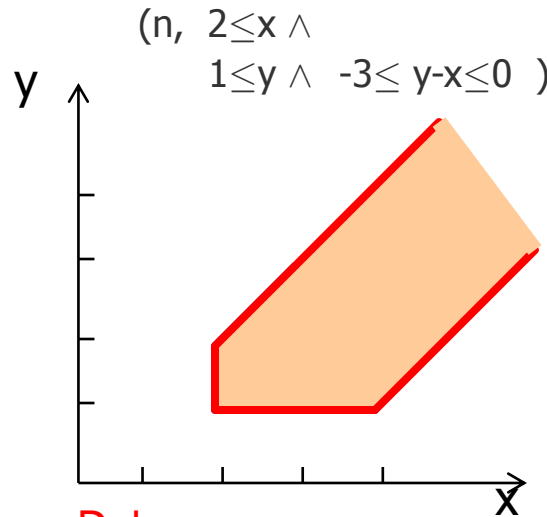
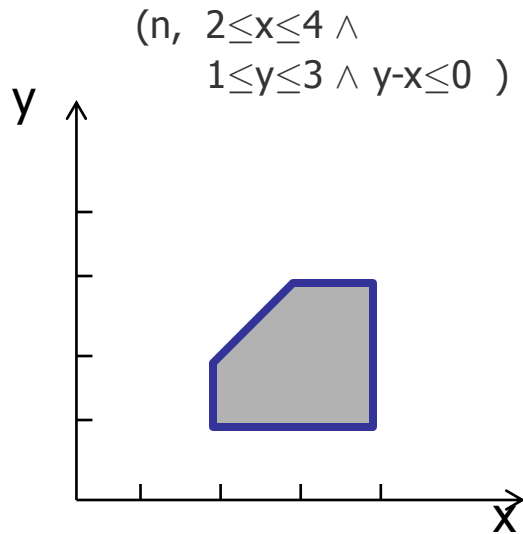
The UPPAAL Verification Engine



# Zones – From Finite to Efficiency

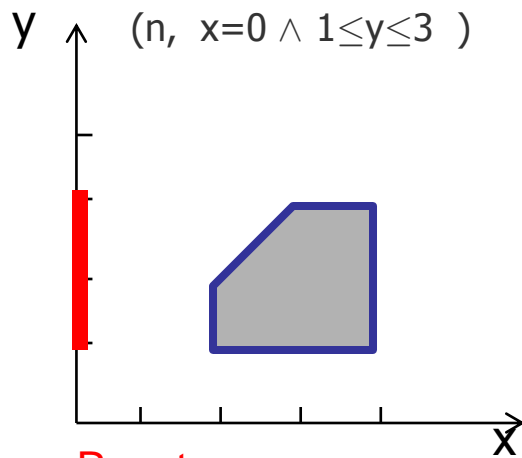


# Zones – Operations

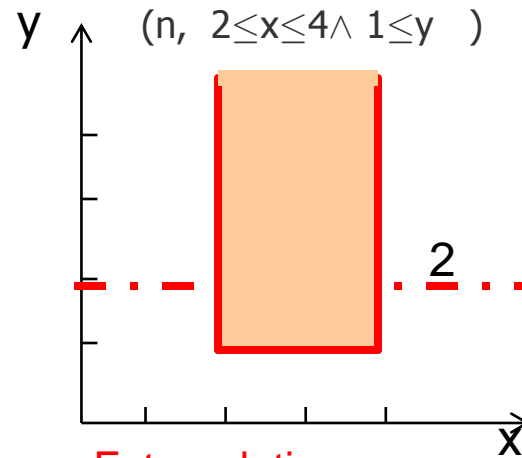


Delay

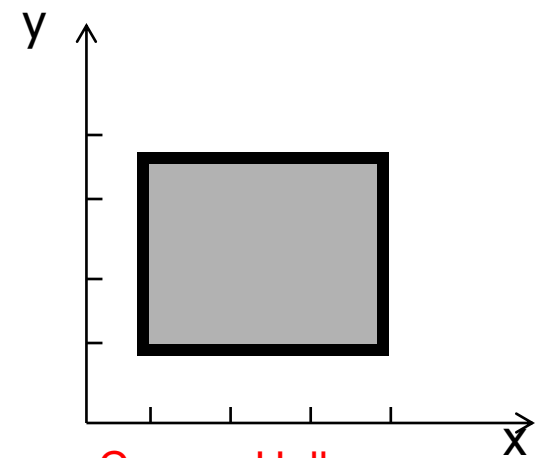
Delay (stopwatch)



Reset



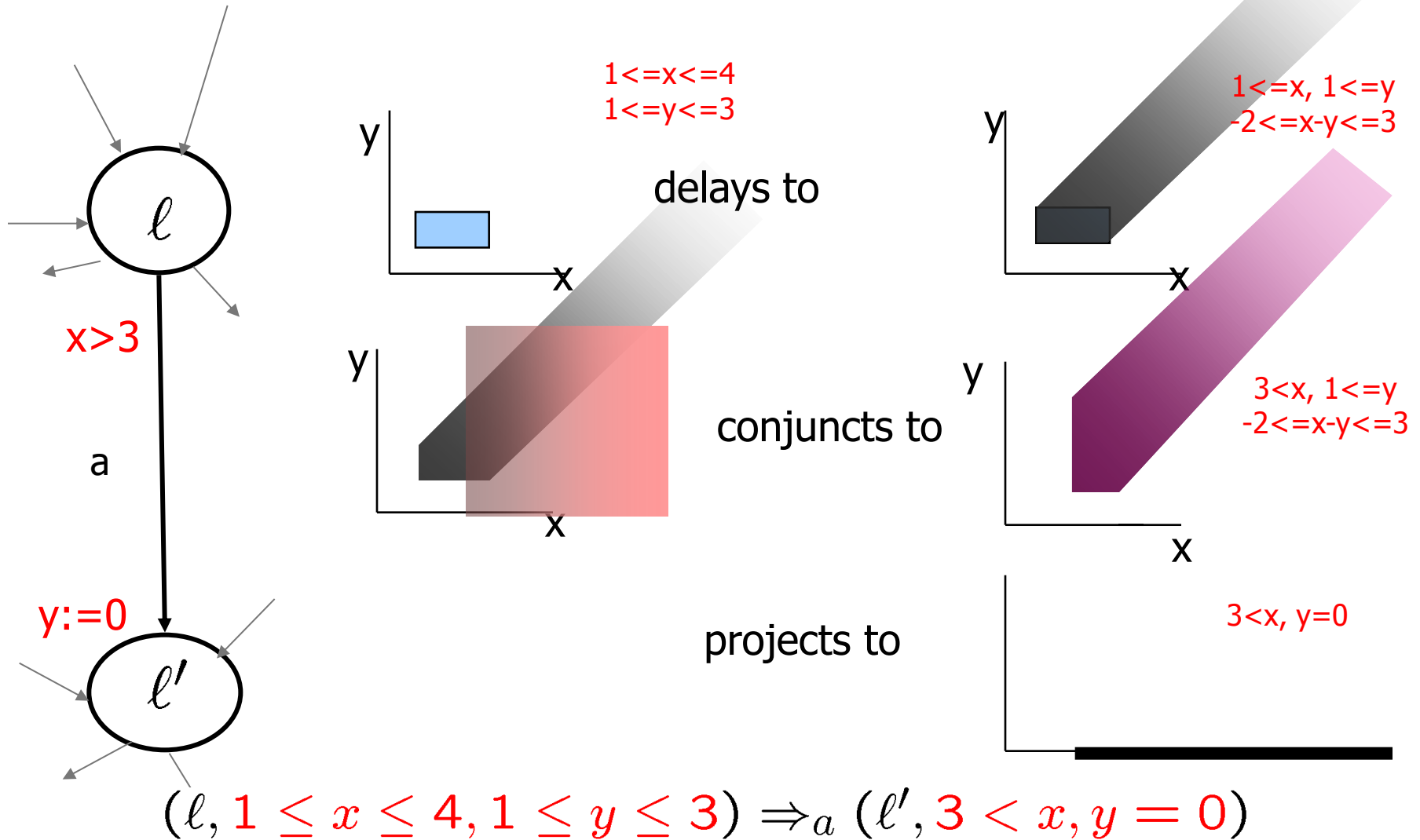
Extrapolation



Convex Hull

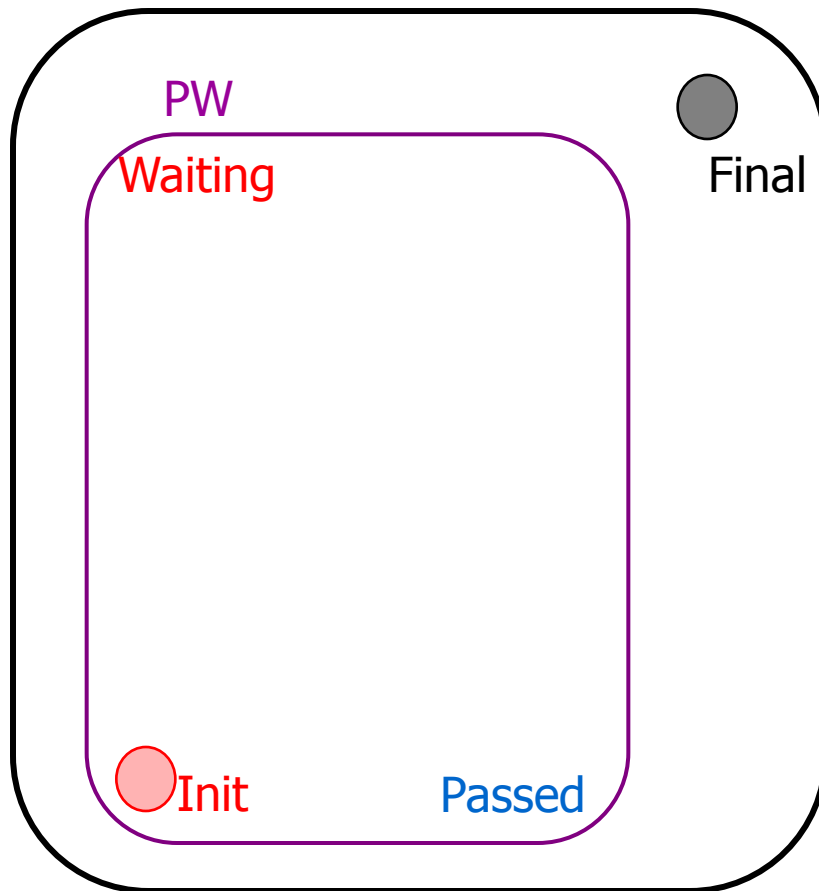


# Symbolic Transitions



# Forward Reachability

Init  $\rightarrow$  Final ?



INITIAL  $Passed := \emptyset;$   
 $Waiting := \{(n_0, Z_0)\}$

REPEAT

pick  $(n, Z)$  in  $Waiting$

if  $(n, Z) = Final$  return true

for all  $(n, Z) \rightarrow (n', Z')$ :

if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue

else add  $(n', Z')$  to  $Waiting$

move  $(n, Z)$  to  $Passed$

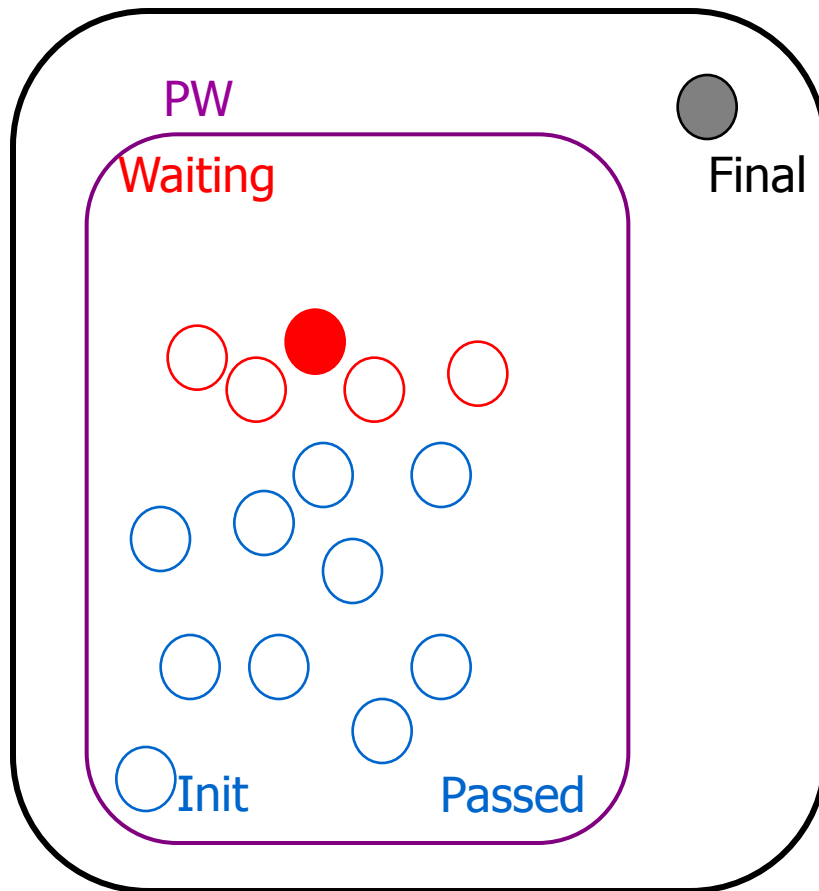
UNTIL  $Waiting = \emptyset$

return false



# Forward Reachability

Init  $\rightarrow$  Final ?



INITIAL **Passed** :=  $\emptyset$ ;  
**Waiting** :=  $\{(n_0, Z_0)\}$

REPEAT

pick  $(n, Z)$  in **Waiting**

if  $(n, Z) = \text{Final}$  return true

for all  $(n, Z) \rightarrow (n', Z')$ :

if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue

else add  $(n', Z')$  to **Waiting**

move  $(n, Z)$  to **Passed**

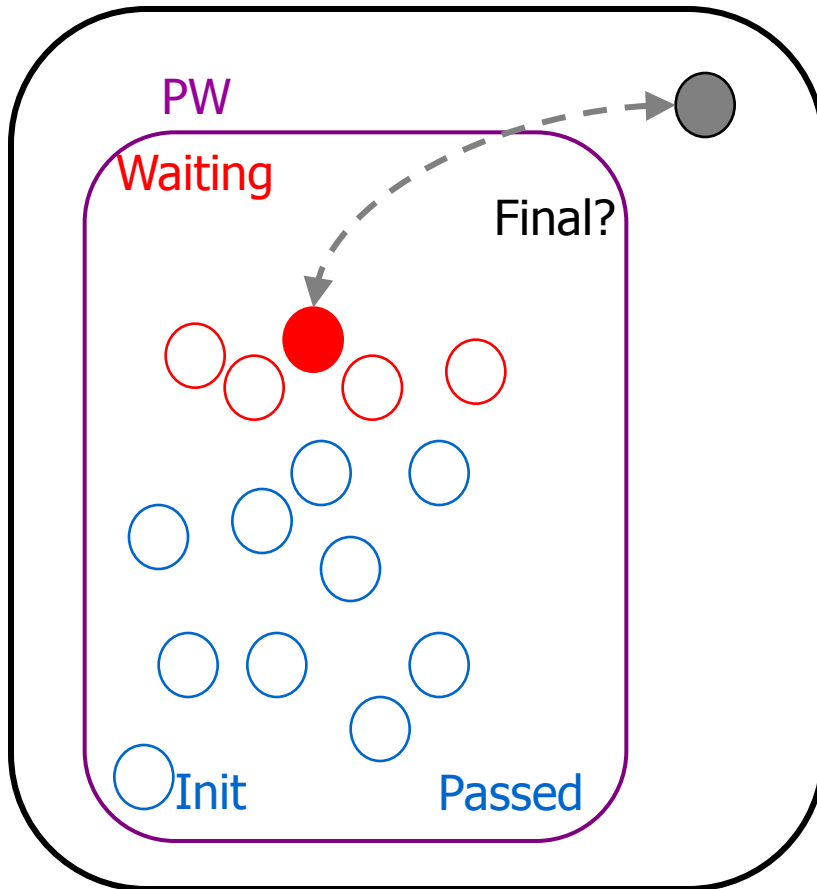
UNTIL **Waiting** =  $\emptyset$

return false



# Forward Reachability

Init  $\rightarrow$  Final ?



INITIAL **Passed** :=  $\emptyset$ ;  
**Waiting** :=  $\{(n_0, Z_0)\}$

REPEAT

pick  $(n, Z)$  in **Waiting**

if  $(n, Z) = \text{Final}$  return true

for all  $(n, Z) \rightarrow (n', Z')$ :

if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue

else add  $(n', Z')$  to **Waiting**

move  $(n, Z)$  to **Passed**

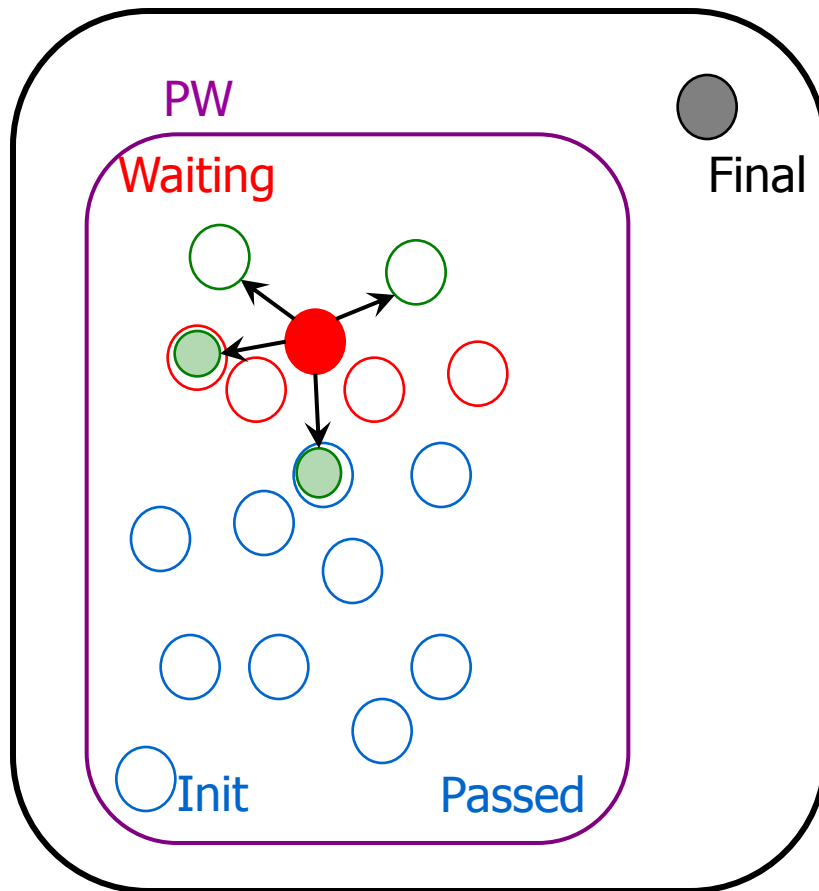
UNTIL **Waiting** =  $\emptyset$

return false



# Forward Reachability

Init  $\rightarrow$  Final ?



INITIAL **Passed** :=  $\emptyset$ ;  
**Waiting** :=  $\{(n_0, Z_0)\}$

REPEAT

pick  $(n, Z)$  in **Waiting**

if  $(n, Z) = \text{Final}$  return true

for all  $(n, Z) \rightarrow (n', Z')$ :

if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue

else add  $(n', Z')$  to **Waiting**

move  $(n, Z)$  to **Passed**

UNTIL **Waiting** =  $\emptyset$

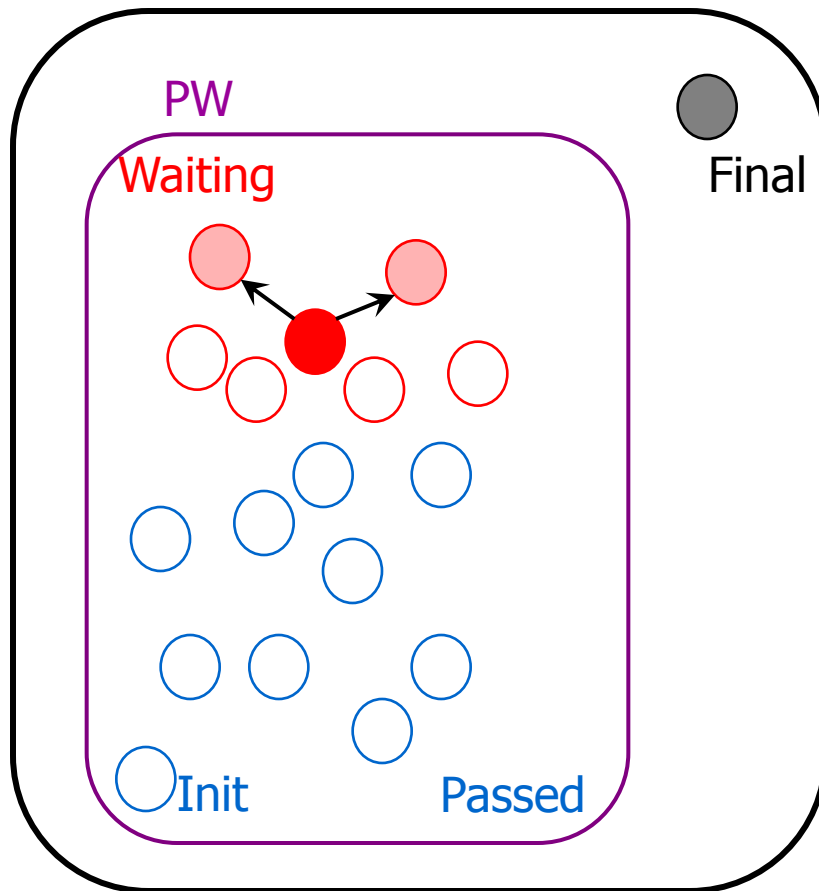
return false





# Forward Reachability

Init  $\rightarrow$  Final ?



INITIAL **Passed** :=  $\emptyset$ ;  
**Waiting** :=  $\{(n_0, Z_0)\}$

REPEAT

pick  $(n, Z)$  in **Waiting**

if  $(n, Z) = \text{Final}$  return true

for all  $(n, Z) \rightarrow (n', Z')$ :

if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue

else add  $(n', Z')$  to **Waiting**

move  $(n, Z)$  to **Passed**

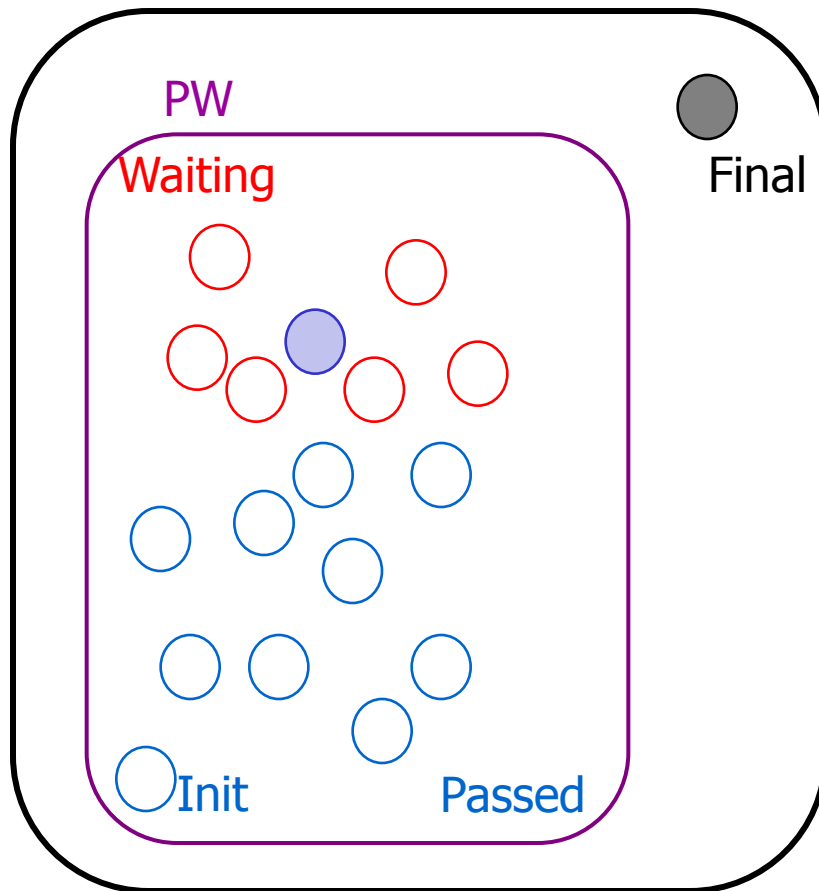
UNTIL **Waiting** =  $\emptyset$

return false



# Forward Reachability

Init  $\rightarrow$  Final ?



INITIAL **Passed** :=  $\emptyset$ ;  
**Waiting** :=  $\{(n_0, Z_0)\}$

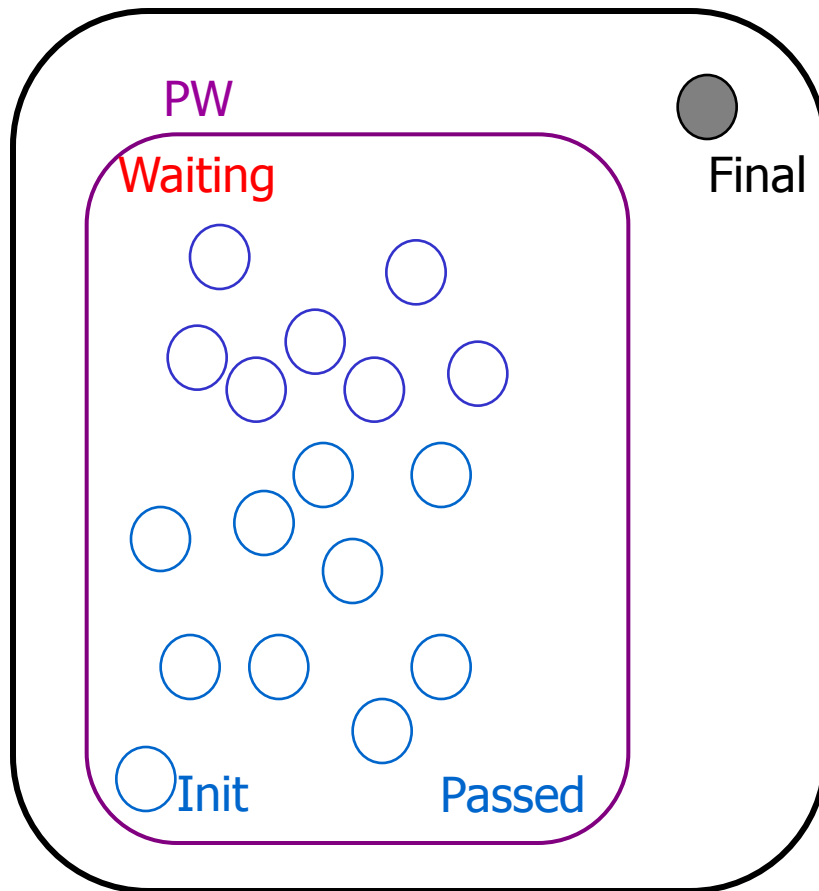
REPEAT  
pick  $(n, Z)$  in **Waiting**  
if  $(n, Z) = \text{Final}$  return true  
for all  $(n, Z) \rightarrow (n', Z')$ :  
if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue  
else add  $(n', Z')$  to **Waiting**  
move  $(n, Z)$  to **Passed**

UNTIL **Waiting** =  $\emptyset$   
return false



# Forward Reachability

Init  $\rightarrow$  Final ?



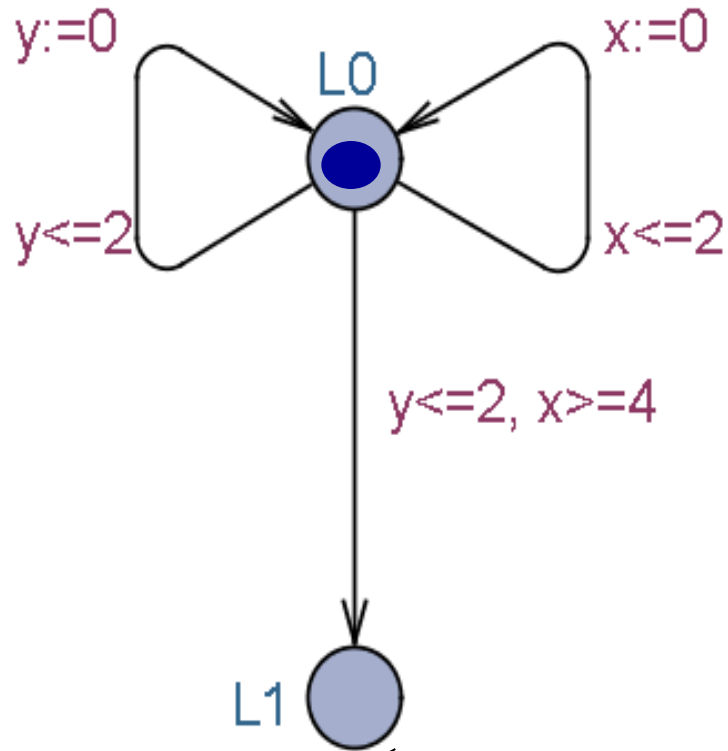
INITIAL **Passed** :=  $\emptyset$ ;  
**Waiting** :=  $\{(n_0, Z_0)\}$

REPEAT  
pick  $(n, Z)$  in **Waiting**  
if  $(n, Z) = \text{Final}$  return true  
for all  $(n, Z) \rightarrow (n', Z')$ :  
if for some  $(n', Z'')$   $Z' \subseteq Z''$  continue  
else add  $(n', Z')$  to **Waiting**  
move  $(n, Z)$  to **Passed**

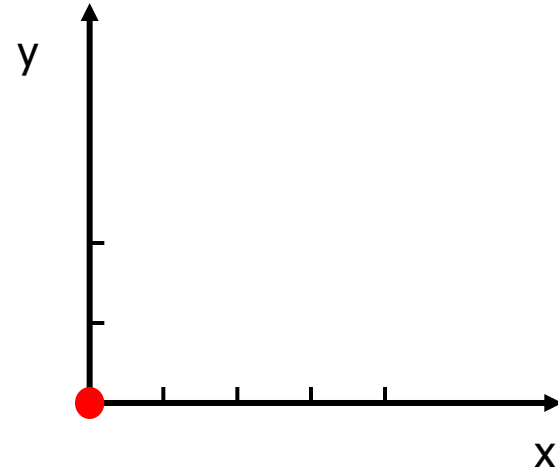
UNTIL **Waiting** =  $\emptyset$   
return false



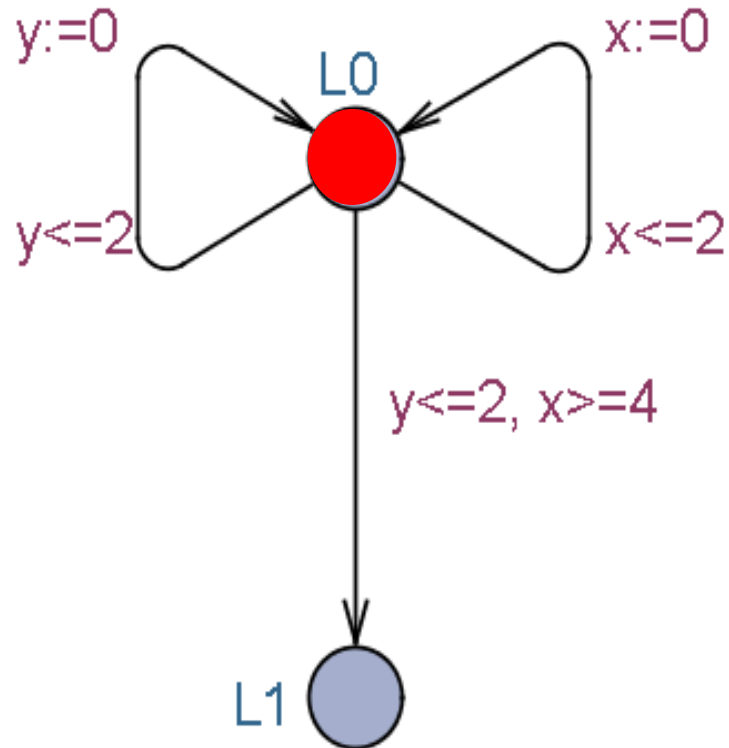
# Symbolic Exploration



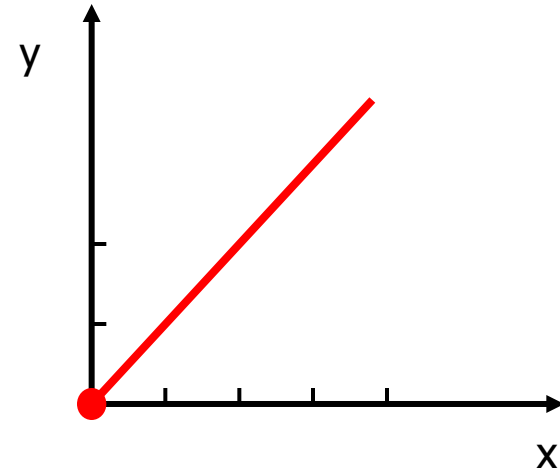
Reachable?



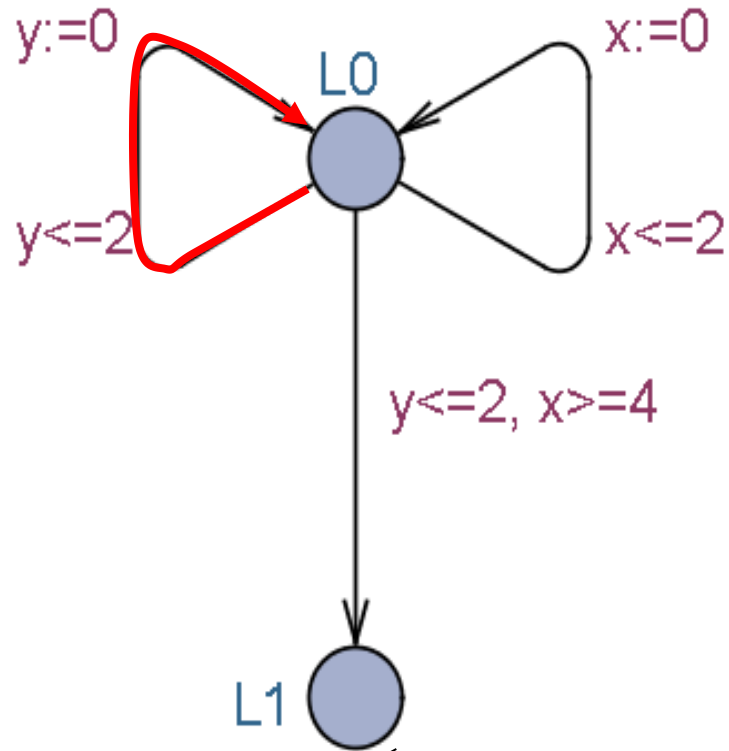
# Symbolic Exploration



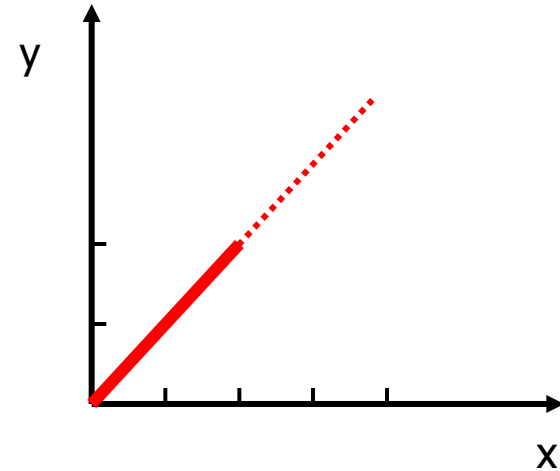
Reachable?



# Symbolic Exploration



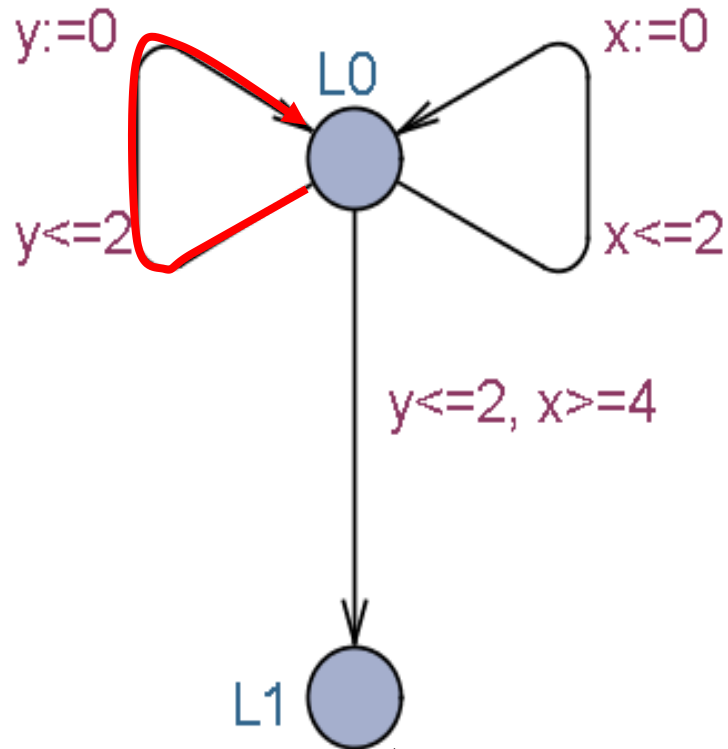
Reachable?



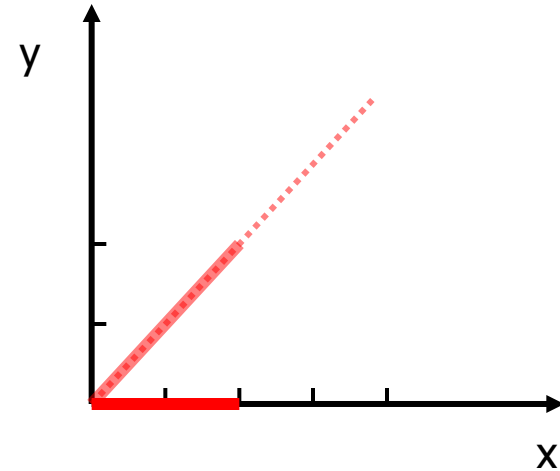
Left



# Symbolic Exploration



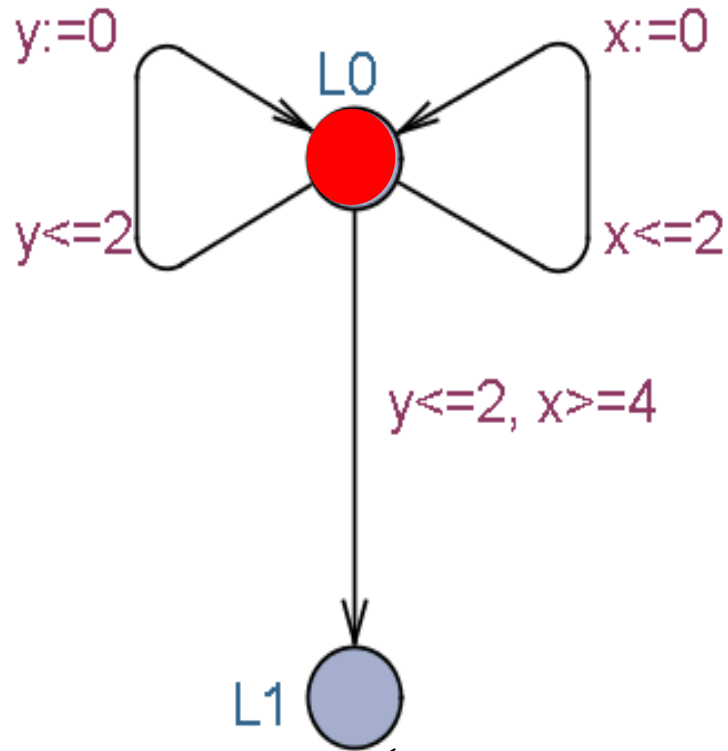
Reachable?



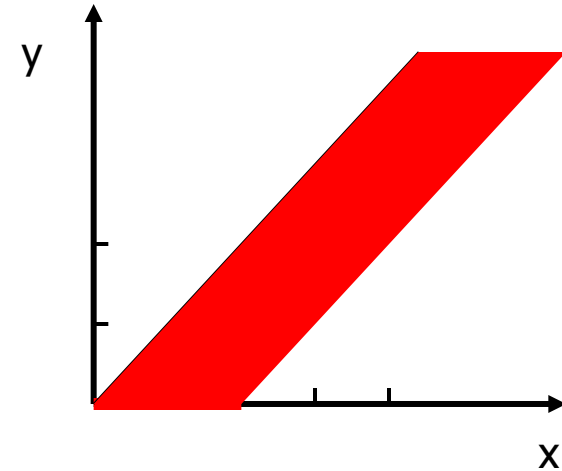
Left



# Symbolic Exploration

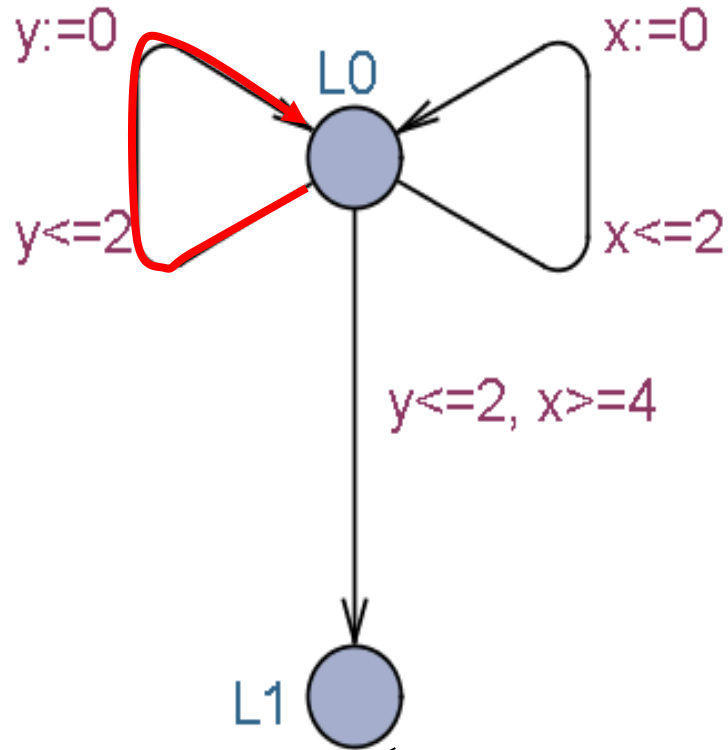


Reachable?

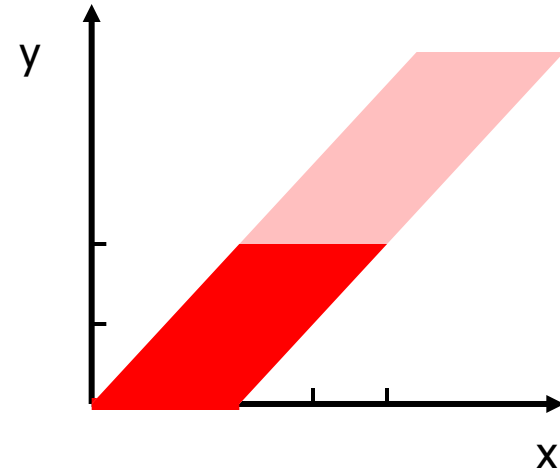




# Symbolic Exploration



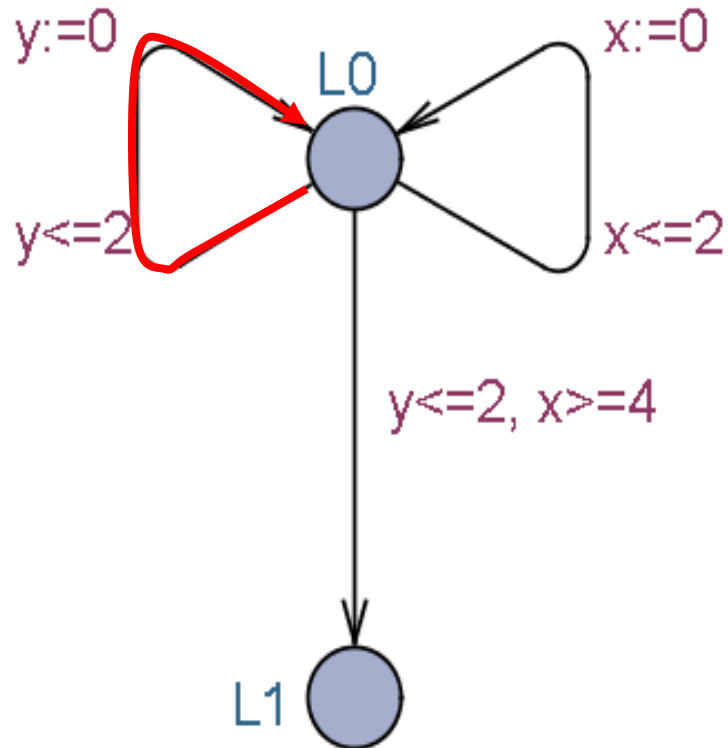
Reachable?



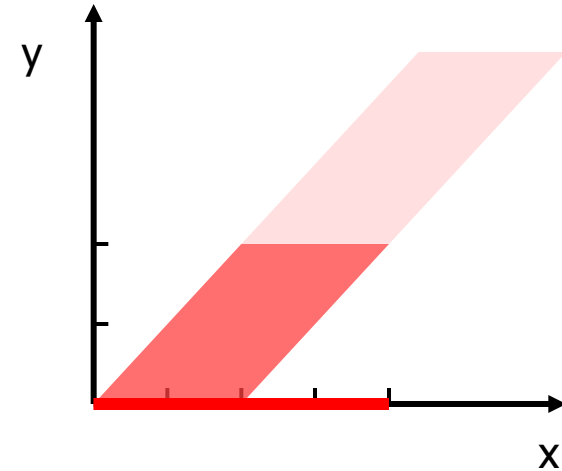
Left



# Symbolic Exploration



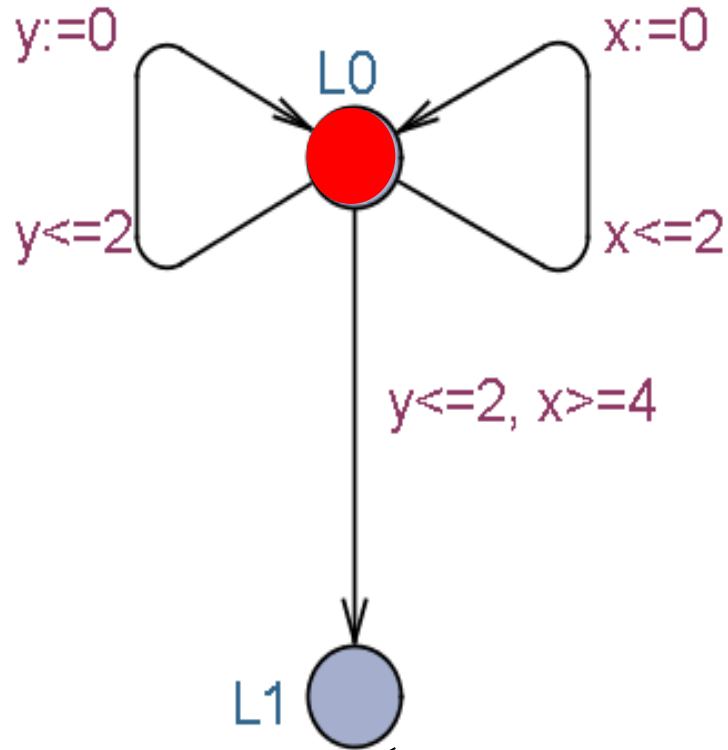
Reachable?



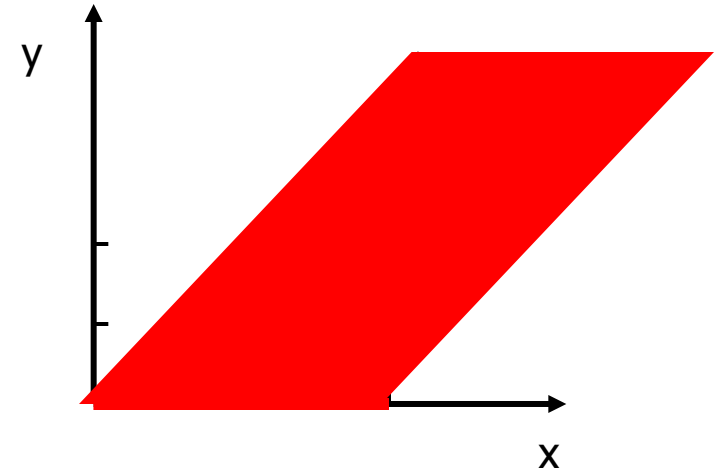
Left



# Symbolic Exploration



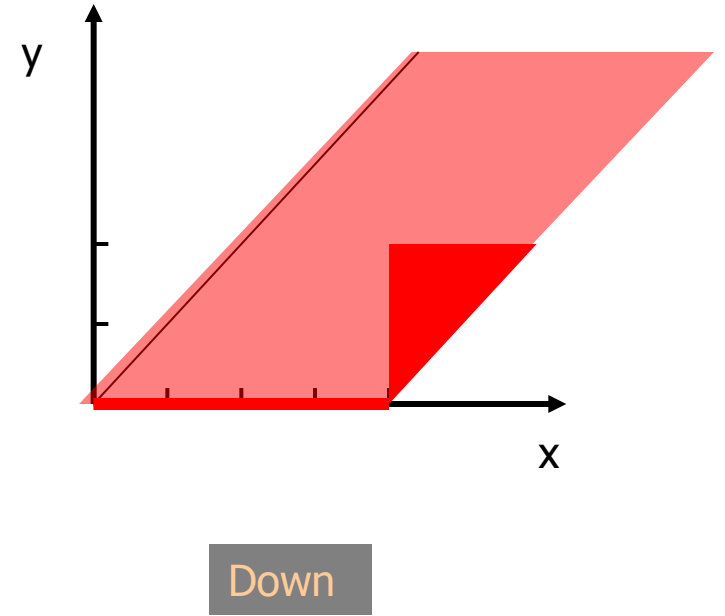
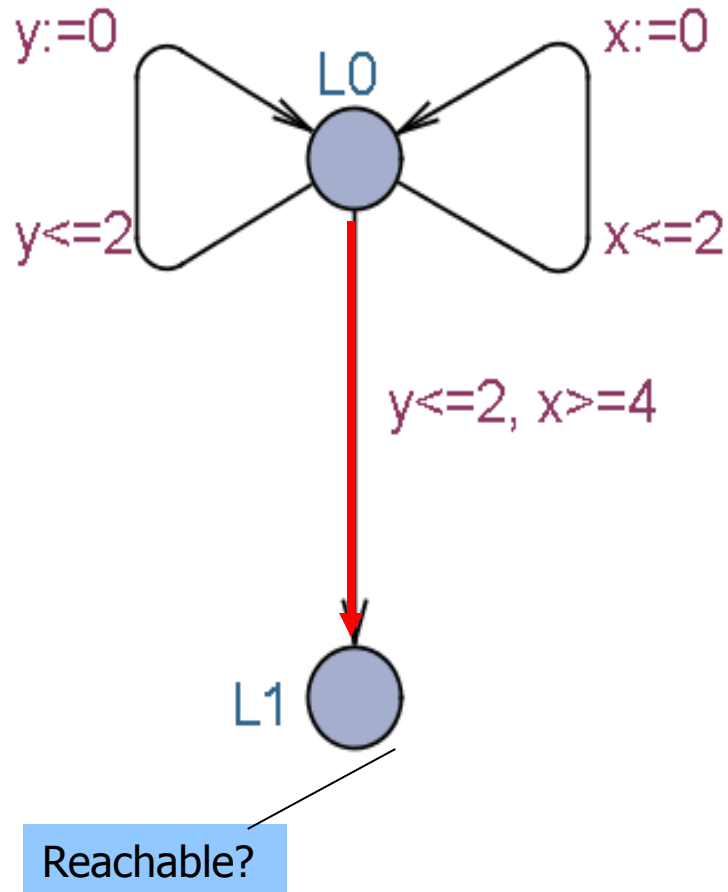
Reachable?



Delay



# Symbolic Exploration

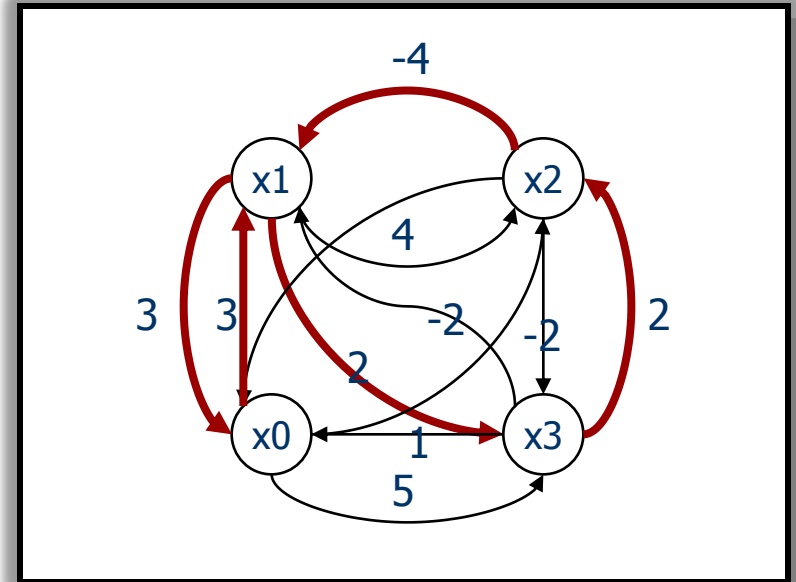


# Datastructures for Zones

- Difference Bounded Matrices (DBMs)

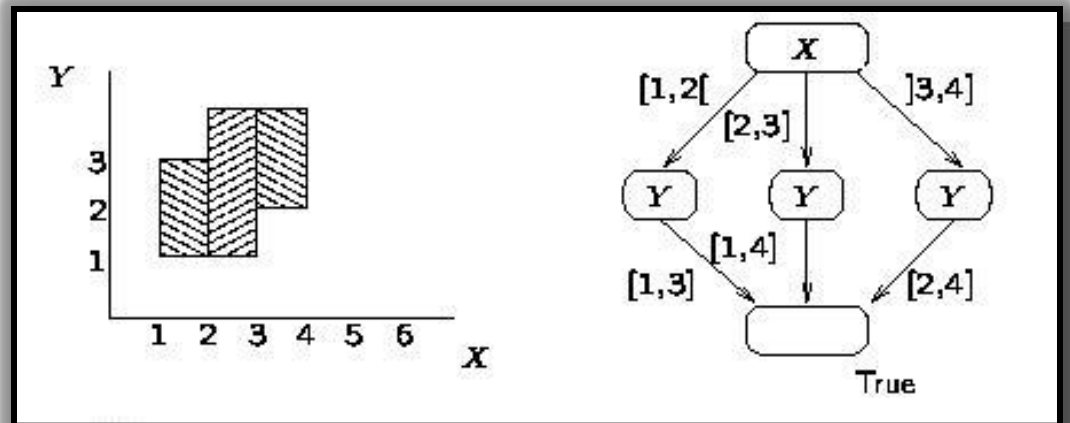
- Minimal Constraint Form

[RTSS97]



- Clock Difference Diagrams

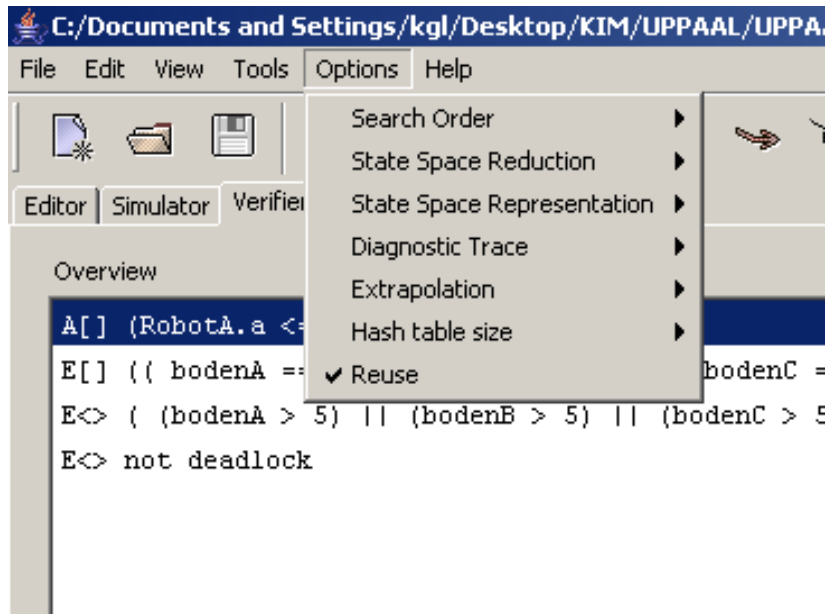
[CAV99]



# Verification Options



# Verification Options



## Search Order

- Depth First
- Breadth First

## State Space Reduction

- None
- Conservative
- Aggressive

## State Space Representation

- DBM
- Compact Form
- Under Approximation
- Over Approximation

## Diagnostic Trace

- Some
- Shortest
- Fastest

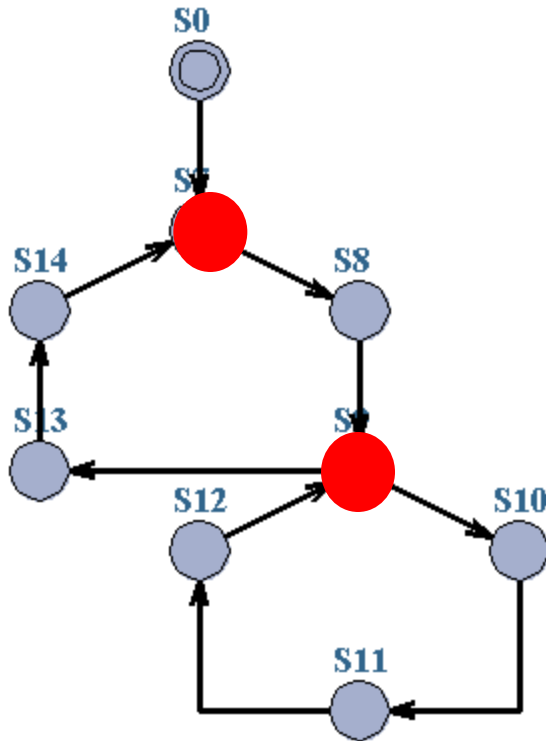
## Extrapolation

## Hash Table size

- Reuse



# State Space Reduction



## Cycles:

Only symbolic states involving loop-entry points need to be saved on **Passed** list

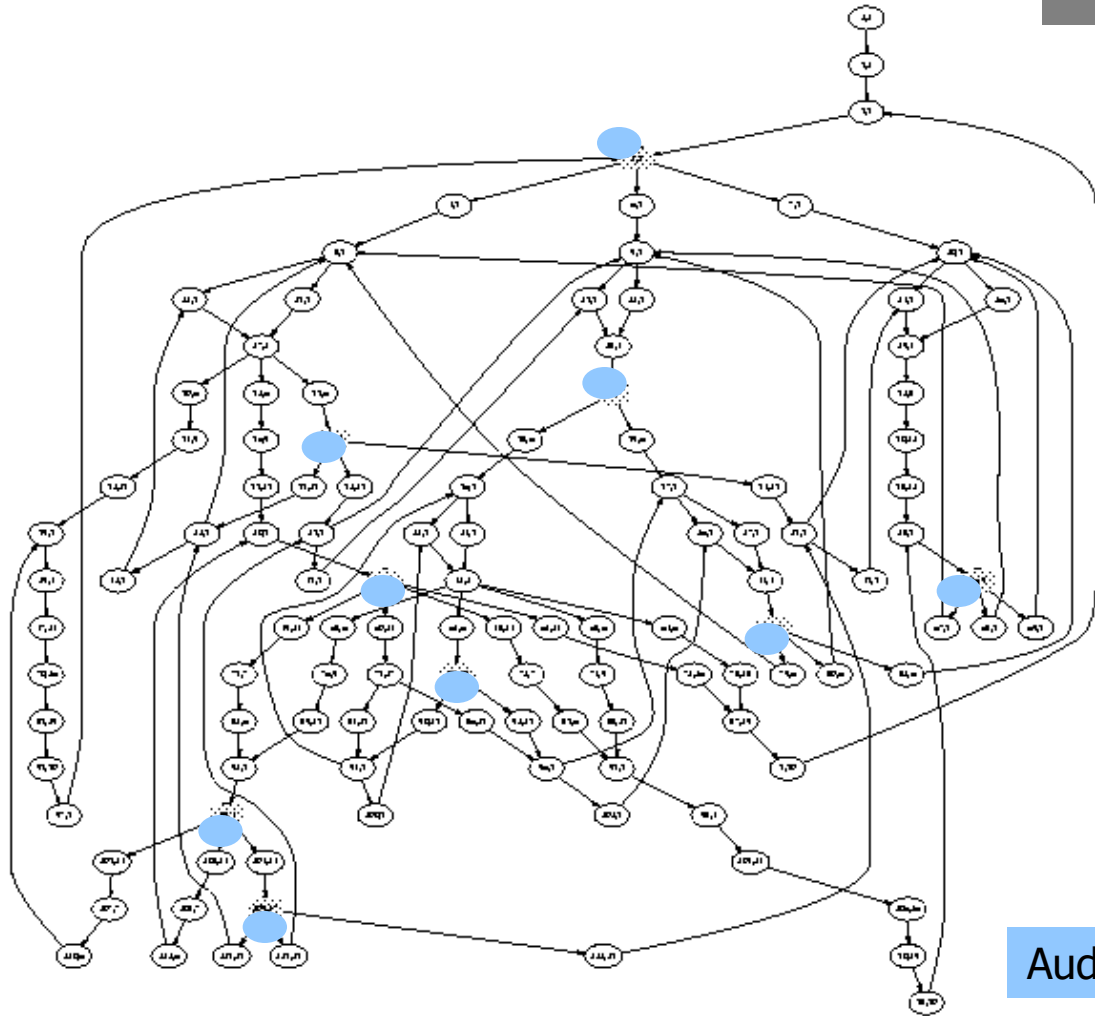




# To Store or Not To Store

Behrmann, Larsen, Pelanek 2003

117 states<sub>total</sub>  
→  
81 states<sub>entrypoint</sub>  
→  
9 states

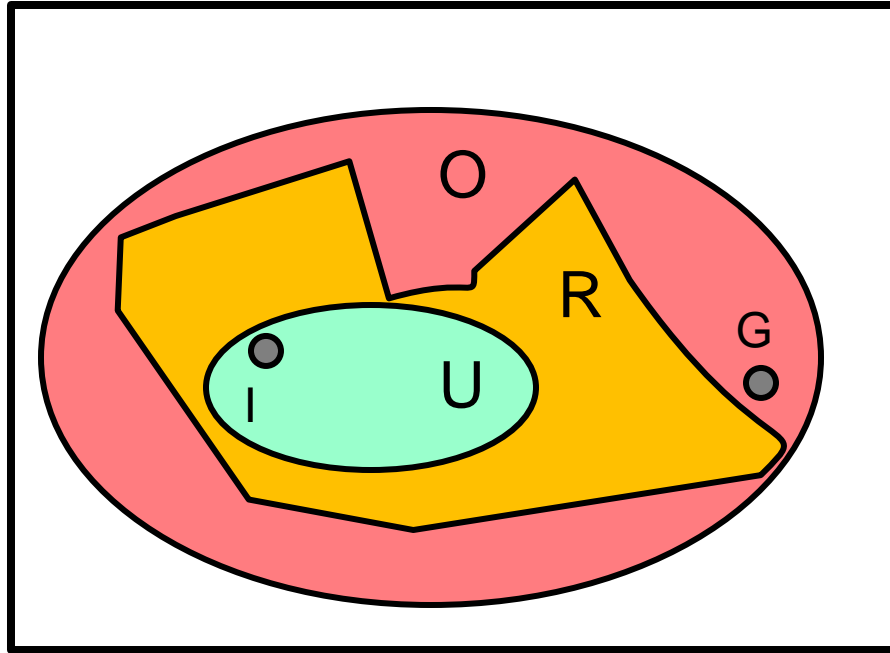


Time OH  
less than 10%

Audio Protocol



# Over/Under Approximation



Declared State Space

Question:

$$G \in R ?$$

How to use:

$$G \in O ?$$

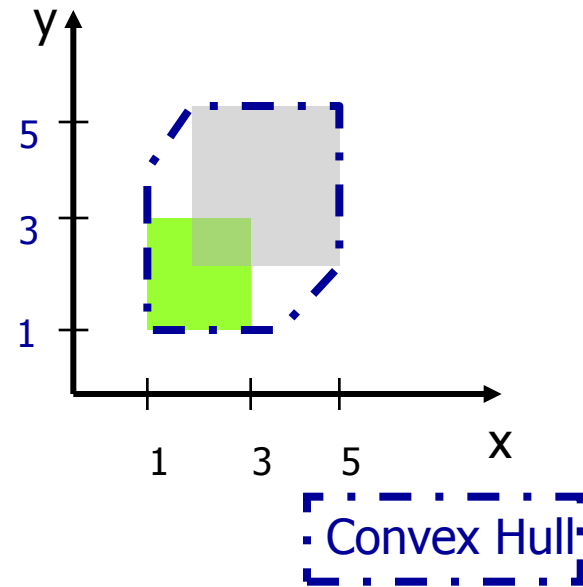
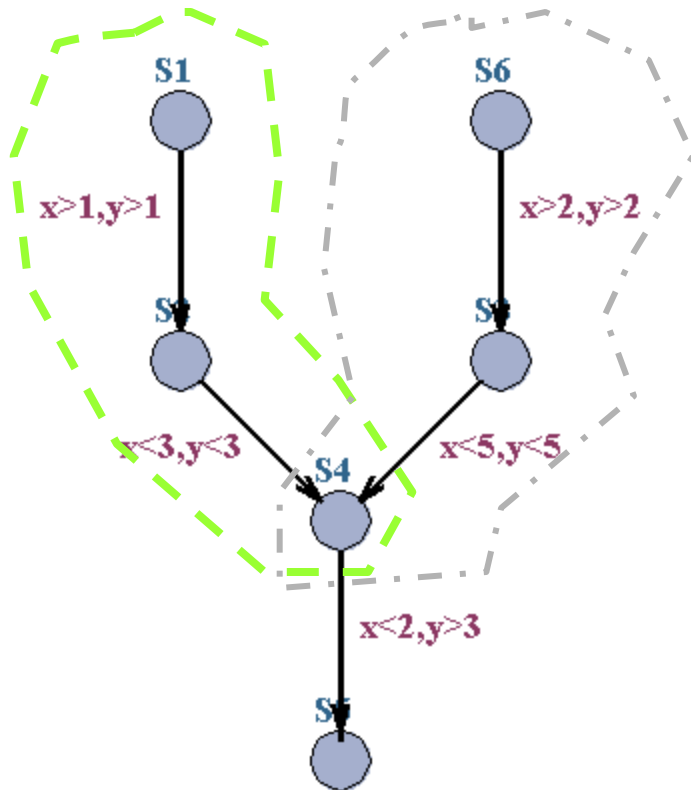
$$G \in U ?$$

$$G \in U \Rightarrow G \in R$$

$$\neg(G \in O) \Rightarrow \neg(G \in R)$$



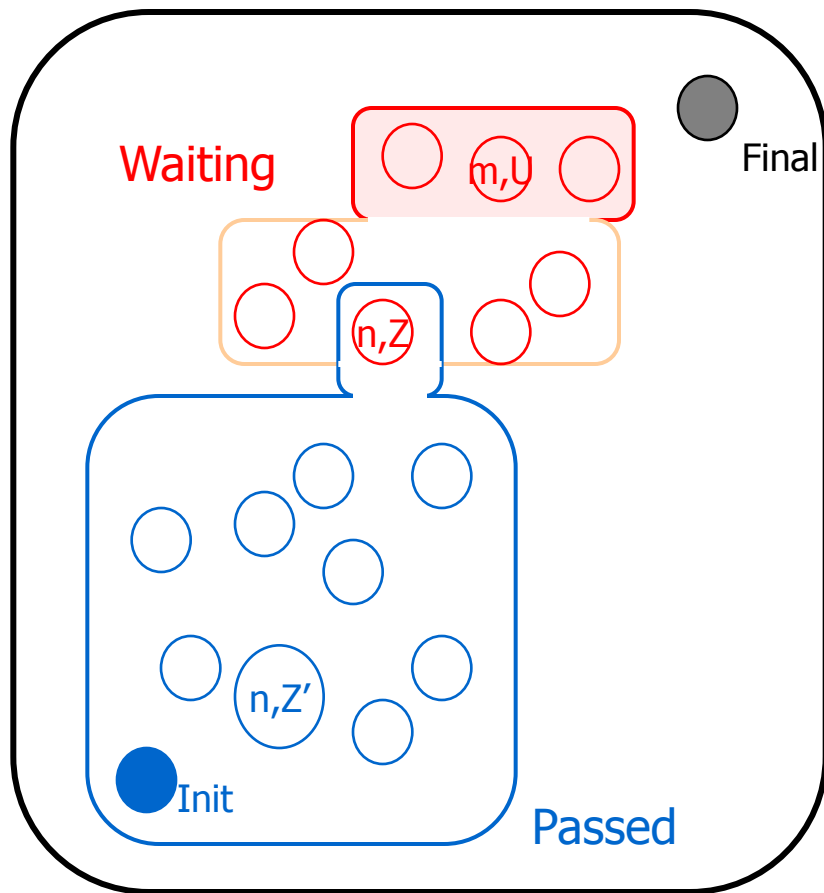
# Over-approximation Convex Hull



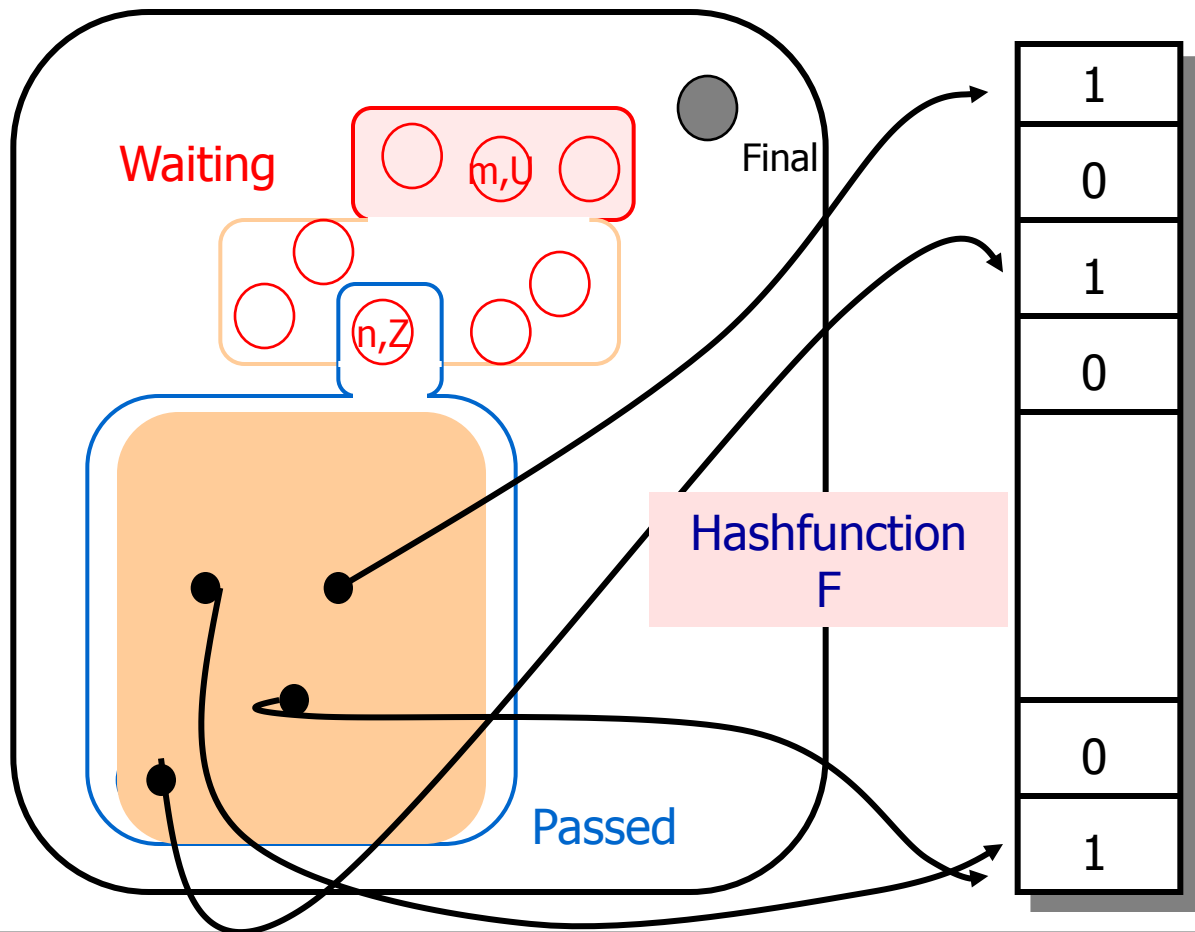
TACAS04: An **EXACT** method performing as well as Convex Hull has been developed based on abstractions taking max constants into account distinguishing between clocks, locations and  $\leq$  &  $\geq$

A. David [39]

# Under-approximation Bitstate Hashing



# Under-approximation Bitstate Hashing

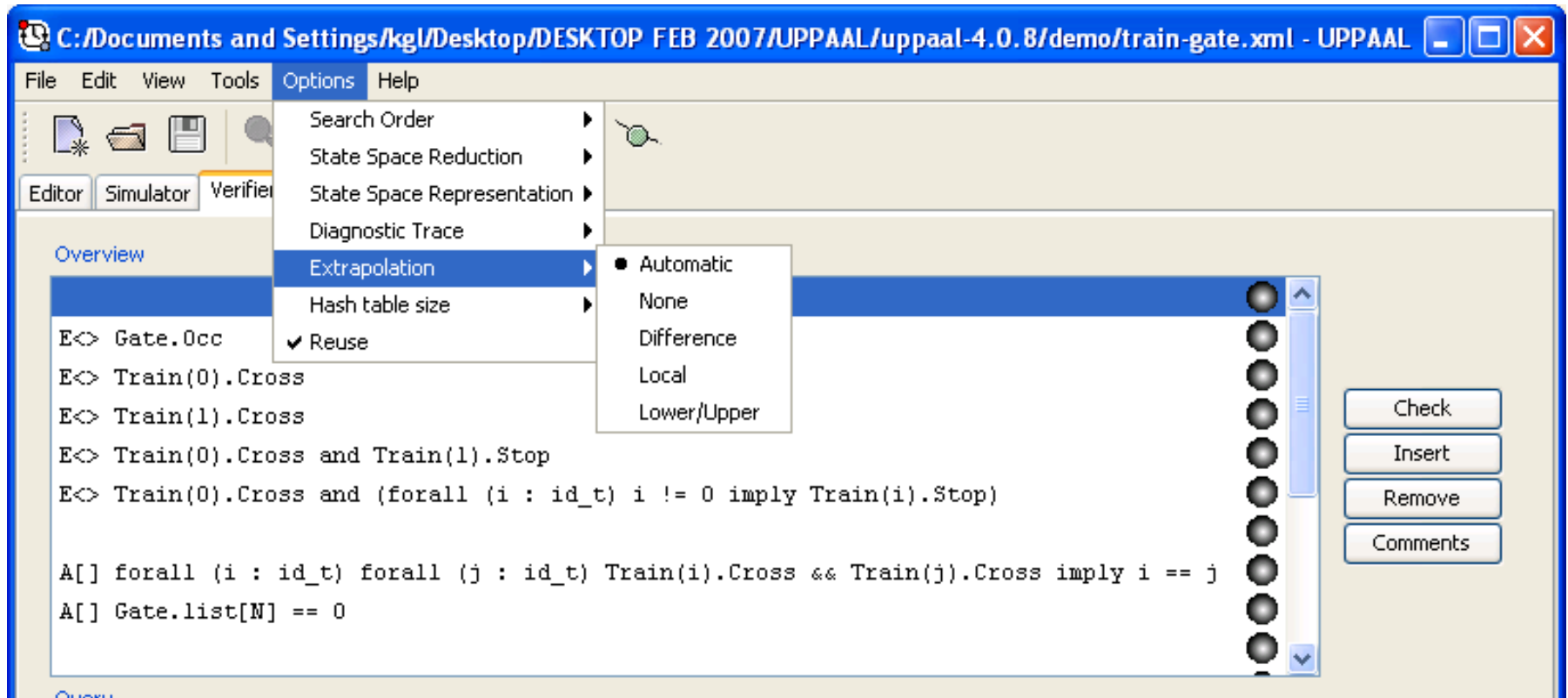


Passed=  
Bitarray

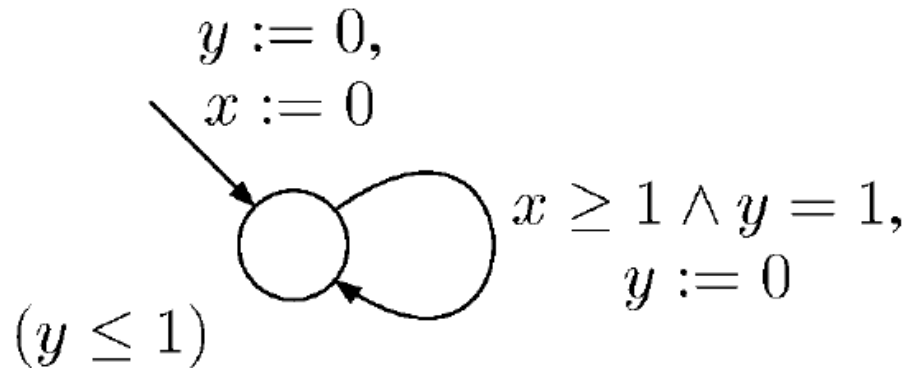
UPPAAL  
4 - 512 Mbits



# Extrapolation

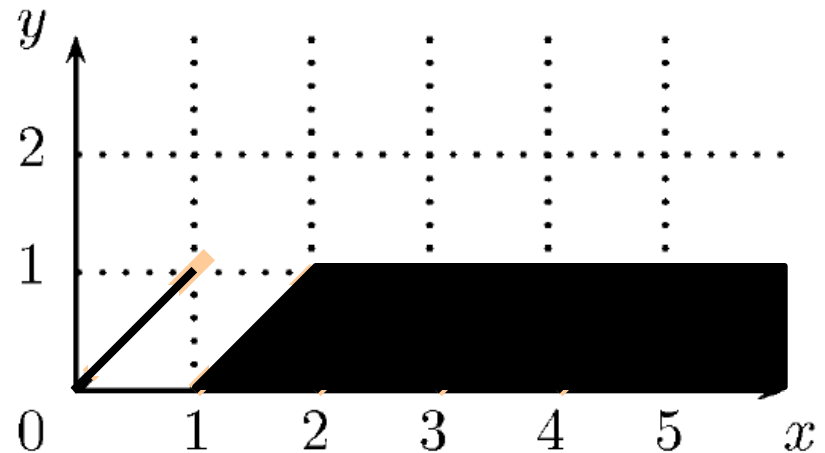


# Forward Symbolic Exploration



TERMINATION  
not  
garanteed

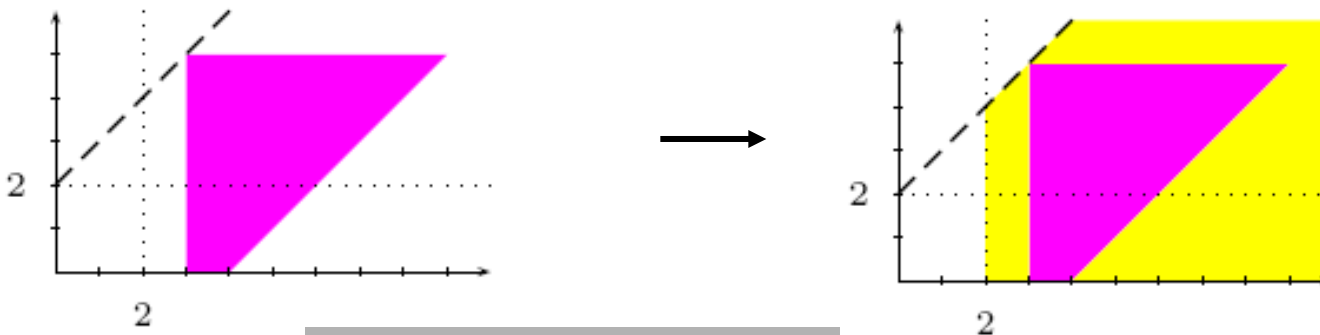
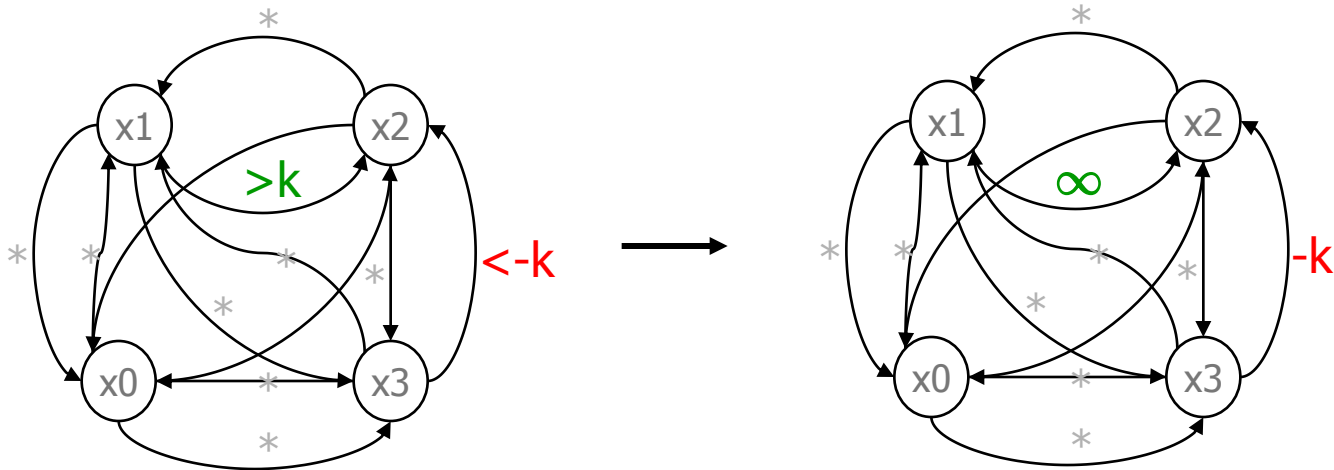
Need for  
Finite  
Abstractions



# Abstraction by Extrapolation

[Daws, Tripakis 98]

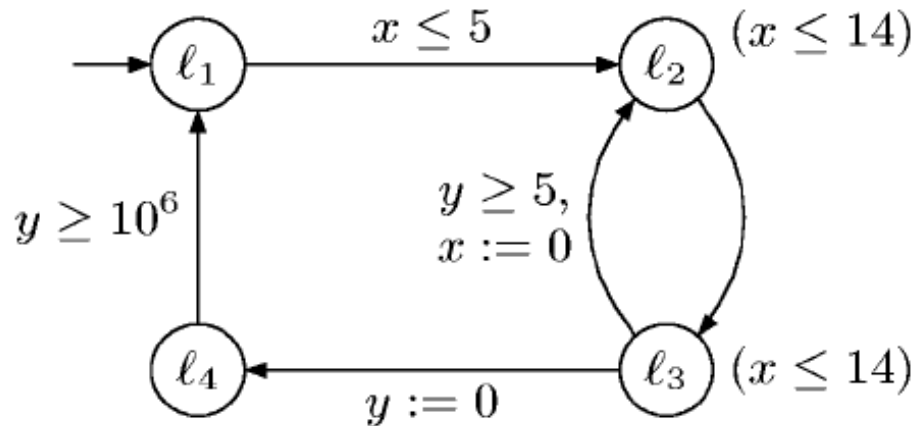
Let  $k$  be the largest constant appearing in the TA





# Location Dependency

[Behrmann, Bouyer,  
Fleury, Larsen 03]



$$k_x = 5 \quad k_y = 10^6$$

Will generate all symbolic states of the form

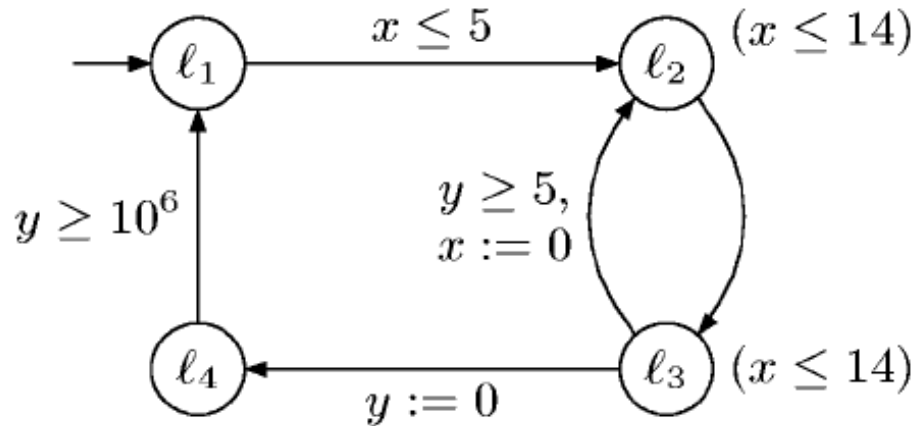
$$(l_2, x \in [0, 14], y \in [5, 14n], y - x \in [5, 14n - 14])$$

for  $n \leq 10^6/14$  !!

But  $y \geq 10^6$  is not RELEVANT in  $l_2$



# Location Dependent Constants



$$k_x = 5 \quad k_y = 10^6$$

$$\begin{array}{ll}
 k_x^i & = 14 \quad \text{for } i \in \{1, 2, 3, 4\} \\
 k_y^i & = 5 \quad \text{for } i \in \{1, 2, 3\} \\
 & k_y^4 = 10^6
 \end{array}$$

$k_j^i$  may be found as solution to simple linear constraints!

Active Clock Reduction:

$$k_j^i = -\infty$$



# Experiments

Active by default

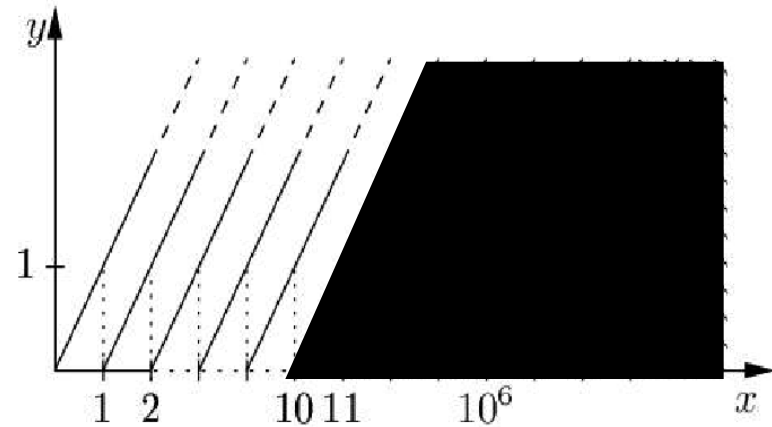
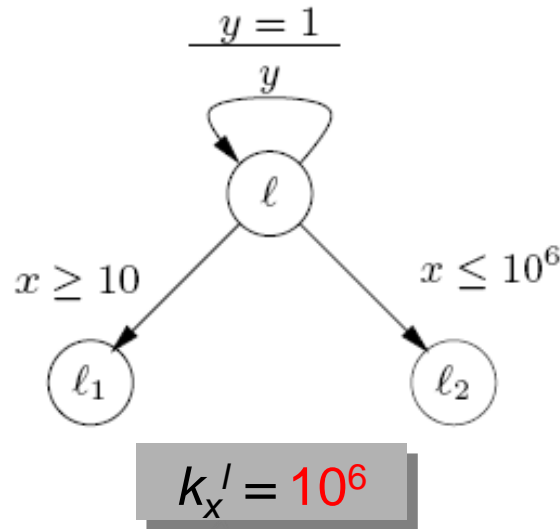


	<i>Constant BIG</i>	<i>Global Method</i>	<i>Active-clock Reduction</i>	<i>Local Constants</i>
<i>Naive Example</i>	$10^3$	0.05s/1MB	0.05s/1MB	0.00s/1MB
	$10^4$	4.78s/3MB	4.83s/3MB	0.00s/1MB
	$10^5$	484s/13MB	480s/13MB	0.00s/1MB
	$10^6$	stopped	stopped	0.00s/1MB
<i>Two Processes</i>	$10^3$	3.24s/3MB	3.26s/3MB	0.01s/1MB
	$10^4$	5981s/9MB	5978s/9MB	0.37s/2MB
	$10^5$	stopped	stopped	72s/5MB
<i>Asymmetric Fischer</i>	$10^3$	0.01s/1MB	0.01s/1MB	0.01s/1MB
	$10^4$	2.20s/3MB	2.20s/3MB	0.85s/2MB
	$10^5$	333s/19MB	333s/19MB	160s/13MB
	$10^6$	33307s/122MB	33238s/122MB	16330s/65MB
<i>Bang &amp; Olufsen</i>	25000	stopped	159s/243MB	123s/204MB



# Lower and Upper Bounds

[Behrmann, Bouyer,  
Larsen, Pelanek 04]



Given that  $x \leq 10^6$  is an *upper* bound implies that

$(l, v_x, v_y)$  *simulates*  $(l, v'_x, v_y)$

whenever  $v'_x \geq v_x \geq 10$ .

For reachability downward  
closure wrt **simulation**  
suffices!

# Additional “secrets”

- Sharing among symbolic states
  - location vector / discrete values / zones
- Symmetry Reduction
- Sweep Line Method
- Guiding wrt Heuristic Value (CORA)
  - User-supplied / Auto-generated
- “Manual” tricks:
  - active variable reduction
  - Value passing using arrays of channels



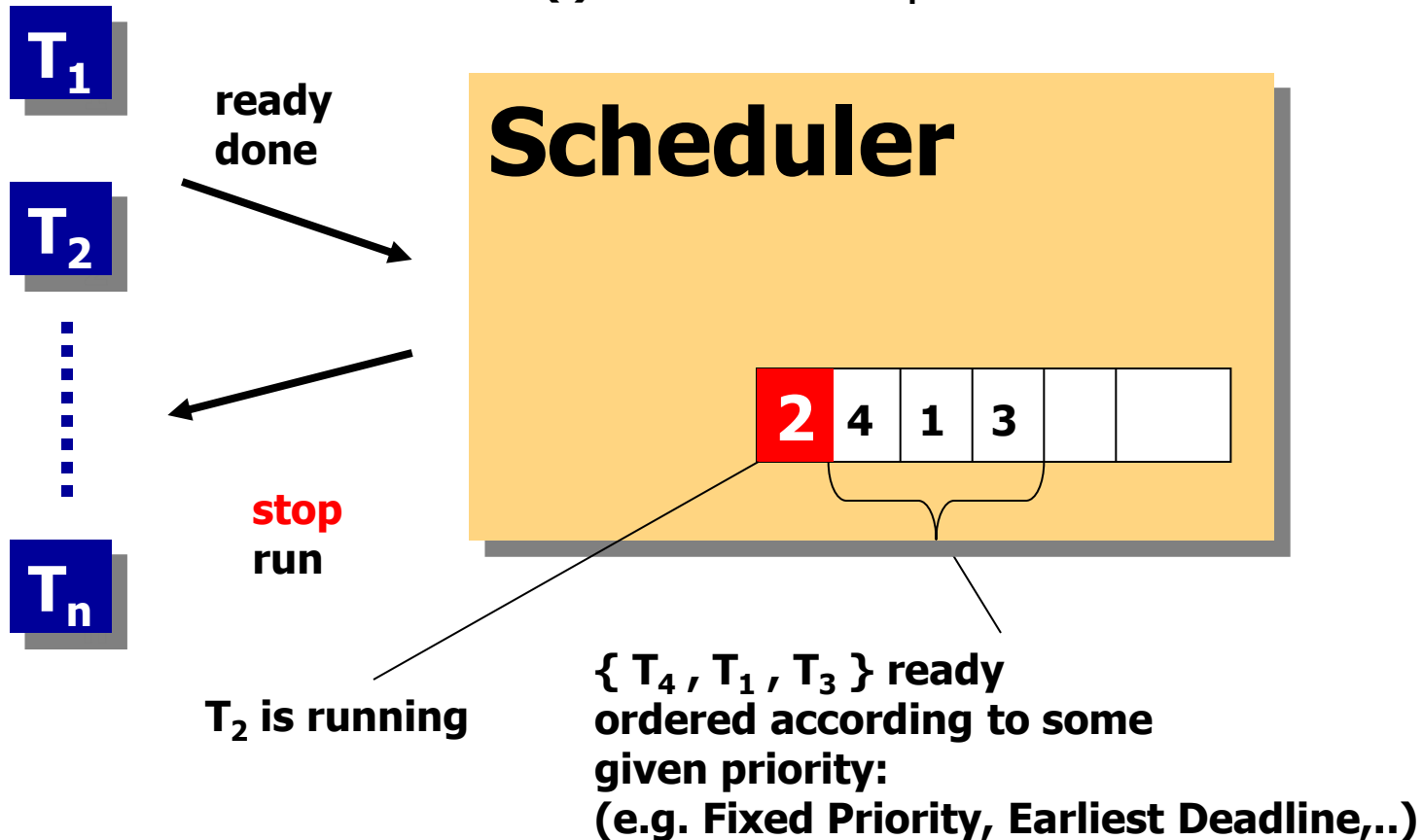
# Application: Schedulability Analysis



# Task Scheduling

*utilization of CPU*

$P(i), [E(i), L(i)], ..$  : period or  
earliest/latest arrival or .. for  $T_i$   
 $C(i)$ : execution time for  $T_i$   
 $D(i)$ : deadline for  $T_i$



# Classical Scheduling Theory

## Utilisation-Based Analysis

- A simple **sufficient but not necessary** schedulability test exists

$$U \equiv \sum_{i=1}^N \frac{C_i}{T_i} \leq N (2^{1/N} - 1)$$

$$U \leq 0.69 \text{ as } N \rightarrow \infty$$

Where C is WCET and T is period

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## Response Time Equation

$$R_i = C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

Where  $hp(i)$  is the set of tasks with priority higher than task  $i$

Solve by forming a recurrence relationship:

$$w_i^{n+1} = C_i + \sum_{j \in hp(i)} \left\lceil \frac{w_i^n}{T_j} \right\rceil C_j$$

The set of values  $w_i^0, w_i^1, w_i^2, \dots, w_i^n, \dots$  is monotonically non decreasing  
When  $w_i^n = w_i^{n+1}$  the solution to the equation has been found,  $w_i^0$  must not be greater than  $R_i$  (e.g. 0 or  $C_i$ )

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## Classical WCRT Analysis



- "Classical" scheduling analysis technique
- For all tasks  $i$ :  $WCRT_i \leq \text{Deadline}_i$

$$R_i = B_i + C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

Blocking times for priority inheritance protocol (BSW):

- $\text{Blocking}(i) = \sum_{r=1}^R \text{usage}(r, i) WCET_{\text{CriticalSection}(r)}$

Blocking times for priority ceiling protocol (ASW):

$$\text{Blocking}(i) = \max_{r=1}^R \text{usage}(r, i) WCET_{\text{CriticalSection}(r)}$$

Quasimodo Workshop, Eindhoven, Nov 6, 2009

Page 21

✓ Simple to perform

- Overly conservative

- Limited settings

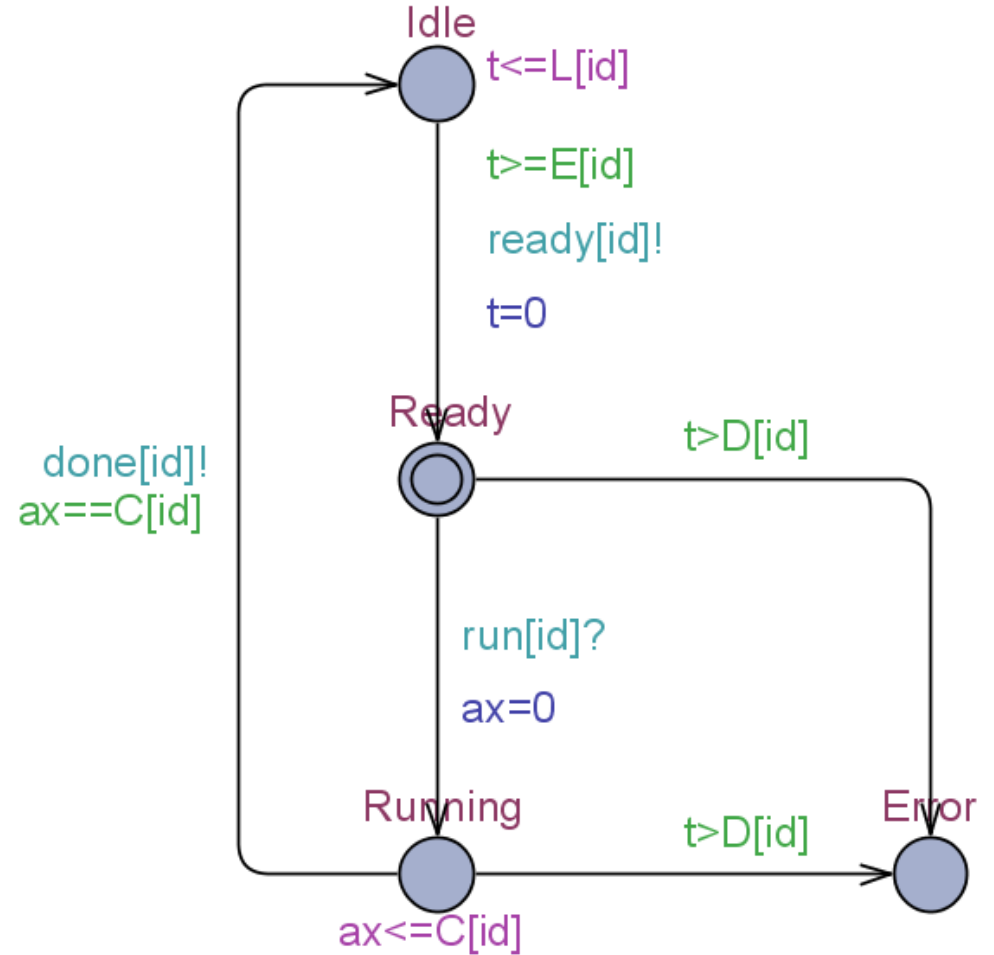
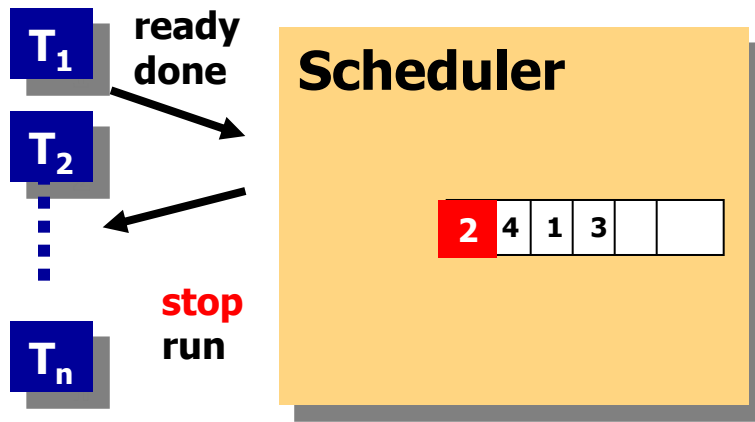
- Single-processor

⇒ Do it in UPPAAL!

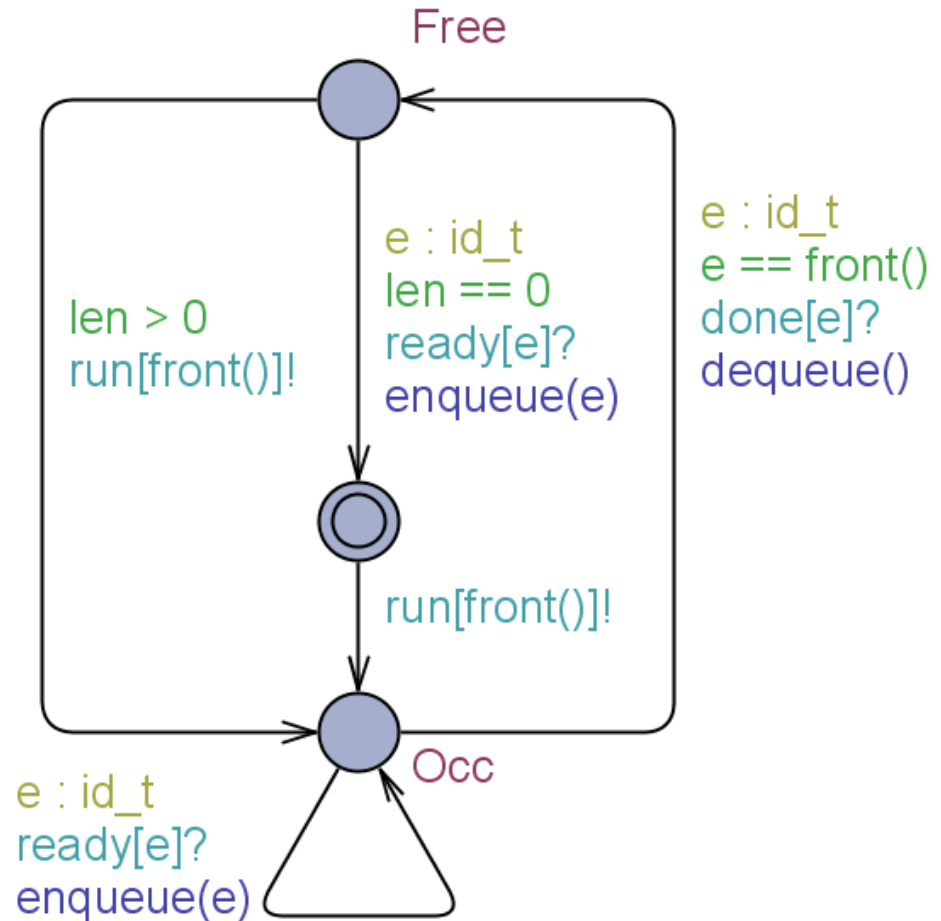
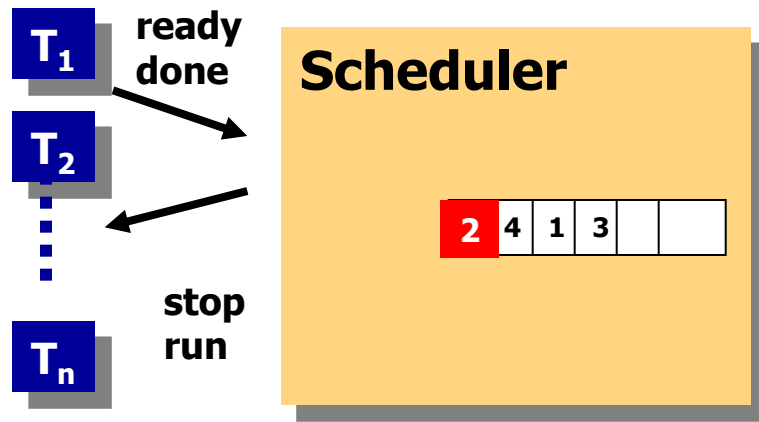




# Modeling Task



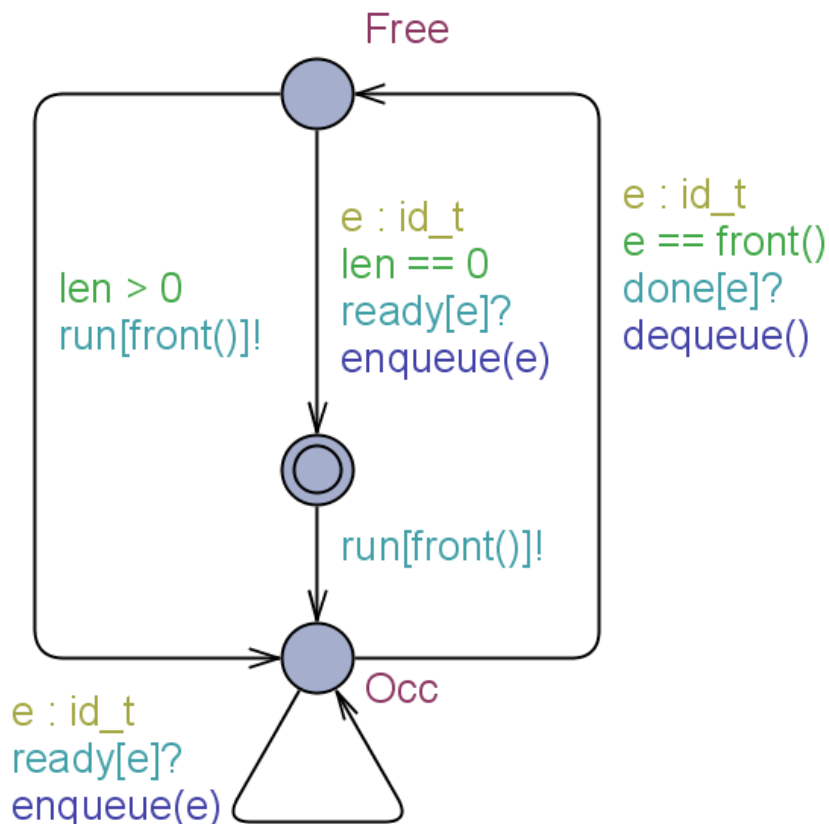
# Modeling Scheduler



Implementation of enqueue/dequeue  
⇒ scheduling policy

# Modeling Queue

In UPPAAL 4.0  
User Defined Function

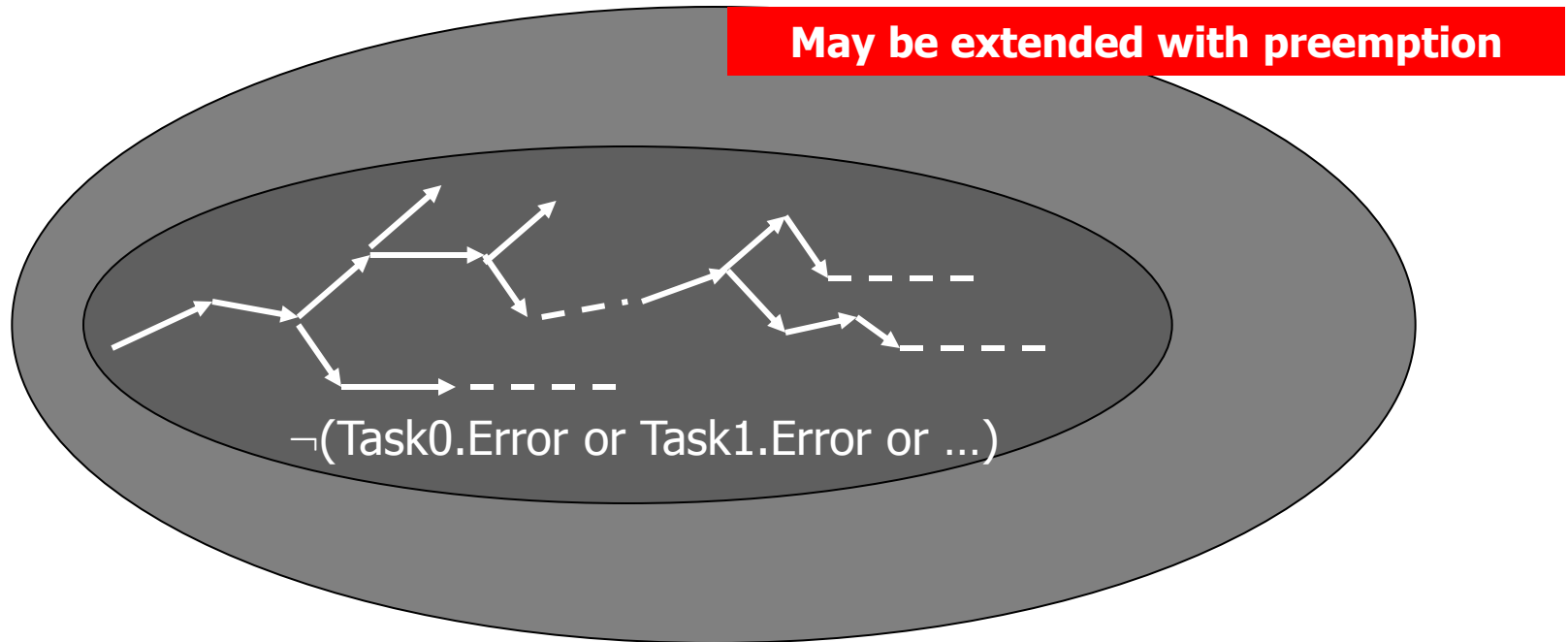


```
// Put an element at the end of the queue
void enqueue(id_t element)
{
  int tmp=0;
  list[len++] = element;
  if (len>0)
  {
    Sort by priority
    int i=len-1;
    while (i>1 && P[list[i]]>P[list[i-1]])
    {
      tmp = list[i-1];
      list[i-1] = list[i];
      list[i] = tmp;
      i--;
    }
  }
}
```

```
// Remove the front element of the queue
void dequeue()
{
  .....
}
```

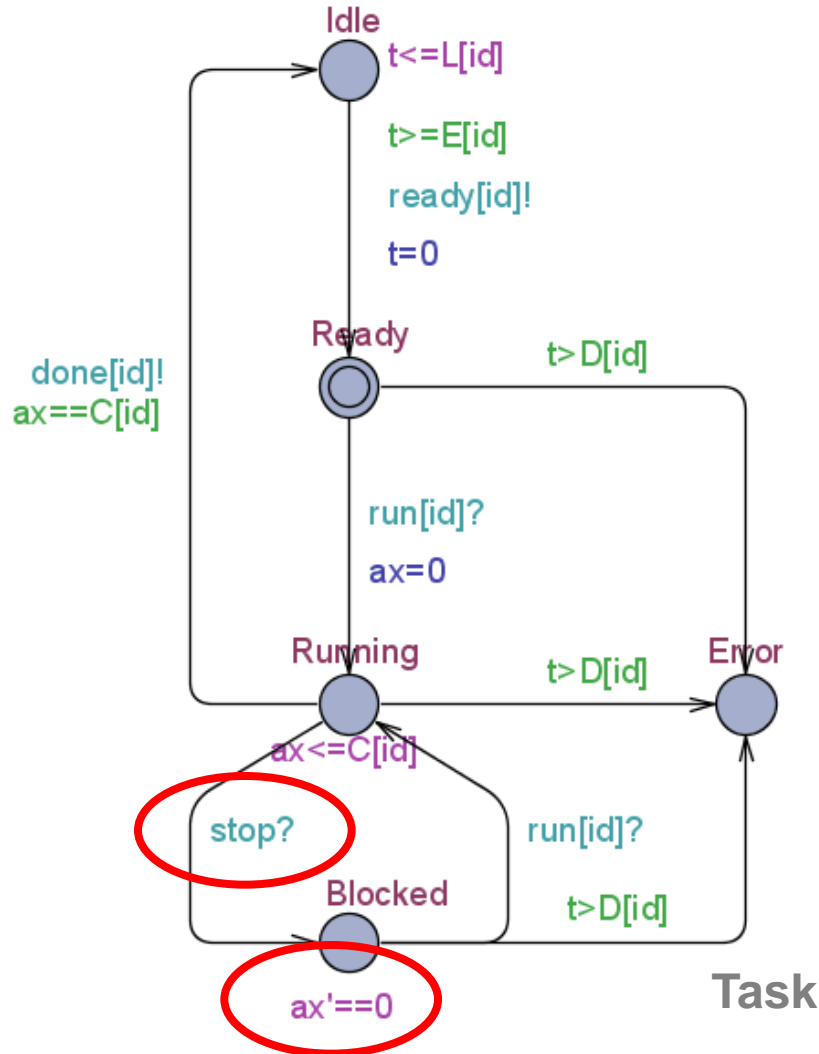


# Schedulability = Safety Property

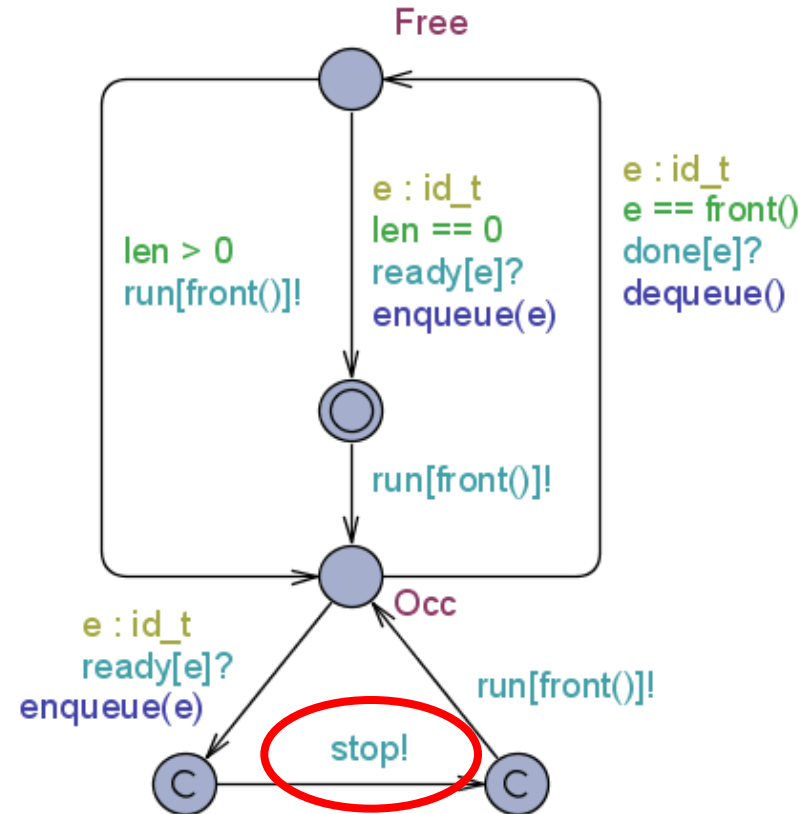


**$A \square \neg(\text{Task0.Error or Task1.Error or ...})$**

# Preemption – Stopwatches!



## Scheduler



Defeating undecidability 😊



# Stop-Watches

- Make reachability undecidable.
- Over-approximation used in UPPAAL
  - $\Rightarrow$  Safe for positive schedulability results!
- What to do if you violate deadlines?
  - Try to validate the trace using other techniques, e.g., polyhedra.
  - Use SMC!

