Correlated Time Series Forecasting using Multi-Task Deep Neural Networks

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ABSTRACT
Cyber-physical systems often consist of entities that interact with each other over time. Meanwhile, as part of the continued digitization of industrial processes, various sensor technologies are deployed that enable us to record time-varying attributes (a.k.a., time series) of such entities, thus producing correlated time series. To enable accurate forecasting on such correlated time series, this paper proposes two models that combine convolutional neural networks (CNNs) and recurrent neural networks (RNNs). The first model employs a CNN on each individual time series, combines the convoluted features, and then applies an RNN on top of the convoluted features in the end to enable forecasting. The second model adds additional auto-encoders into the individual CNNs, making the second model a multi-task learning model, which provides accurate and robust forecasting. Experiments on a large real-world correlated time series data set suggest that the proposed two models are effective and outperform baselines in most settings.

KEYWORDS
Correlated time series; Deep learning; Multi-Task Learning

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1 INTRODUCTION
Complex cyber-physical systems (CPSs) often consist of multiple entities that interact with each other. With the continued digitization, various sensor technologies are deployed to record time-varying attributes of such entities, thus producing correlated time series.

For example, in a vehicular transportation system, traffic sensors (e.g., loop detectors) are able to capture time-varying traffic information (e.g., in the form of average speeds) of different road segments [6], which produces traffic time series [5]. Since the traffic on a road segment affects the traffic on other road segments, traffic time series on different road segments correlate with each other [2, 13].

Accurate forecasting of correlated time series have the potential to reveal holistic system dynamics of the underlying CPSs, including identifying trends, predicting future behavior [14], and detecting anomalies [9], which are important to enable effective operations of the CPSs. For example, in an sewage system, time series forecasting enables identifying the changing trends of different chemicals, early warning of high concentrations of toxic chemicals, and predicting the effect of incidents, e.g., drought or rains, which enables more effective operations of the sewage system. Similarly, in an intelligent transportation system, analyzing traffic time series enables travel time forecasting, early warning of congestion, and predicting the effect of incidents, which benefit drivers and fleet owners.

To enable accurate and robust correlated time series forecasting, we propose two novel non-linear forecasting algorithms based on deep neural networks—a Convolutional Recurrent Neural Network (CRNN) and an Auto-Encoder Convolutional Recurrent Neural Network (AECRNN). In CRNNs, we first consider each of the correlated time series independently and feed each time series into a 1-dimensional convolutional neural network (CNN). The usage of the CNNs helps us learn features for each individual time series. Next, the convoluted time series features are merged together, which is then fed into a Recurrent Neural Network (RNN), with the aim of learning the sequential information while considering the correlations among different time series.

In AECRNNs, we add additional auto-encoders into CRNN. The output of convoluted time series are not only merged together to be fed into the RNN. In addition, each convoluted time series is also reconstructed back to the original time series. Then, the objective function combines the error of the prediction by the RNN and the reconstruction discrepancy by the auto-encoders, making AECRNN a multi-task learning model. The use of auto-encoders makes the CNNs also learn representative features of each time series, but not only distinct features for predicting future values.

In other words, the auto-encoders work as an extra regularization which avoids overfitting, disregards outliers, and thus provides more robust forecasting.

To the best of our knowledge, this is the first study that combines CNNs and RNNs in a unified framework with the help of multi-task learning to enable accurate forecasting for correlated time series. Experiments on a large real-world chemical concentration time series from a sewage treatment plant offer evidence that the proposed methods are accurate and robust.
2 PRELIMINARIES

A time series \(X(t) = (X_1(t), X_2(t), \ldots, X_m(t))\) is a time-ordered sequence of measurements, where measurement \(X_i(t)\) is recorded at time stamp \(t_k\) and we have \(t_j < t_k\) if \(1 \leq j < k \leq m\). Usually, the time interval between two consecutive measurements is constant, i.e., \(t_{j+1} - t_j = t_{k+1} - t_k, 1 \leq j, k < m\). A correlated time series set is denoted as \(X = (X^{(1)}, X^{(2)}, \ldots, X^{(n)})\), where time series in \(X\) are correlated with each other. For example, in the sewage treatment example, we have \(X = (X^{(1)}, X^{(2)}, X^{(3)})\), where \(X^{(1)}, X^{(2)}, \) and \(X^{(3)}\) represent the time series of NH4, NO3, and O2, respectively.

**Problem statement:** Given a correlated time series set \(X = (X^{(1)}, X^{(2)}, X^{(3)})\), where \(X^{(1)}, X^{(2)}, \) and \(X^{(3)}\) represent the time series of NH4, NO3, and O2, respectively. We aim at predicting the future measurements of a specific, target time series in \(X\). Without loss of generality, the first time series \(X^{(1)}\) is chosen as the target time series. More specifically, we assume that the given time series in \(X\) have measurements covering a window \([t_{a+1}, t_{a+l+1}]\) that contains \(l\) time stamps, and we aim at predicting time series \(X^{(1)}\)'s measurements in a future window \([t_{a+l+1}, t_{a+l+p}]\). We call this problem **p-step ahead forecasting**.

3 RELATED WORK

We categorize related studies w.r.t. two dimensions—linear vs. non-linear forecasting and single vs. multiple time series (see Table 1).

<table>
<thead>
<tr>
<th></th>
<th>Single</th>
<th>Multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>[7]</td>
<td>[11, 14]</td>
</tr>
<tr>
<td>Non-Linear</td>
<td>NN</td>
<td>RNN, LSTM, [10, 12], CRNN, AECRNN</td>
</tr>
</tbody>
</table>

Table 1: Related Work

We first consider linear methods for single time series. Here, Autoregressive Integrated Moving Average (ARIMA) [7] is widely used as a baseline method. Linear methods for multiple time series also exist, e.g., multiple linear regression [11] and spatio-temporal hidden Markov models [14]. Neural networks (NN) are able to model non-linear relationships, which are often employed to enable non-linear forecasting models. For example, Recurrent Neural Networks (RNN) and Long Short Term Memory (LSTM) are able to provide non-linear time series forecasting models.

In this paper, we aim at providing non-linear forecasting models that are able to support multiple time series, i.e., the right bottom cell of Table 1. Multivariate time series convolutional neural network (MTCNN) [12] uses a CNN to extract features from a multivariate time series and then a fully connected neural network layer to predict. We propose CRNN and AECRNN which outperform existing methods in most settings. Similar to MTCNN, we first use CNNs to extract features from multiple time series. However, we instead use RNN in the proposed CRNN and AECRNN. In addition, AECRNN also incorporates auto-encoders, making it a multi-task learning model, which enables effective and robust forecasting. A similar multi-task learning model T2Net [8] employs CNN and auto-encoders to enable classification and clustering, but not for prediction. Non-deep learning methods [10] also exist.

4 PROPOSED MODELS

Convolutional Recurrent Neural Network (CRNN): We propose a CRNN that utilizes a combination of Convolutional Neural Networks (CNNs) and Recurrent Neural Networks (RNNs) to enable p-step ahead forecasting for a set of correlated time series. CNNs are used successfully for classifying images by learning the features and patterns of the images. RNNs are able to capture dependencies of a sequence of values, thus being good at forecasting future values. This motivates us to first use CNNs to extract distinctive features of each of the correlated multiple time series, and then to apply RNNs on top of the combined output of CNNs to predict near future values of the target time series. We call this model CRNN (cf. Fig. 1).

Specifically, CRNN takes as input multiple time series, where each input time series contains \(l\) measurements. Part A of Fig 1 shows an example where \([X] = 3\) correlated time series \(X^{(1)}, X^{(2)}, \) and \(X^{(3)}\) are fed into CRNN as input. CRNN outputs a single time series, say \(Z = (z_1, z_2, \ldots, z_p)\), that contains \(p\) measurements which are the predicted \(p\) measurements of the target time series \(X^{(1)}\) in the near future (see Part G of Fig 1).

In the CRNN, we first treat each time series in \(X\) independently. In particular, we treat each time series as a \(1 \times l\) matrix, as shown in part B of Fig. 1. Next, we apply convolutions on each time series at the convolutional layer (see part C of Fig. 1). In particular, we apply \(a\), e.g., 3, in Fig. 1, filters to conduct convolutions, with the aim to extract distinctive features in the individual input time series. This produces \(a\) matrices with size of \(1 \times l\). Next, in the pooling layer (see part D of Fig 1), for each matrix, we apply a max-pooling operator to capture the most representative features of the time series as a \(1 \times \alpha\) matrix by using a \(1\times 2\) window with a stride set to 2. Thus, we obtain a total of \(a\) matrices of size \(1 \times \alpha\) for each input time series. Note that we may apply the convolution and pooling layers multiple times. After convolution and pooling, we have \([X]\) cubes of size \(a \times 1 \times \frac{1}{2}\) as the output of the pooling layers (see part D Fig 1-D). So far, the work of CNNs finishes.

Next, the \([X]\) cubes are concatenated into an \(n\)-dimensional vector (see part E of Fig 1), where \(n = [X] \times a \times 1 \times \frac{1}{2}\) to be fed into an RNN (see part F of Fig 1) for the predicting purpose. We obtain \(Z\) as the near future measurements of the target time series (see Part G of Fig 1). The objective function of CRNN is

\[
J_t = \frac{1}{p} \sum_{i=1}^{p} Error(z_i, x_{a+l+i}^{(1)})
\]

Here, \(Error(\cdot, \cdot)\) is an error function that measures the discrepancy (e.g., mean square error) between the predicted measurement \(z_i\) and the ground truth measurement \(x_{a+l+i}^{(1)}\) at time stamp \(a + l + i\).

Note that the RNN can be easily extended to enable forecasting for all time series, but not just a single target time series.

**Auto Encoder CRNN (AECRNN):** In AECRNN, we incorporate an auto-encoder to each CNN (see Fig 2). The intuition behind AECRNN is to use auto-encoders to learn robust features and ignore features that represent outliers. The auto-encoders also work as an additional regularization to enable learning most representative features for all input time series but not overfit to features that are specific to forecasting the target time series in the training data.

After the pooling layer, we not only concatenate the output cubes of the pooling layer, but also feed the output cubes to an additional deconvolution layer (see Part E of Fig 2). In particular, each cube is deconvoluted into \(a\) matrices with the same size as the matrices in part C. Then, we obtain \([X]\) groups of matrices, where each group has \(a\) matrices. Next, we apply Sigmoid activation function to each matrix group to produce a \(1 \times l\) matrix. This
corresponds \(|X|\) reconstructed time series, e.g., \(\hat{X}^{(1)}\), \(\hat{X}^{(2)}\), and \(\hat{X}^{(3)}\), where \(\hat{X}^{(1)} = \langle \hat{x}_{a+1}^{(1)}, \hat{x}_{a+2}^{(1)}, \ldots, \hat{x}_{a+l}^{(1)} \rangle\). The objective function of the additional auto encoders is

\[
J_2 = \frac{1}{l} \frac{1}{|X|} \sum_{k=1}^{l} \sum_{i=1}^{|X|} Error(\hat{x}_{a+i}^{(k)}, x_{a+i}^{(k)}).
\]

It measures the discrepancy between the reconstructed measurement \(\hat{x}_{a+i}^{(k)}\) and the original, ground-truth measurement \(x_{a+i}^{(k)}\) at the \((a + i)\)-th time stamp over all \(|X|\) time series.

The final objective function of AECRNN is \(J = J_1 + J_2\). This makes AECRNN a multi-task learning model, where one task is to forecast the \(p\) future measurements of the target time series (i.e., \(J_1\)) and the other tasks are to reconstruct \(|X|\) time series’ \(l\) known measurements (i.e., \(J_2\)).

5 EMPIRICAL STUDY

Data Sets: We use a large time series data set provided by a sewage treatment center from Aalborg, Denmark. The sewage treatment center has 6 tanks and three different sensors are deployed in each tank to measure the concentrations of three different chemicals, NH4, NO3, and O2, every 2 minutes. The data covers a period of 3 years in total. We consider the three time series of three chemicals from a specific tank as correlated time series.

We choose multiple windows over the 3-year period to test the proposed methods. For each window, we use the first 84% of the data for training and validation, and the remaining 16% of data for testing. When learning, we further segment the training data into multiple training cases using sliding windows. In particular, in each segment, we use a sequence of \(l\) measurements as the input data to the proposed models and use the immediately following \(p\) measurements as the ground truth data to calculate the prediction errors (i.e., \(J_1\)) to enable back-propagation.

Parameters: We vary three problem parameters in the experiments according to Table 2, where the default values are shown in bold. Note that the problem parameters define different problem settings and are independent of solutions.

Specifically, we vary the number of time series, i.e., the cardinality of the correlated time series set \(|X|\), from 1, 2, to 3. When \(|X| = 1\), we only consider a single time series, which is also the target time series. Then, we add 1 and 2 additional correlated time series with the target time series, respectively. Next, we vary the length \(l\) of time series when training. We also vary \(p\) to enable \(p\)-step ahead forecasting, where a large \(p\) means forecasting far into the future.

In addition, we vary solution parameters (a.k.a. hyper-parameters) that are specific to solutions. In particular, for the proposed deep learning based methods, we vary the number of convolution and pooling layers (1, 2, and 3), number of filters per convolution layer (2, 3, 4, 5, 8, 10, and 16), the filter size (1, 2, 3, 5, and 10), and hidden state size in RNNs (3, 4, 5, 6). We identify the optimal solution parameters for each problem parameter setting.

Baseline: We consider 5 baselines. Yesterday is a simple, linear method that propagates the last known value of the time series for the entire prediction window, which is commonly used in financial
time series prediction. Methods ARIMA, RNN, LSTM, and MTCNN are covered in Section 3.

Implementation Details: All methods are implemented in Python 3.6 where the deep learning methods are implemented using TensorFlow 1.7. A computer with Intel i7-4700MQ CPU, 4 cores, 16 GB RAM is used to conduct all experiments.

Experimental Results: We use both root mean square error (RMSE) and mean absolute percentage error (MAPE) to evaluate methods. This suggests that CRNN and AECRNN are able to take advantage when having correlated time series, meaning that they take advantage of correlated times series, and RNN and LSTM do not.

<table>
<thead>
<tr>
<th>$X$, $I$, $P$</th>
<th>YESTERDAY</th>
<th>ARIMA</th>
<th>RNN</th>
<th>LSTM</th>
<th>MTCNN</th>
<th>CRNN</th>
<th>AECRNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 200, 100</td>
<td>16.59±2.70</td>
<td>18.77±2.65</td>
<td>9.50±1.39</td>
<td>10.16±2.60</td>
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<tr>
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<td>16.59±2.70</td>
<td>18.77±2.65</td>
<td>9.43±1.45</td>
<td>10.79±1.61</td>
<td>11.08±0.91</td>
<td>7.91±0.37</td>
<td>8.45±0.61</td>
</tr>
<tr>
<td>3, 200, 100</td>
<td>16.59±2.70</td>
<td>18.77±2.65</td>
<td>9.59±1.85</td>
<td>10.77±0.57</td>
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<td>9.59±0.93</td>
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<td>2, 10, 100</td>
<td>18.23±2.93</td>
<td>20.85±3.98</td>
<td>9.86±2.77</td>
<td>9.12±1.93</td>
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<td>9.48±2.44</td>
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<tr>
<td>50, 100</td>
<td>16.41±2.25</td>
<td>20.75±5.37</td>
<td>10.77±3.10</td>
<td>11.90±3.15</td>
<td>12.20±5.66</td>
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<tr>
<td>100, 100</td>
<td>17.90±1.58</td>
<td>36.99±24.37</td>
<td>12.62±2.39</td>
<td>12.92±2.39</td>
<td>12.43±1.84</td>
<td>12.78±3.30</td>
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<tr>
<td>200, 100</td>
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<td>11.08±0.91</td>
<td>7.91±0.37</td>
<td>8.45±0.61</td>
</tr>
</tbody>
</table>

Table 2: Problem Parameters

| Number of time series $|X|$ | $I$, $P$ | $\rho$-step ahead prediction | $|X|$, $I$, $P$ |
|------------------------|-------|-----------------------------|----------------|
| 1, 2, 3                |       | 1, 25, 50, 100              |

Table 3: Accuracy, MAPE with Standard Deviations

<table>
<thead>
<tr>
<th>CRNN</th>
<th>AECRNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single target TS</td>
<td>10.8</td>
</tr>
<tr>
<td>With a correlated TS</td>
<td>8.3</td>
</tr>
<tr>
<td>With an uncorrelated TS</td>
<td>13.0</td>
</tr>
</tbody>
</table>

Table 4: Dealing with uncorrelated time series, MAPE

6 CONCLUSION AND OUTLOOK

We propose two deep learning models, CRNN and AECRNN, to enable accurate correlated time series forecasting. Experiments on a large real world time series show promising results. In the future, it is of interest to verify the proposed models on time series from different domains such as transportation [3, 4] and to study the scalability of the proposed models for large time series data, e.g., by exploring parallel computing frameworks [15].

REFERENCES