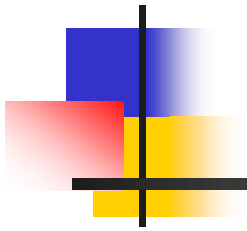

Reasoning about Performance

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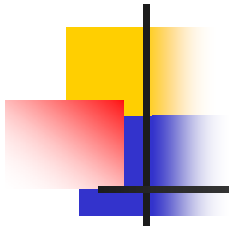
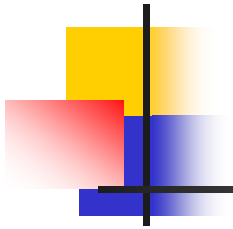


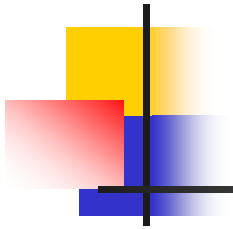
Basics

- A sequential algorithm is evaluated by its runtime in function of its input size.
  - O(f(n)), Ω(f(n)), Θ(f(n)).
- The asymptotic runtime is independent of the platform. Analysis "at a constant factor".
- A parallel algorithm has more parameters.



Basics

- A parallel algorithm is evaluated by its runtime in function of
  - the input size,
  - the number of processors,
  - the communication parameters.
- Which performance measures?
- Compare to which (serial version) baseline?



Sources of Performance Loss

- In practice, E<1.
  - Overhead (sync, communication, threads).
  - W_S – Amdahl's law.
  - Idling & contention.
  - Overhead function: $T_0=pT_P-T_S$
- Ex. count3s:
  - false sharing (communication) was avoidable
    - at the cost of memory
  - incrementing global counter unavoidable

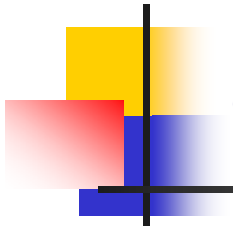
Example – Amdahl's Law

- Assume 20% not parallelizable.
- For P processors

  $T_P = 1/S * T_S + (1-1/S) * T_S/P$

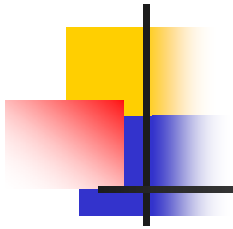
  $T_2 = 0.2T_S + 0.8T_S/2 = 0.6T_S$
  $S = 1/0.6$
  $E_2 = S/2 = 0.83$

  $T_{10} = 0.2T_S + 0.8T_S/10 = 0.28T_S$
  $S = 1/0.28$
  $E_2 = S/10 = 0.36$



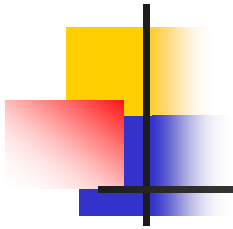
Overheads

- Communication overheads
  - explicit (send/recv) + implicit (threads, false sharing…)
- Synchronization
  - locks, barriers, semaphores
- Extra computations
  - typically reductions
- Memory
  - extra padding, local copies



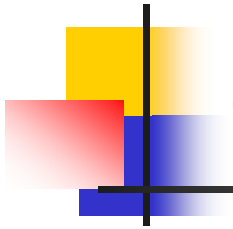
Scalability

- As seen in Amdahl's law:
  - Speedup inherently limited by the problem size.
  - Increase p, lose speedup.
  - Increase the size, gain speedup – in general W_S grows slowly with the size.
- The question is:
  - How much do you need to increase the size in function of p to keep the same efficiency?
  - Measured by isoefficiency function = measure of scalability.



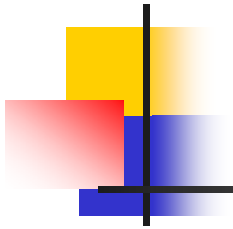
Scalability

- Suppose $T_S=cn^x$.
- We increase the problem size by m and we use p processors. *But we want to keep the same execution time*.
  - $c(mn)^x/p=cn^x$ – optimistic.
    $p=m^x$
    $m=p^{1/x}$.
  - x=4, m=100, we need 10^8 processors.
  - Example in the book is miss-leading: The problem comes from the complexity of the algorithm.



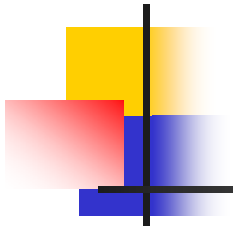
Contention

- Special because not seen directly in the code.
- Comes for behavior.
- Difficult to replicate.
- Sources:
  - excessive loads on memory, e.g., spin-lock
  - central locks – granularity problems
  - effects on the architecture – flood the bus



Idling

- Waiting for locks.
- Data dependencies.
  - See previous examples (sum, prefix).
  - May need algorithm rewrites.
- Load balancing.
- Memory bound computations – locality.
- Detecting termination!

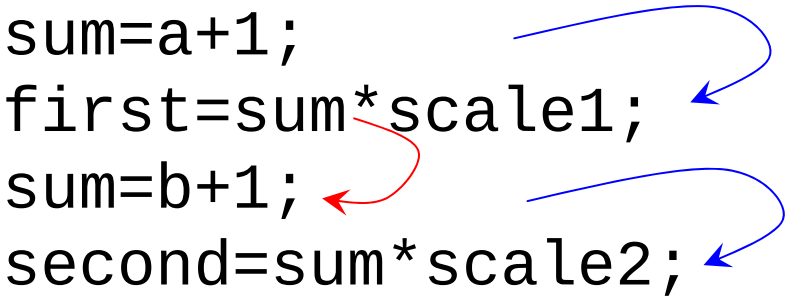


Parallel Structure

- Identify dependencies
  - task dependency graph
  - data dependencies
    flow dependency: read after write – *true dep*.
    anti-dependency: write after read
    output dependency: write after write
    → *memory reuse*

Example

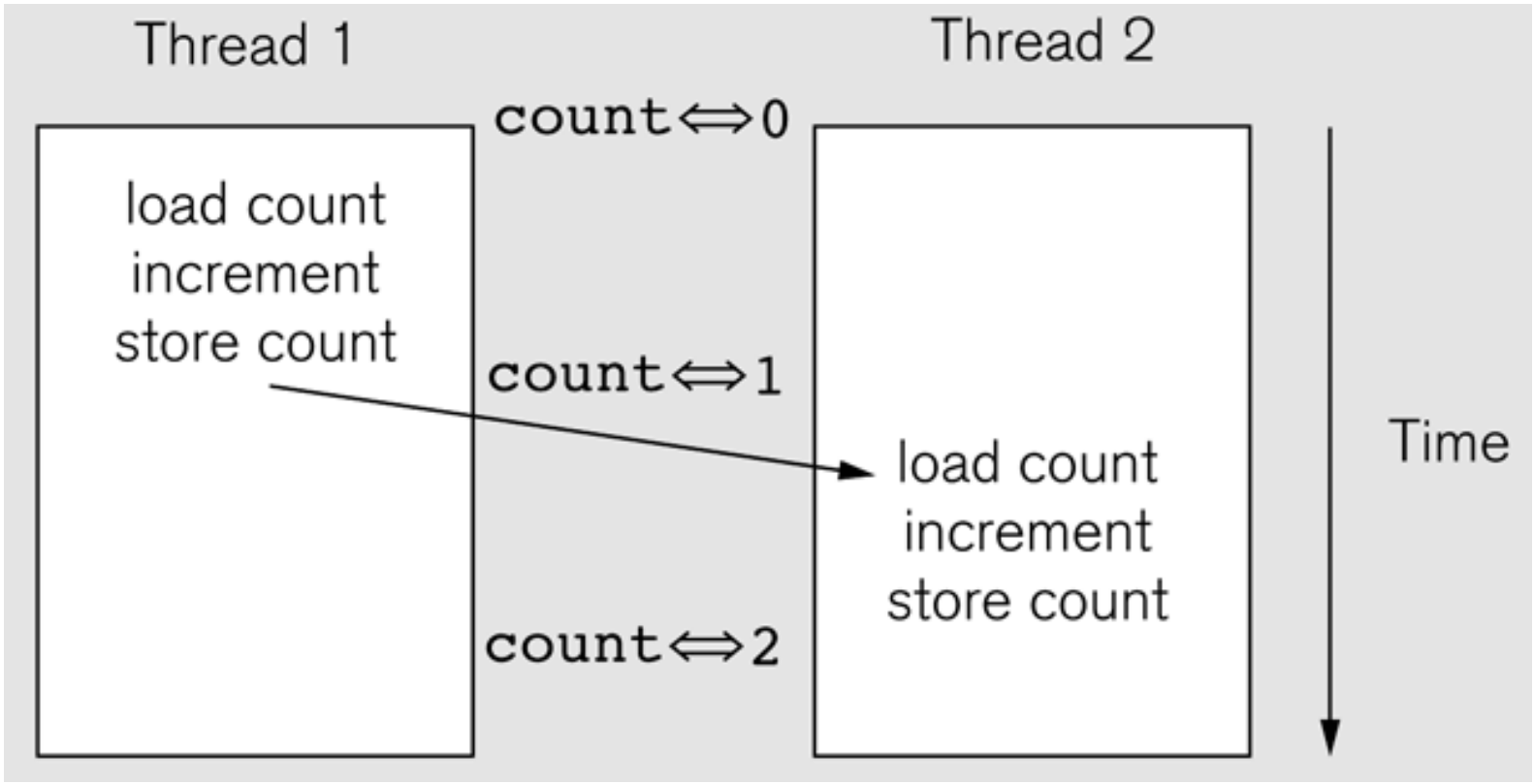
- Flow dependency.
  - Cannot be removed.
- Anti-dependency.
  - Can be removed by renaming/rewriting.



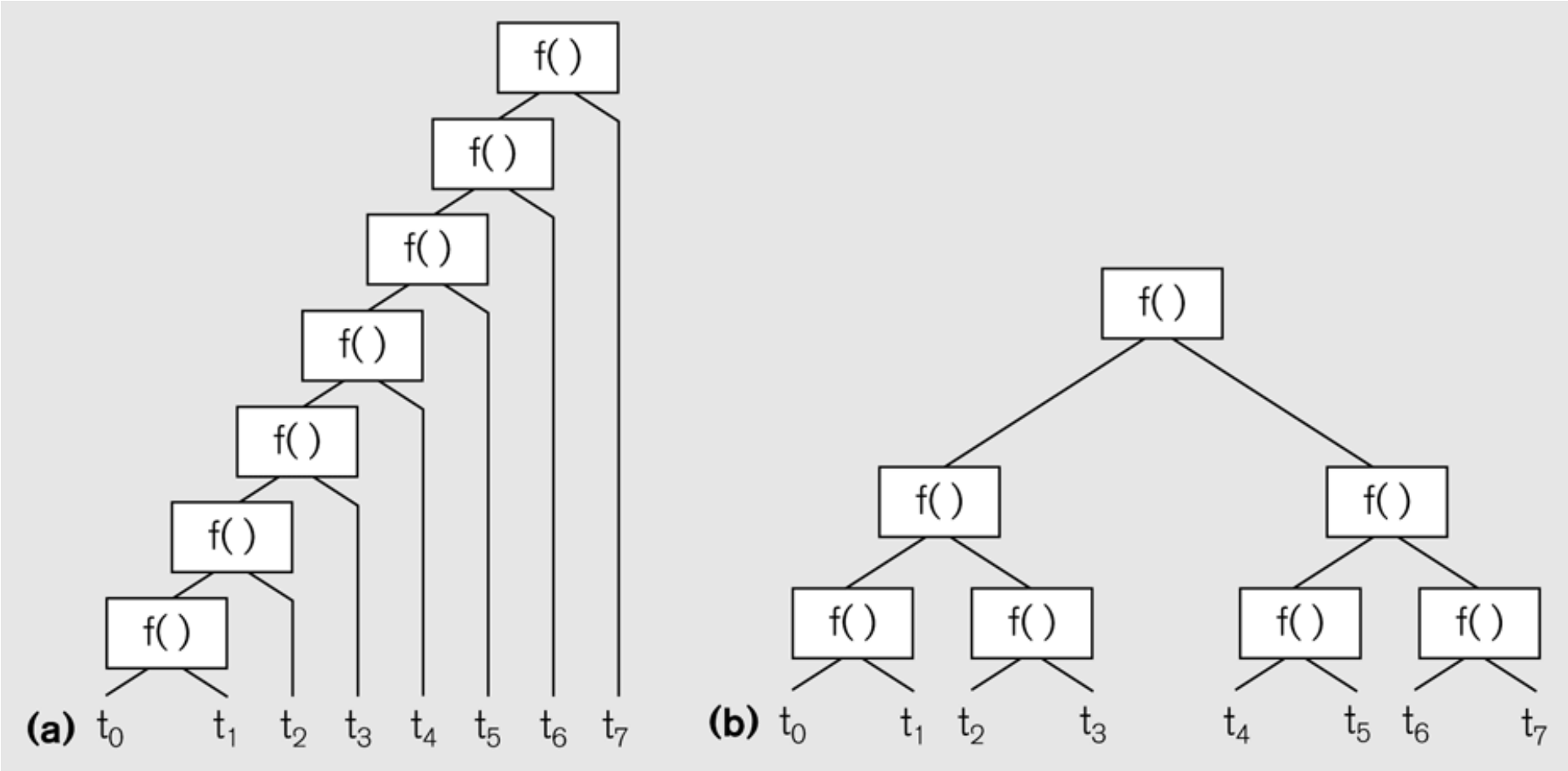
```
sum=a+1;
first=sum*scale1;
sum=b+1;
second=sum*scale2;
```

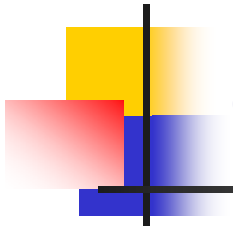
```
first=(a+1)*scale1;
second=(b+1)*scale2;
```

Example


Thread 1
Thread 2
count⇔0
load count
increment
store count
count⇔1
load count
increment
store count
count⇔2
Time

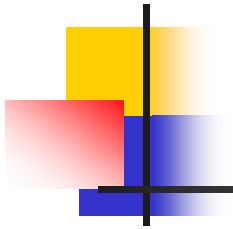
Example





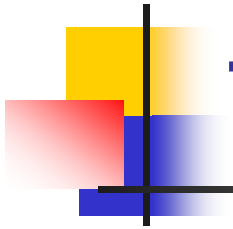
Granularity

- Size of
  - tasks
  - data associated to threads
  - $\rightarrow$ determines frequency of interactions.
- Fine grain: small independent computations/small data sets $\rightarrow$ frequent interactions.
  - Multi-core – hardware support, low latency.
- Coarse grain: large independent computations/large data sets $\rightarrow$ infrequent interactions.
  - MPI – high latency.
- Opposite to load balancing goals.



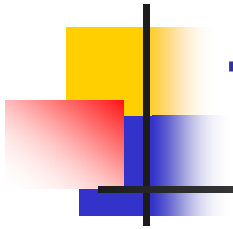
Locality

- Temporal locality.
- Spatial locality.
- Use cache/paging efficiently.
  - Even more important for parallel programs.
- Locality effect: Reduce dependencies.



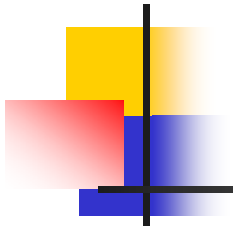
Trade-offs

- Identify the 10% code taking 90% time.
  - Not enough for parallel programs but still useful.
- Computation vs. Communication
  - Overlap com. & comp. – trylocks, async I/O.
  - Redundant comp. – recompute (cheaper than communication).
    - Reduces dependencies & sync. costs.
- Memory vs. Parallelism
  - private memory – copy – improve locality
  - padding – avoid false sharing



Trade-offs

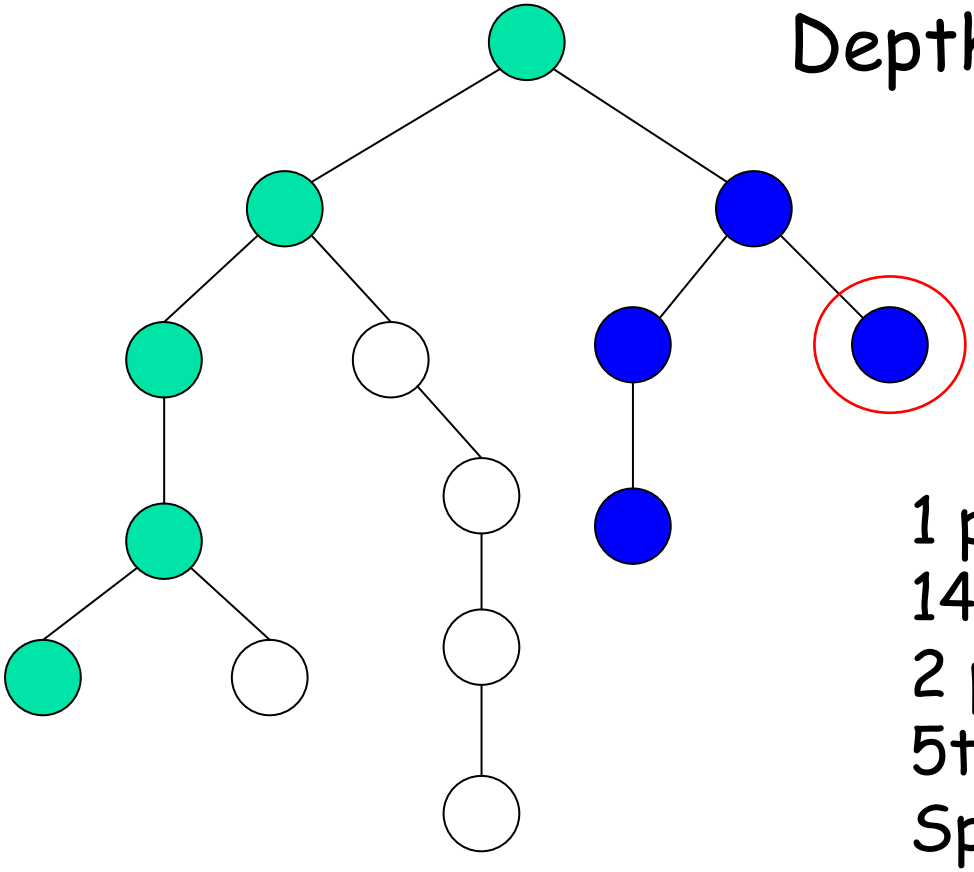
- High parallelism vs. overhead
  - need to combine/reduce results eventually
  - load balance – communication
  - granularity – batching is useful for communication but may result in idling



Measuring Performance

- Execution time, FLOPS – limited.
- Speedup, efficiency – useful.
  - Superlinear speedup: either performance anomalies (different work) or increased locality (caches) that overcomes overheads. Theoretically $S \leq p$.
  - Speedup may vary over different machines.
  - True vs. relative speedup.
  - Cold starts – cache/page issues.
  - I/O activities.

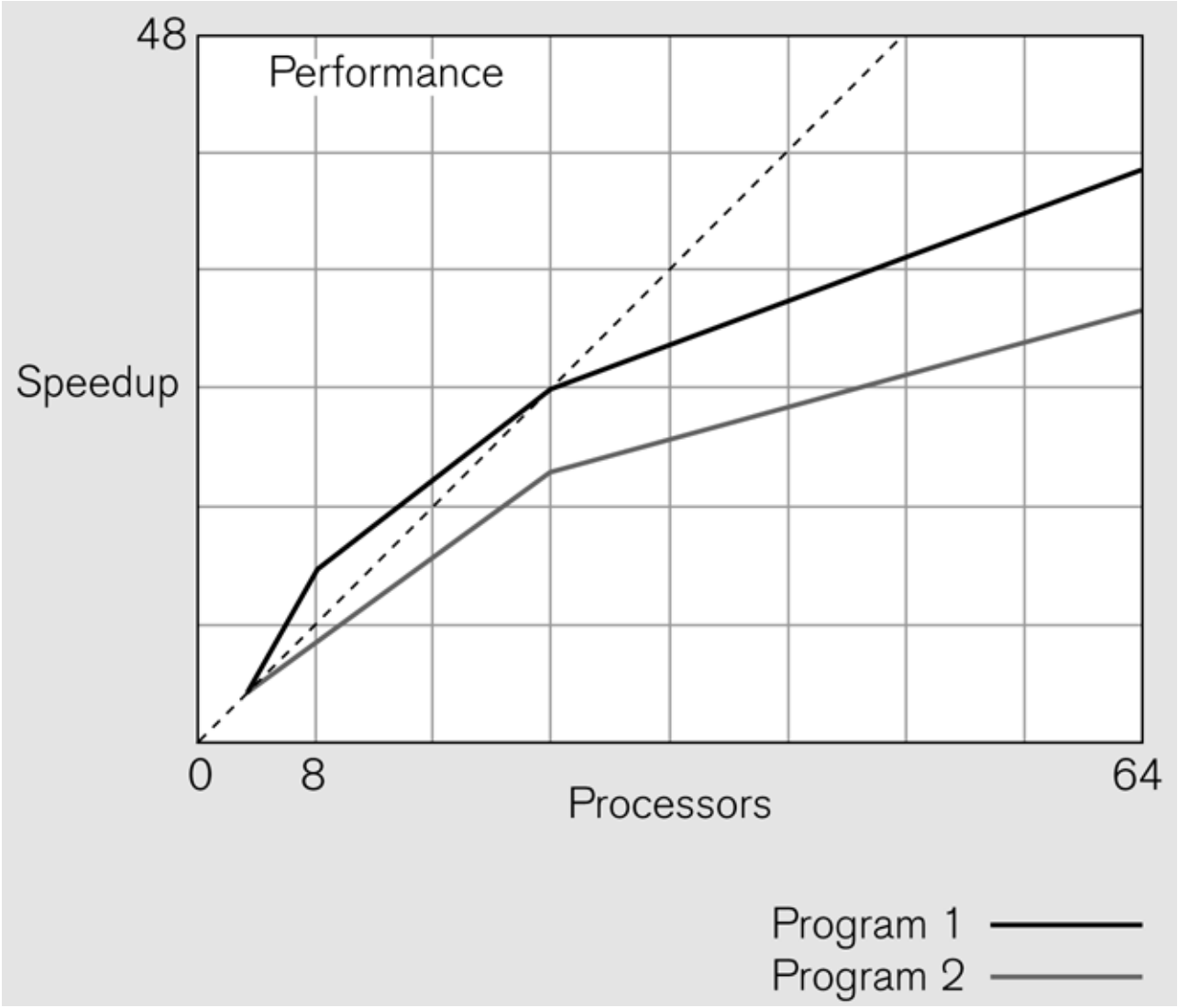
Speedup – Example

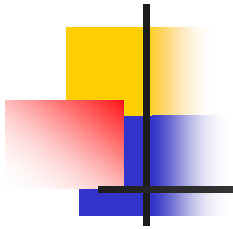


Depth-first Search

1 processing element:
$14t_c$.
2 processing elements:
$5t_c$.
Speedup: 2.8.

Example

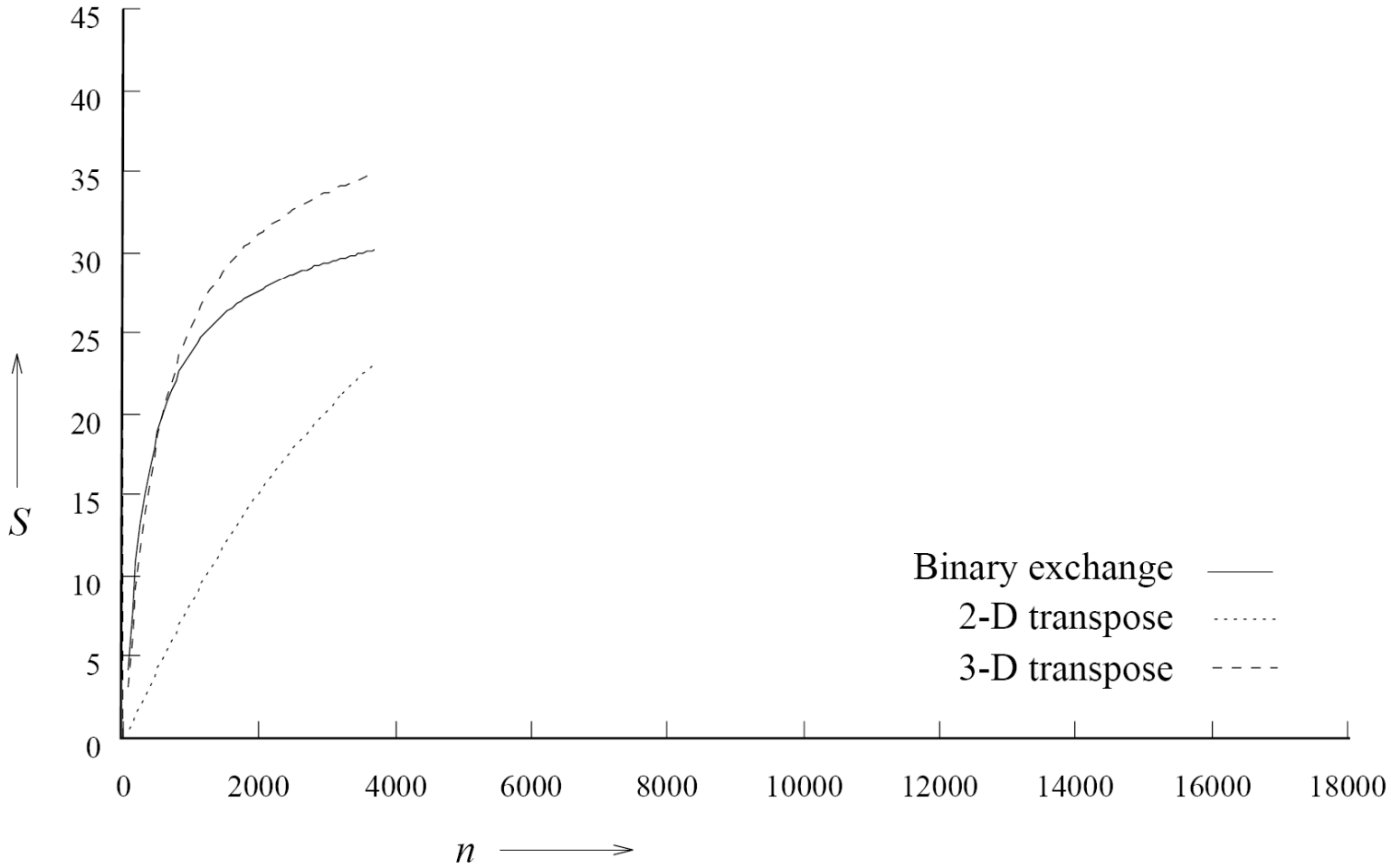

48
Performance
Speedup
0
8
64
Processors
Program 1
Program 2



Scalability of Parallel Systems

- In practice: Develop and test on small systems with small problems.
- Problem: What happens for the real large problems on large systems?
  - Difficult to extrapolate results.

Problem with Extrapolation



Rewriting

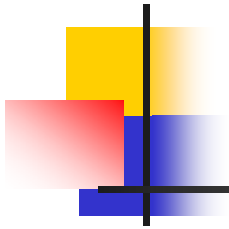
- Rewrite efficiency (E):

$$\begin{cases} E = \dfrac{S}{p} = \dfrac{T_S}{pT_p} \Rightarrow E = \dfrac{1}{1 + \dfrac{T_0}{T_S}} \\ pT_p = T_0 + T_S \end{cases}$$

$T_S = W$

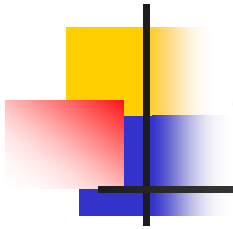
- What does it tell us?

Note: $T_0 = f(p)$ increasing.



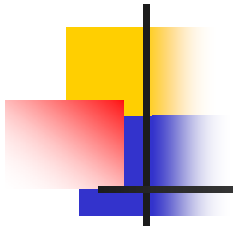
Scalable Parallel System

- Can maintain its efficiency constant when increasing the number of processors and the size of the problem.
- In many cases $T_0=f(T_S,p)$ and grows sub-linearly with T_S. It can be possible to increase p and T_S and keep E constant.
- Scalability measures the ability to increase speedup in function of *p*.



Cost-Optimality

- Cost optimal parallel systems have efficiency $\Theta(1)$.
- Scalability and cost-optimality are linked.
- Adding number example: becomes cost-optimal when $n=\Omega(p \log p)$.



Scalable System

- Efficiency can be kept constant when
  - the number of processors increases and
  - the problem size increases.
- At which rate the problem size should increase with the number of processors?
  - The rate determines the degree of scalability.
- In complexity, problem size = size of the input. Here = number of basic operations to solve the problem. Noted W ($\sim T_s$).

Isoefficiency Function

- For scalable systems efficiency can be kept constant if T_0/W is kept constant.

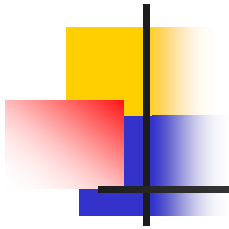
For a target E $$E = \frac{1}{1 + T_o(W,p)/W},$$

Keep this constant $$\frac{T_o(W,p)}{W} = \frac{1-E}{E},$$

Isoefficiency function $$W = \frac{E}{1-E} T_o(W,p).$$

How to increase work in function of p to keep the same efficiency (*iso*)?

$W=KT_0(W,p)$



Example

- Adding number: We saw that $T_0=2p\log p$.
- We get $W=K\,2p\log p$.
- If we increase p to p', the problem size must be increased by $(p'\log p')/(p\log p)$ to keep the same efficiency.
  - Increase p by p'/p.
  - Increase n by $(p'\log p')/(p\log p)$.

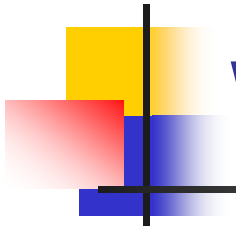
Example

$$T_o - p^{3/2} + p^{3/4}W^{3/4}$$

$$W = Kp^{3/2}.$$

$$\begin{aligned} W &= Kp^{3/4}W^{3/4} \\ W^{1/4} &= Kp^{3/4} \\ W &= K^4p^3 \end{aligned}$$

Isoefficiency = $\Theta(p^3)$.



Why?

- After isoefficiency analysis, we can test our parallel program with few processors and then predict what will happen for larger systems.

Link to Cost-Optimality

A parallel system is cost-optimal iff $pT_p = \Theta(W)$.

$$W + T_o(W,p) = \Theta(W)$$

$$T_o(W,p) = O(W)$$

$$W = \Omega(T_o(W,p))$$

A parallel system is cost-optimal iff its overhead (T_0) does not exceed (asymptotically) the problem size.

Minimum Execution Time

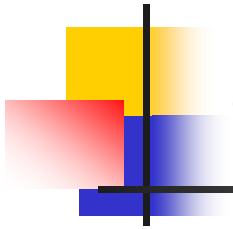
- If T_P ↘↗ in function of p, we want its minimum. Find p_0 s.t. $dT_P/dp=0$.
- Adding n numbers: $T_P=n/p+2\log p$.
  $\rightarrow p_0=n/2$.
  $\rightarrow T_P^{min}=2\log n$.
- Fastest but not necessary cost-optimal.

Asymptotic Analysis of Parallel Programs

Table 5.2 Comparison of four different algorithms for sorting a given list of numbers. The table shows number of processing elements, parallel runtime, speedup, efficiency and the pT_P product.

Best?

Algorithm	A1	A2	A3	A4
p	n^2	$\log n$	n	$\sqrt{n}$
T_P	1	n	$\sqrt{n}$	$\sqrt{n}\log n$
S	$n\log n$	$\log n$	$\sqrt{n}\log n$	$\sqrt{n}$
E	$\frac{\log n}{n}$	1	$\frac{\log n}{\sqrt{n}}$	1
pT_P	n^2	$n\log n$	$n^{1.5}$	$n\log n$



Other Scalability Metrics

- Scaled speedup: speedup when problem size increases linearly in function of p.
  - Motivation: constraints such as memory linear in function of p.
  - Time and memory constrained.