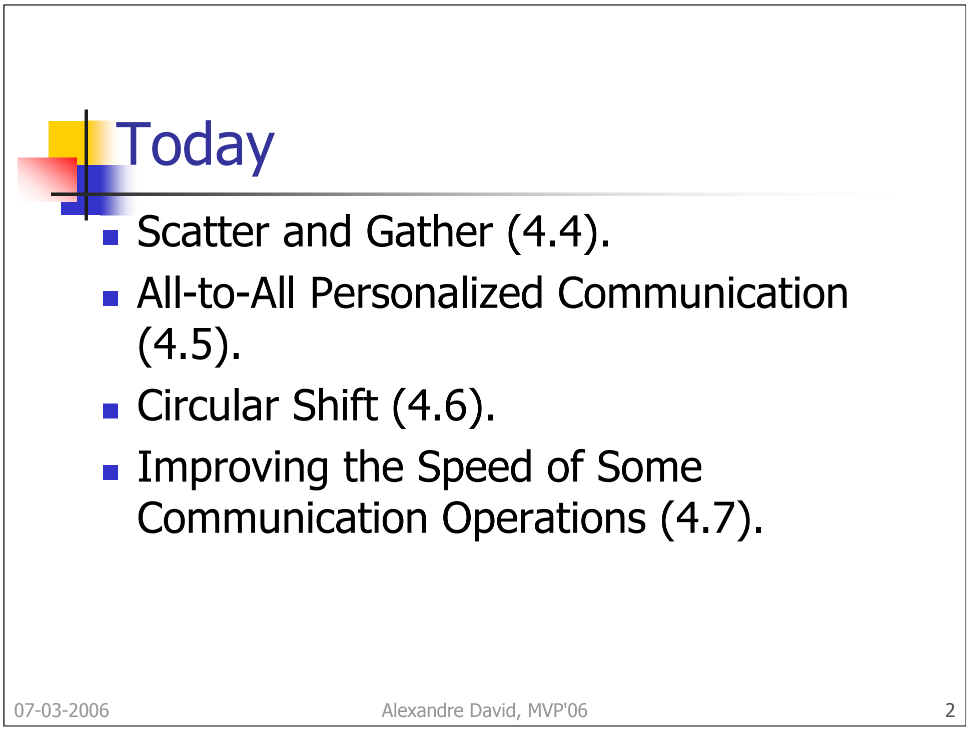
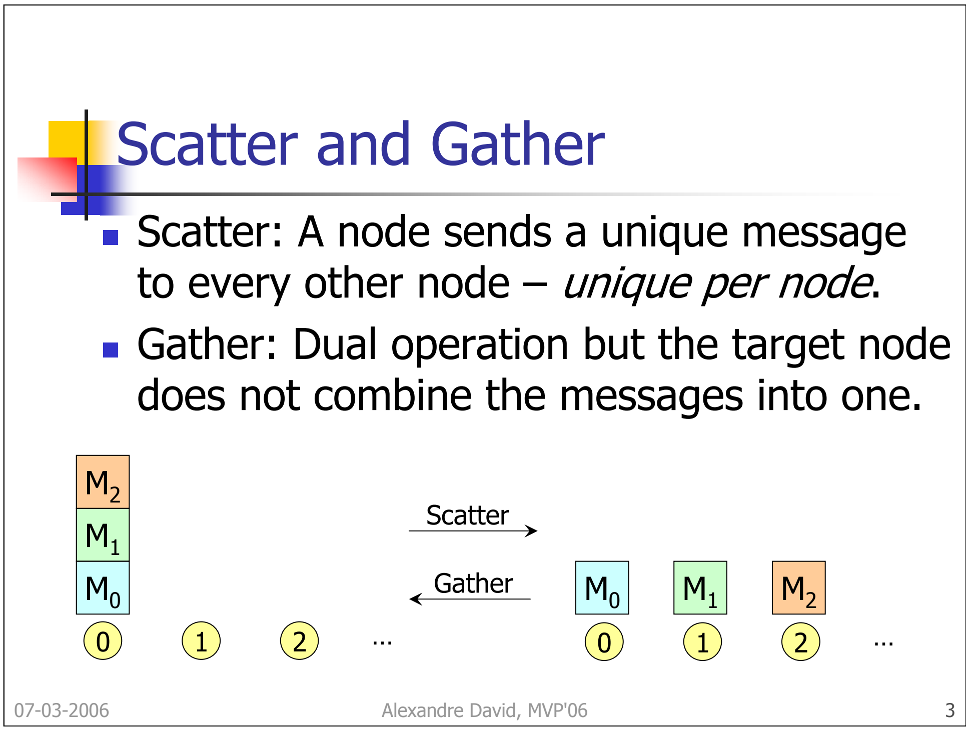
Basic Communication Operations (cont.)

Alexandre David
B2-206



Today

- Scatter and Gather (4.4).
- All-to-All Personalized Communication (4.5).
- Circular Shift (4.6).
- Improving the Speed of Some Communication Operations (4.7).

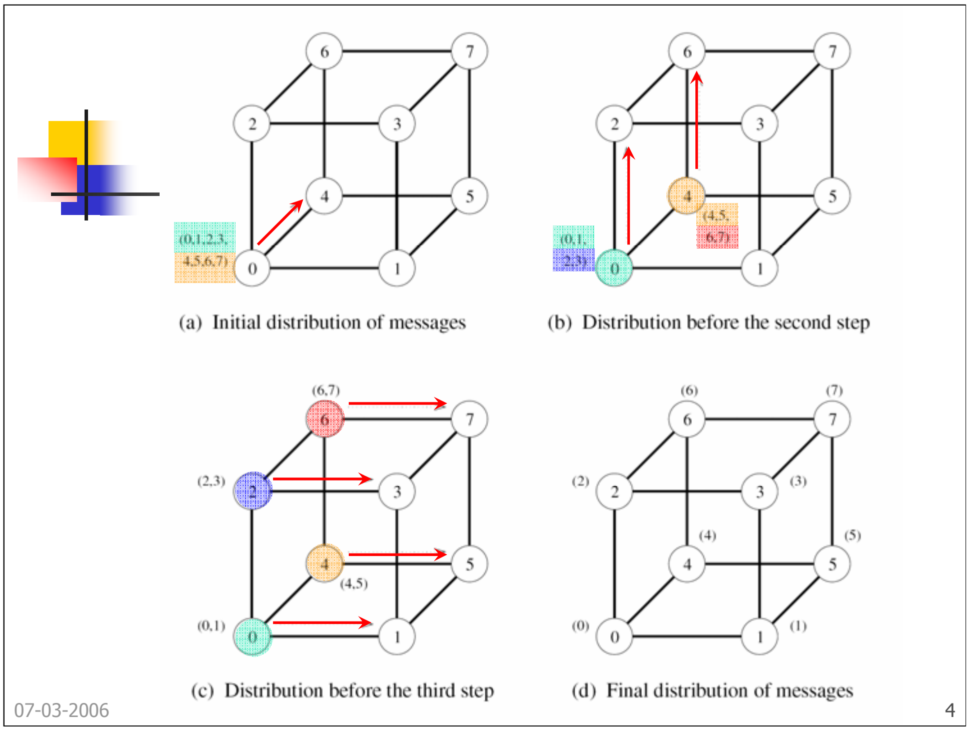


Scatter and Gather

- Scatter: A node sends a unique message to every other node – *unique per node*.
- Gather: Dual operation but the target node does not combine the messages into one.

Do you see the difference with one-to-all broadcast and all-to-one reduce? Communication pattern similar.

Scatter = one-to-all personalized communication.



(a) Initial distribution of messages

(b) Distribution before the second step

(c) Distribution before the third step

(d) Final distribution of messages

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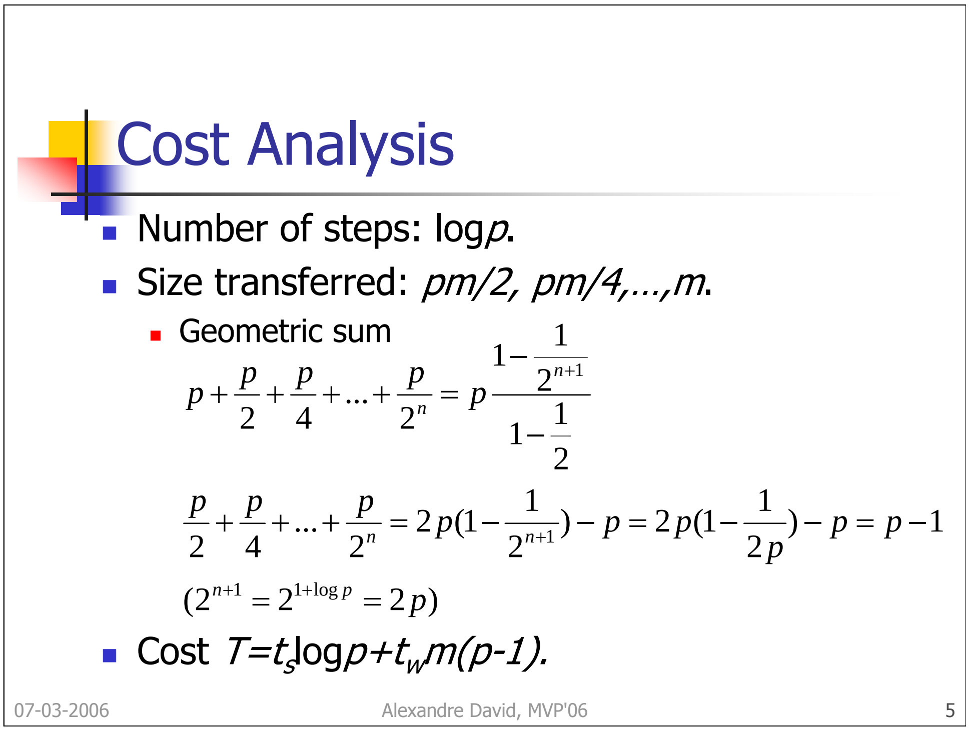
The pattern of communication is identical with one-to-all broadcast but the size and the content of the messages are different. Scatter is the reverse operation. This algorithm can be applied for other topologies.

How many steps? What's the cost?



Cost Analysis

- Number of steps: log*p*.
- Size transferred: *pm/2, pm/4,…,m*.
  - Geometric sum

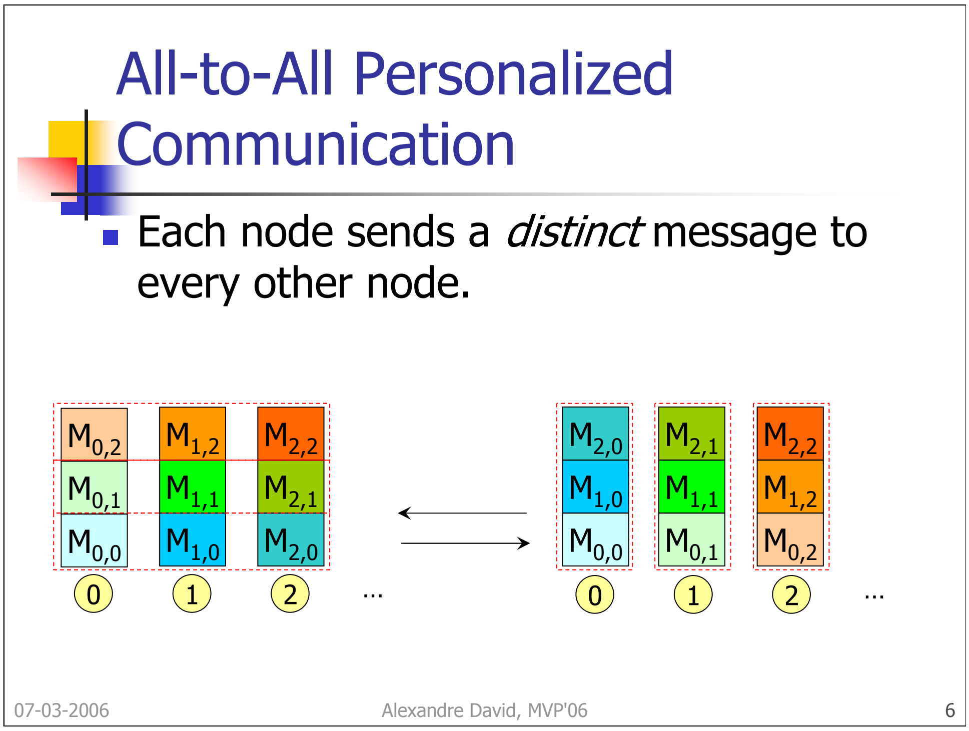
$$p+\frac{p}{2}+\frac{p}{4}+\ldots+\frac{p}{2^n}=p\frac{1-\frac{1}{2^{n+1}}}{1-\frac{1}{2}}$$

$$\frac{p}{2}+\frac{p}{4}+\ldots+\frac{p}{2^n}=2p(1-\frac{1}{2^{n+1}})-p=2p(1-\frac{1}{2p})-p=p-1$$

$$(2^{n+1}=2^{1+\log p}=2p)$$

- Cost $T=t_s\log p+t_w m(p\text{-}1)$.

The term $t_w m(p\text{-}1)$ is a lower bound for any topology because the message of size m has to be transmitted to p-1 nodes, which gives the lower bound of m(p-1) words of data.



All-to-All Personalized Communication

- Each node sends a *distinct* message to every other node.

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See the difference with all-to-all broadcast?

All-to-all personalized communication = total exchange.

Result = transpose of the input (if seen as a matrix).

Example: Transpose

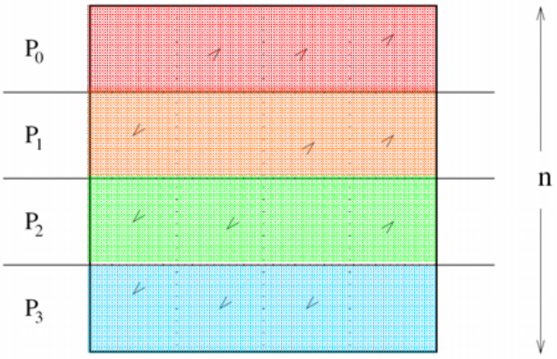
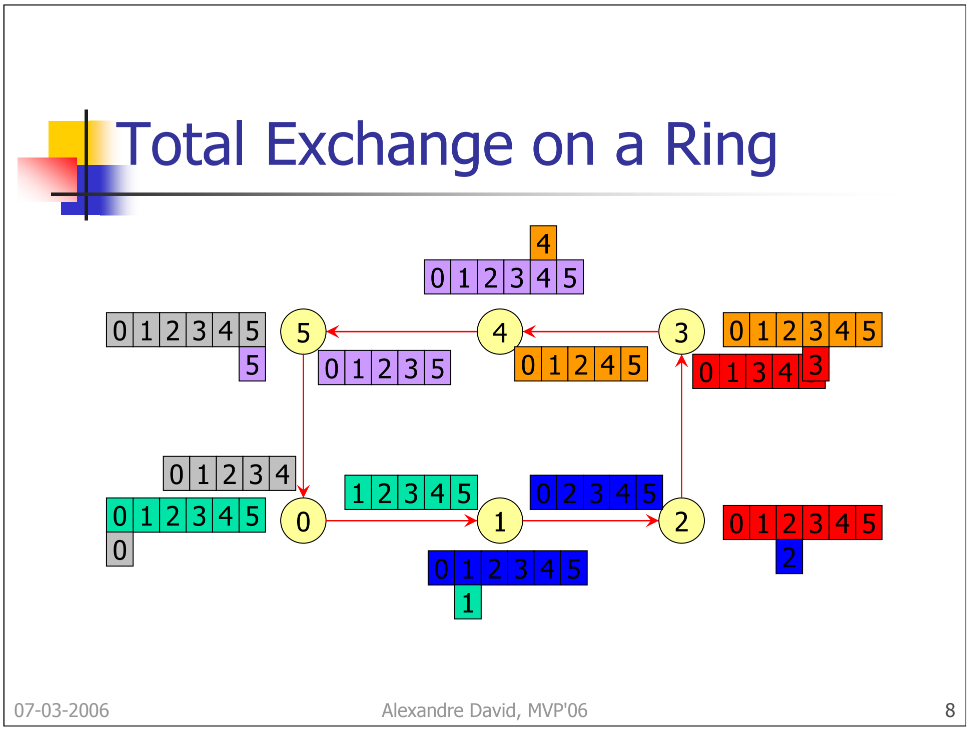


Figure 4.17 All-to-all personalized communication in transposing a 4 × 4 matrix using four processes.



Total Exchange on a Ring

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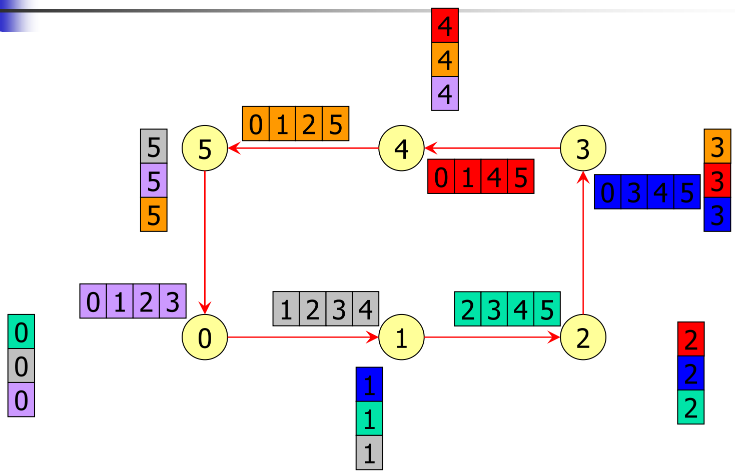
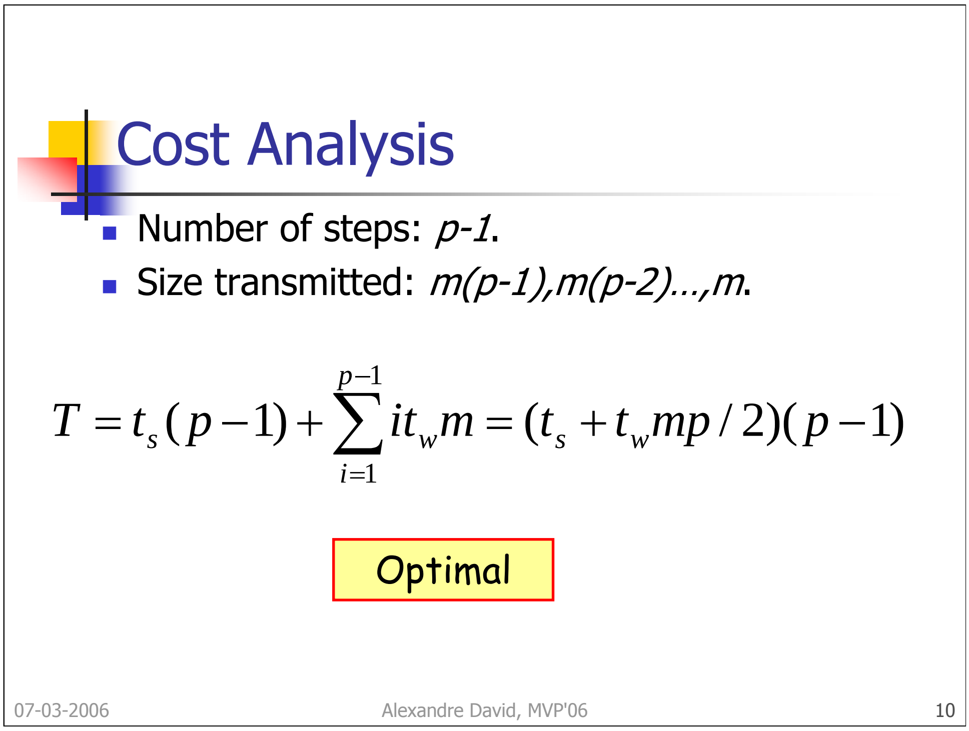
Total Exchange on a Ring





Cost Analysis

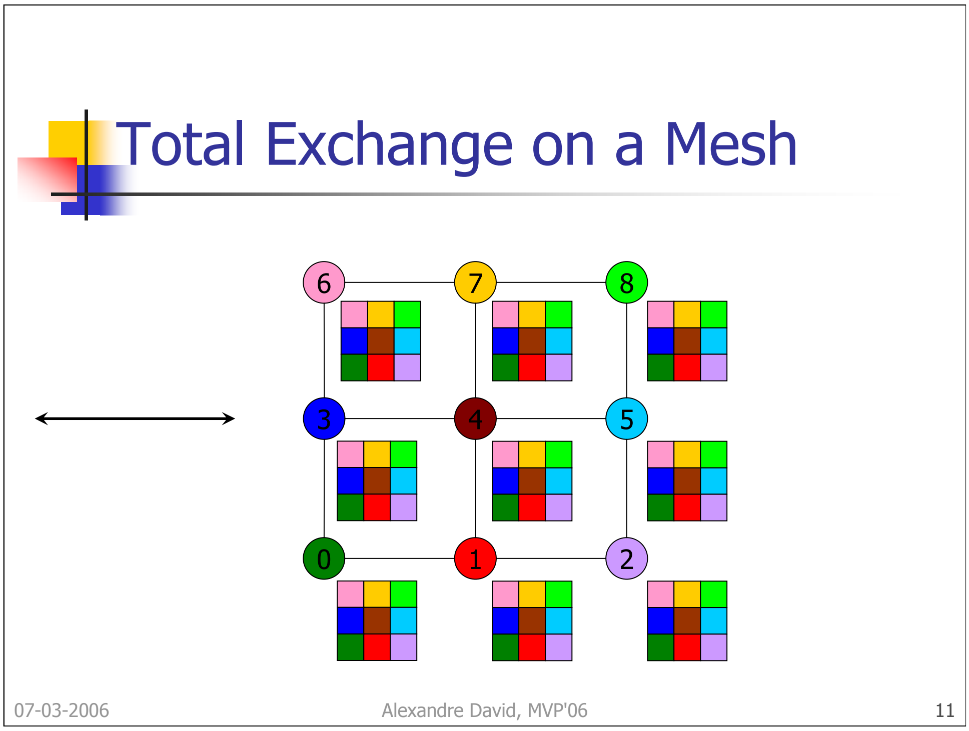
- Number of steps: *p-1*.
- Size transmitted: *m(p-1),m(p-2)…,m*.

$$T = t_s(p-1) + \sum_{i=1}^{p-1} i t_w m = (t_s + t_w mp/2)(p-1)$$

$$\boxed{\text{Optimal}}$$

In average we transmit mp/2 words, whereas the linear all-to-all transmits m words. If we make this substitution, we have the same cost as the previous linear array procedure. To really see optimality we have to check the lowest possible needed data transmission and compare it to T.

Average distance a packet travels = p/2. There are p nodes that need to transmit m(p-1) words. Total traffic = m(p-1)*p/2*p. Number of link that support the load = p, to communication time ≥ t_wm(p-1)p/2.



Total Exchange on a Mesh

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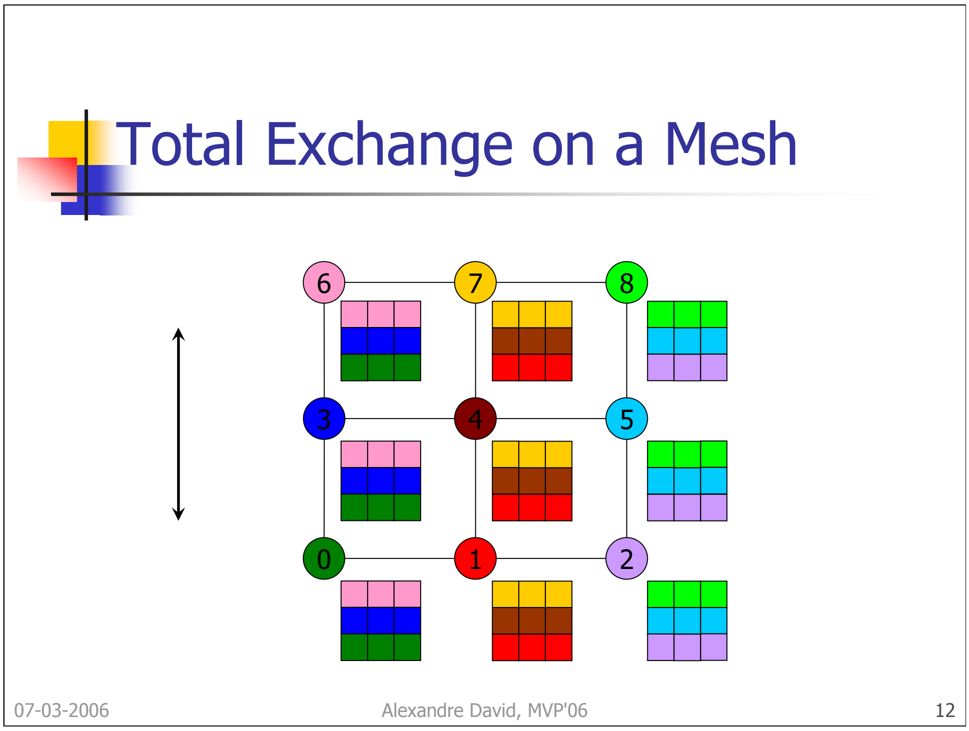
We use the procedure of the ring/array.



Total Exchange on a Mesh

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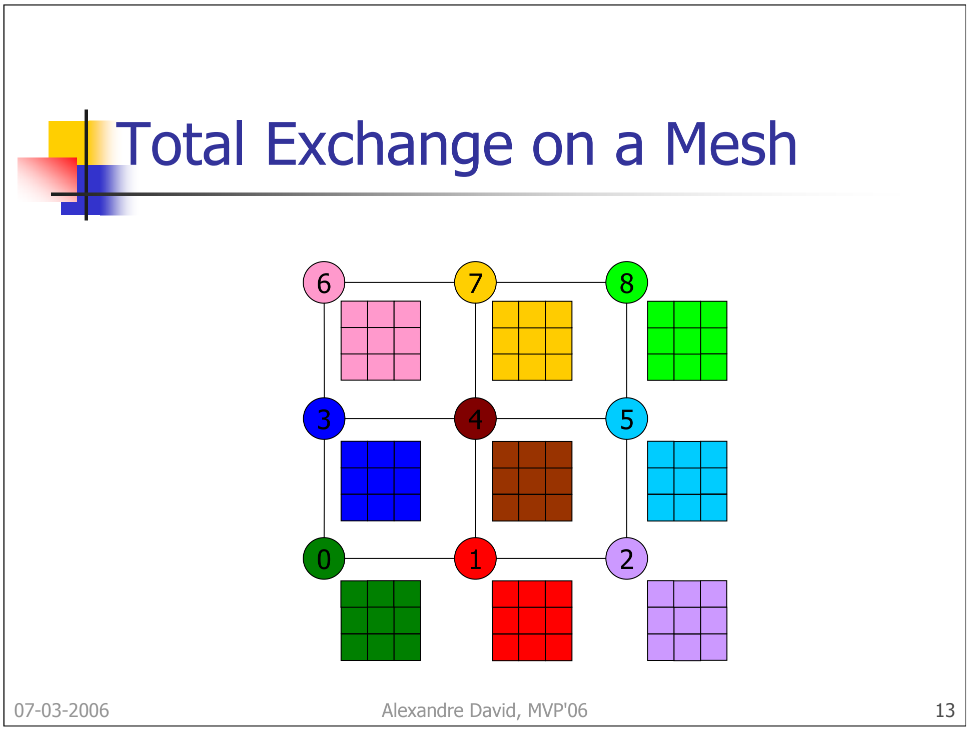

Total Exchange on a Mesh

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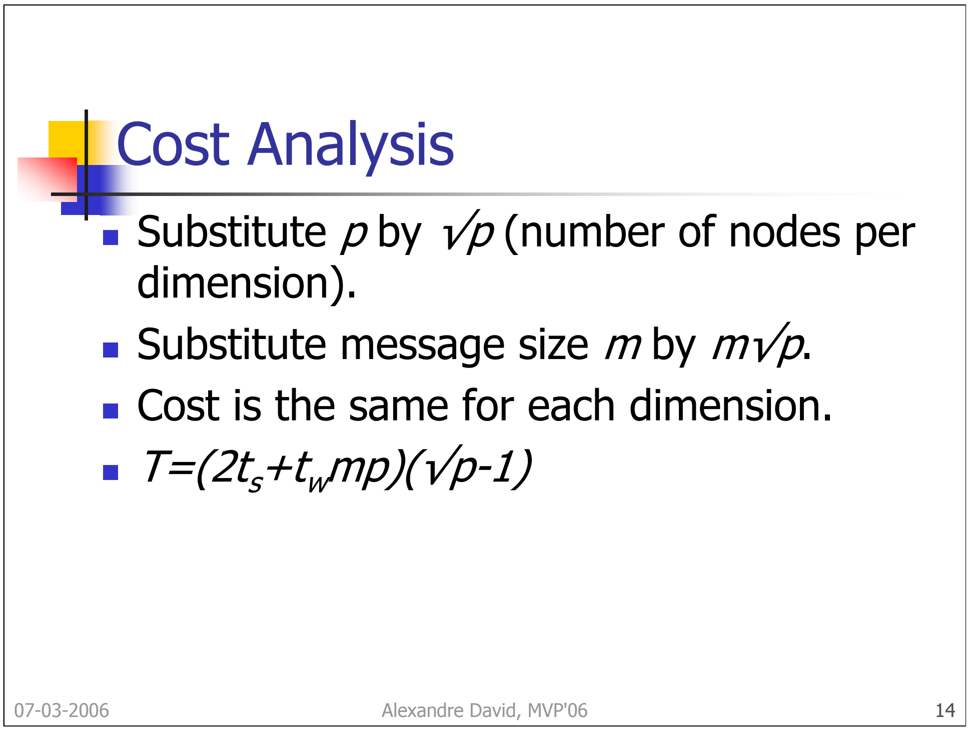
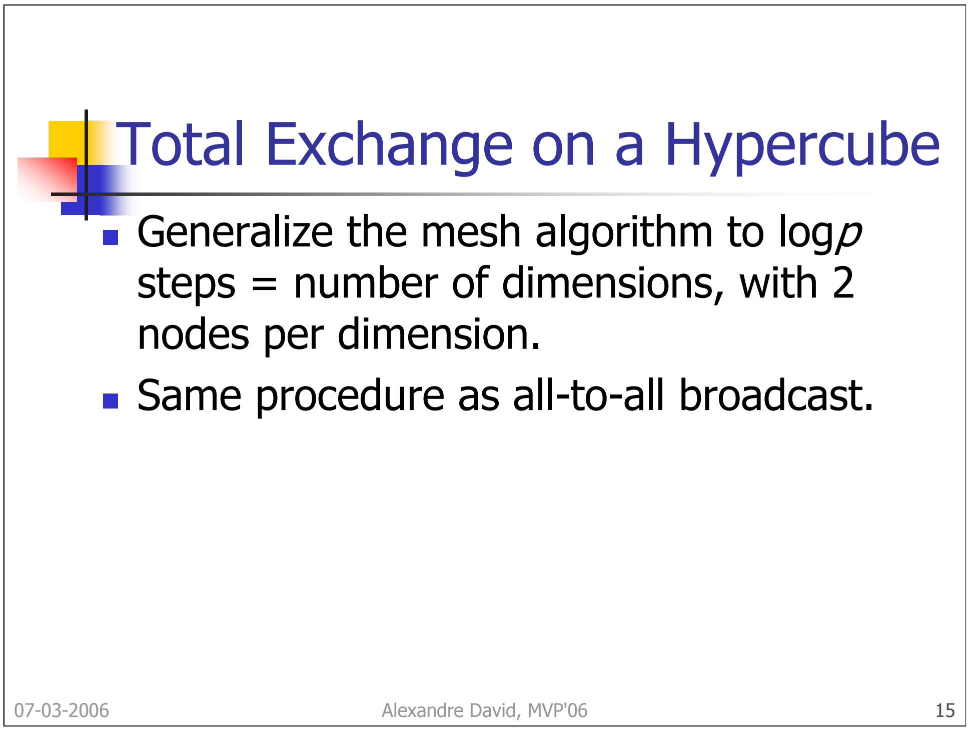
We use the procedure of the ring/array.



Cost Analysis

- Substitute p by $\sqrt{p}$ (number of nodes per dimension).
- Substitute message size m by $m\sqrt{p}$.
- Cost is the same for each dimension.
- $T=(2t_s+t_w mp)(\sqrt{p}-1)$

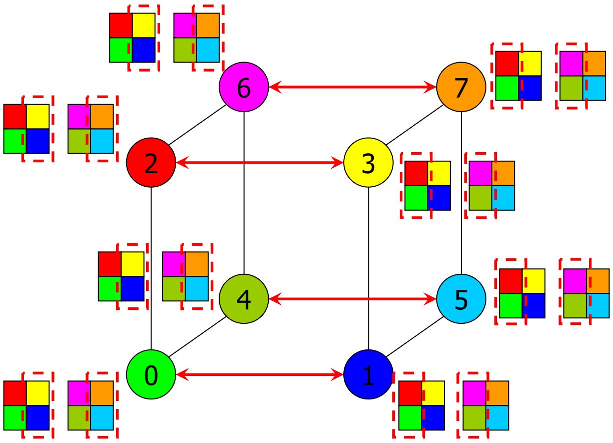
We have p(√p-1)m words transferred, looks worse than lower bound in (p-1)m but no congestion. Notice that the time for data rearrangement is not taken into account. It is almost optimal (by a factor 4), see exercise.



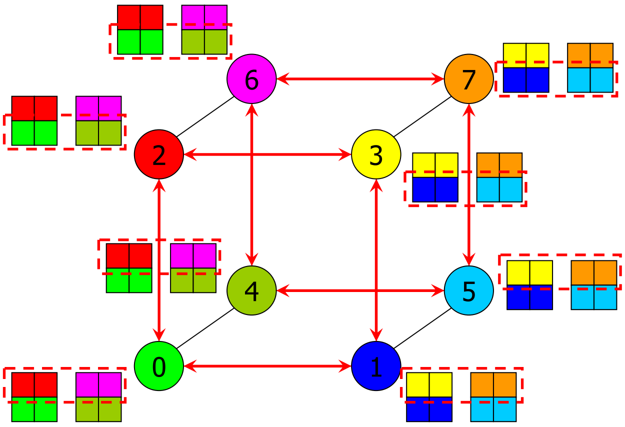
Total Exchange on a Hypercube

- Generalize the mesh algorithm to log$p$ steps = number of dimensions, with 2 nodes per dimension.
- Same procedure as all-to-all broadcast.

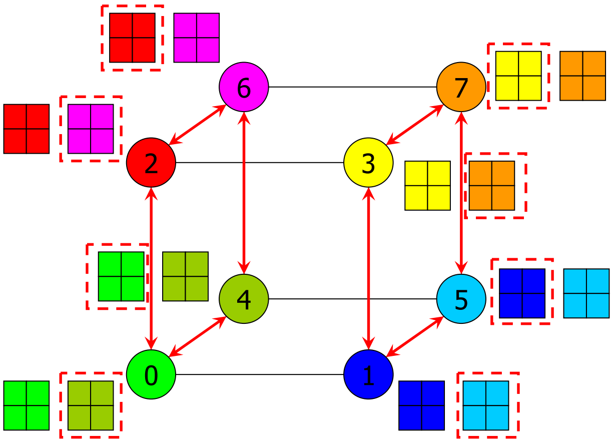
Total Exchange on a Hypercube

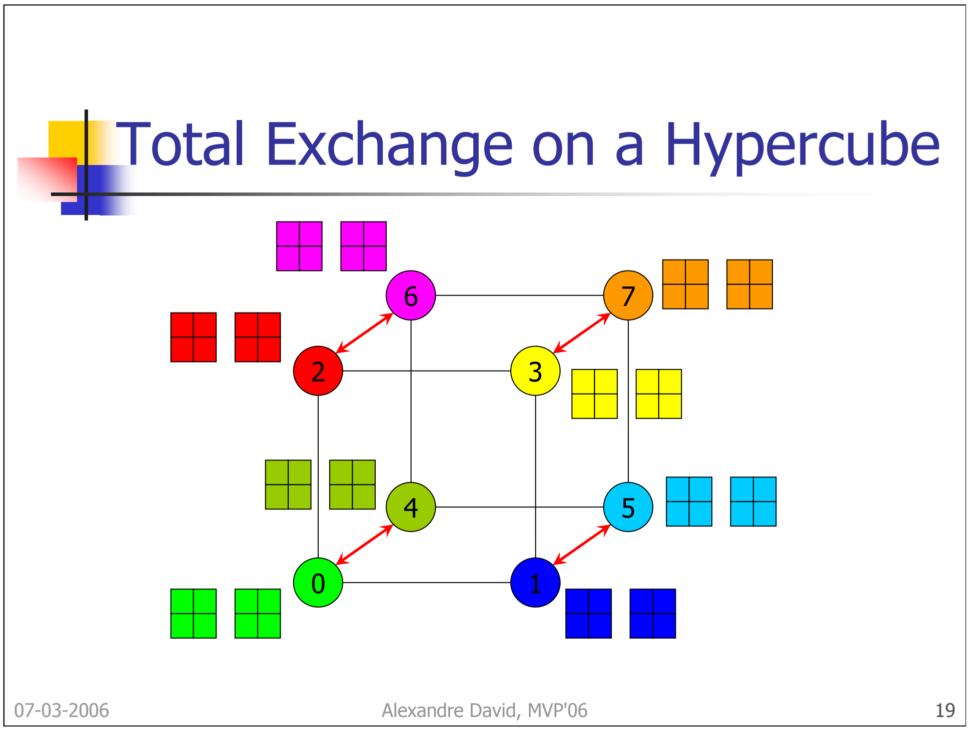


Total Exchange on a Hypercube

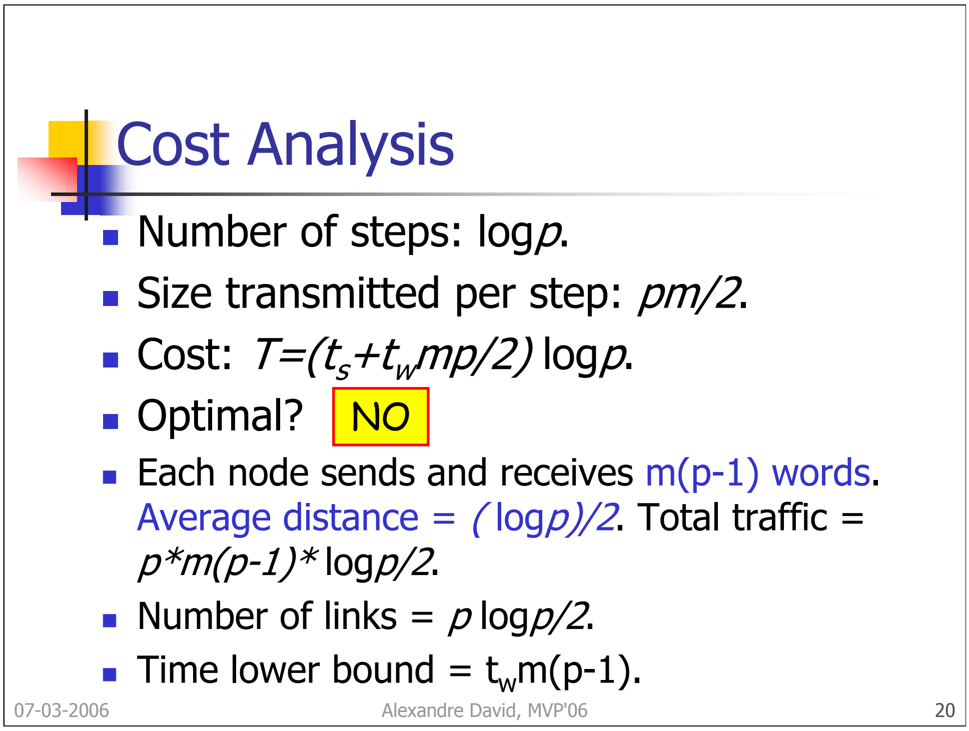


Total Exchange on a Hypercube





Total Exchange on a Hypercube

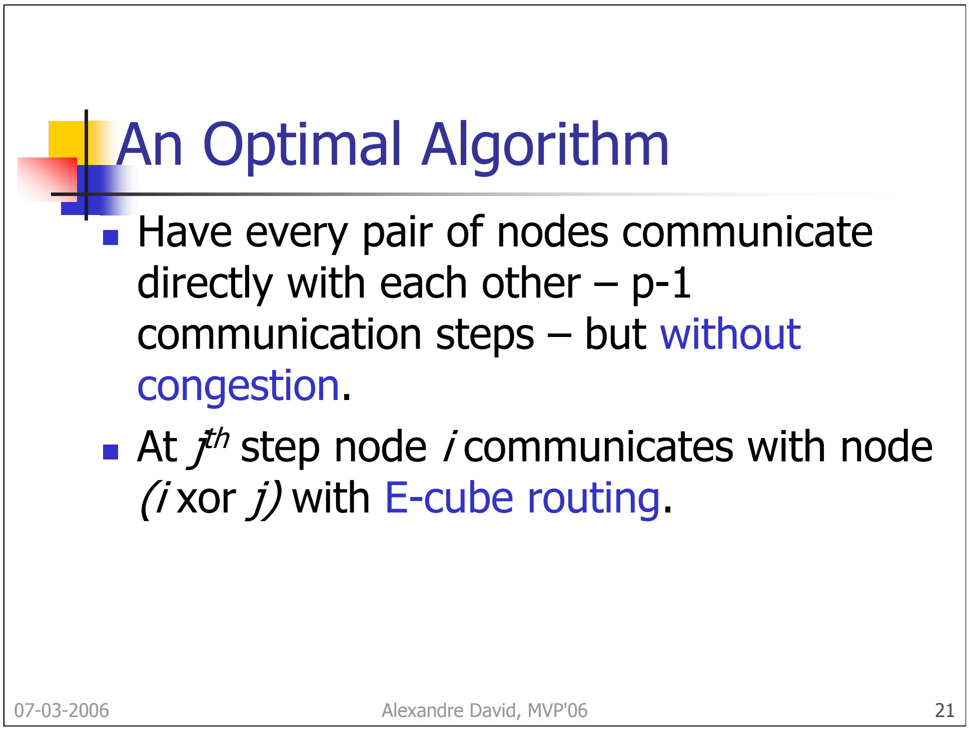


Cost Analysis

- Number of steps: log$p$.
- Size transmitted per step: $pm/2$.
- Cost: $T=(t_s+t_w mp/2)\log p$.
- Optimal? NO
- Each node sends and receives m(p-1) words. Average distance = $(\log p)/2$. Total traffic = $p*m(p-1)*\log p/2$.
- Number of links = $p\log p/2$.
- Time lower bound = $t_w m(p-1)$.

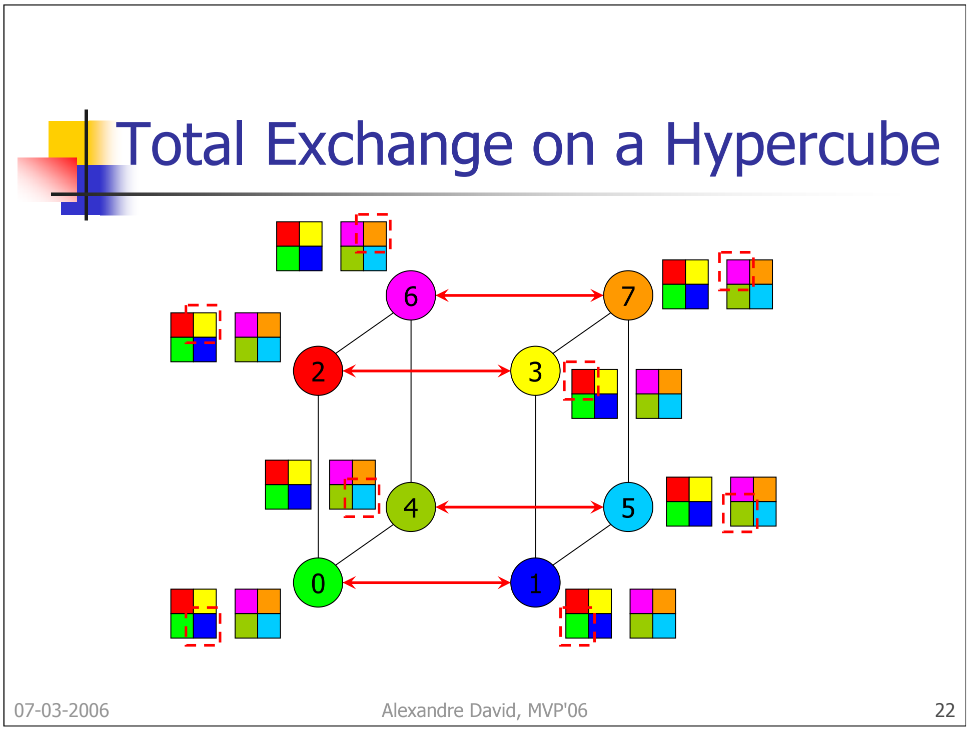
Notes:

1. No congestion.
2. Bi-directional communication.
3. How to conclude if an algorithm is optimal or not: Check the possible lowest bound and see if the algorithm reaches it.



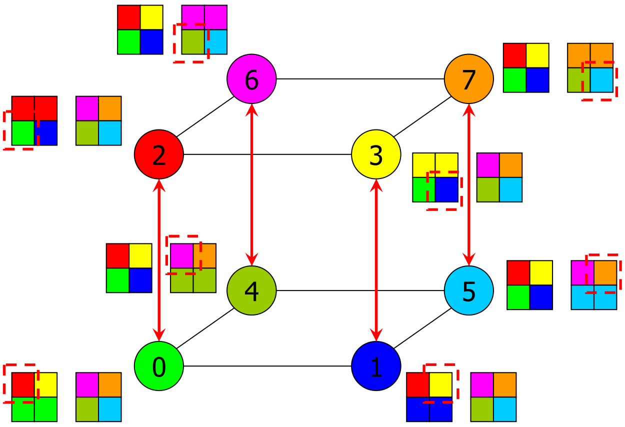
An Optimal Algorithm

- Have every pair of nodes communicate directly with each other – p-1 communication steps – but without congestion.
- At j^{th} step node i communicates with node $(i \text{ xor } j)$ with E-cube routing.

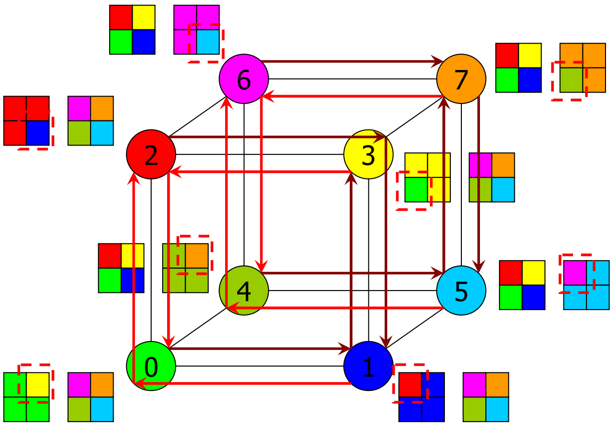


Total Exchange on a Hypercube

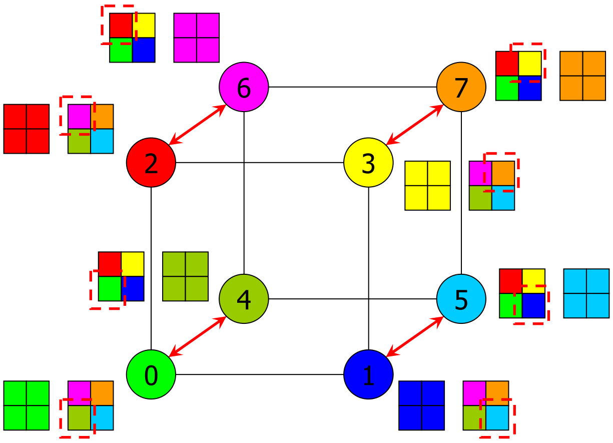
Total Exchange on a Hypercube



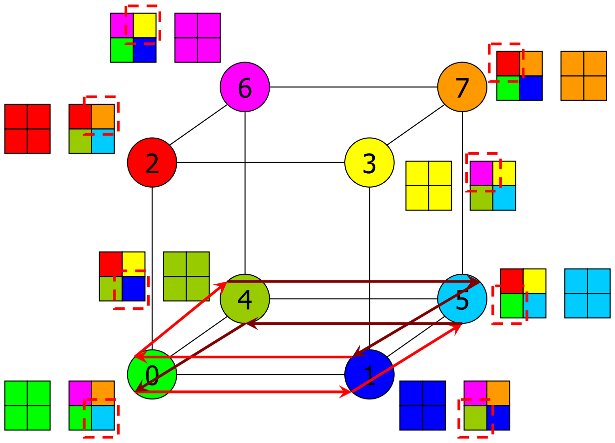
Total Exchange on a Hypercube

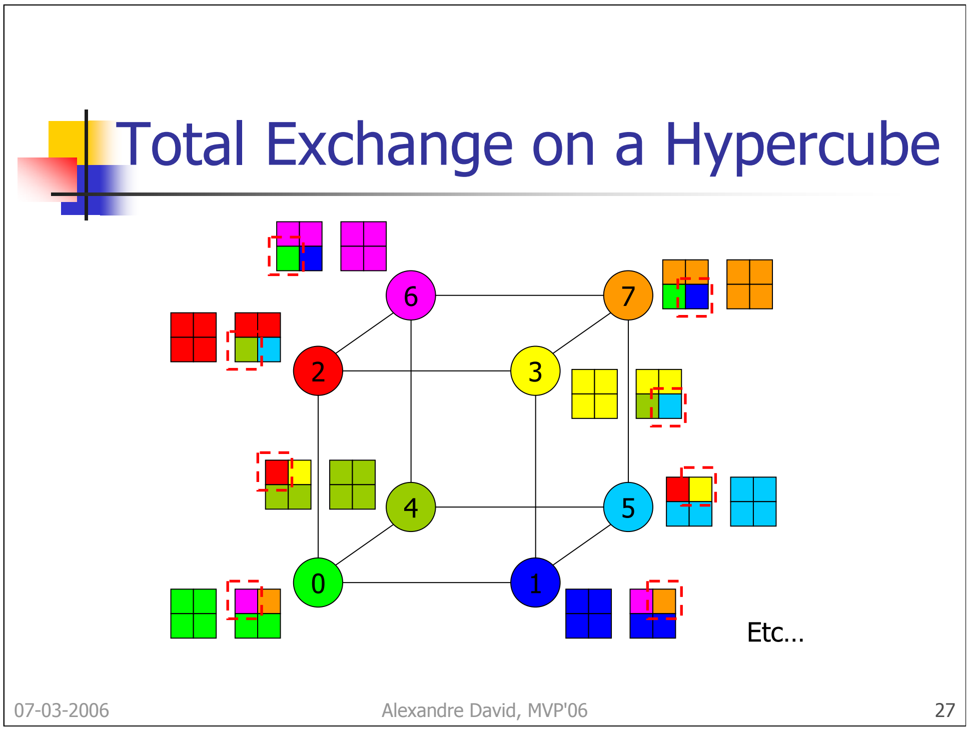


Total Exchange on a Hypercube



Total Exchange on a Hypercube





Total Exchange on a Hypercube

Etc…

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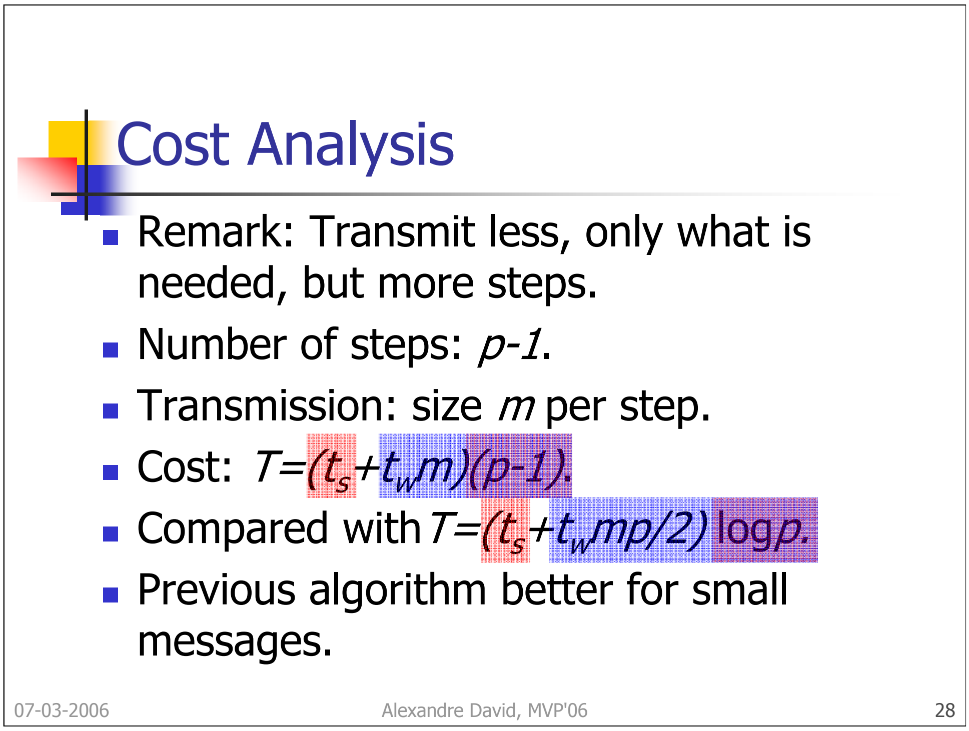
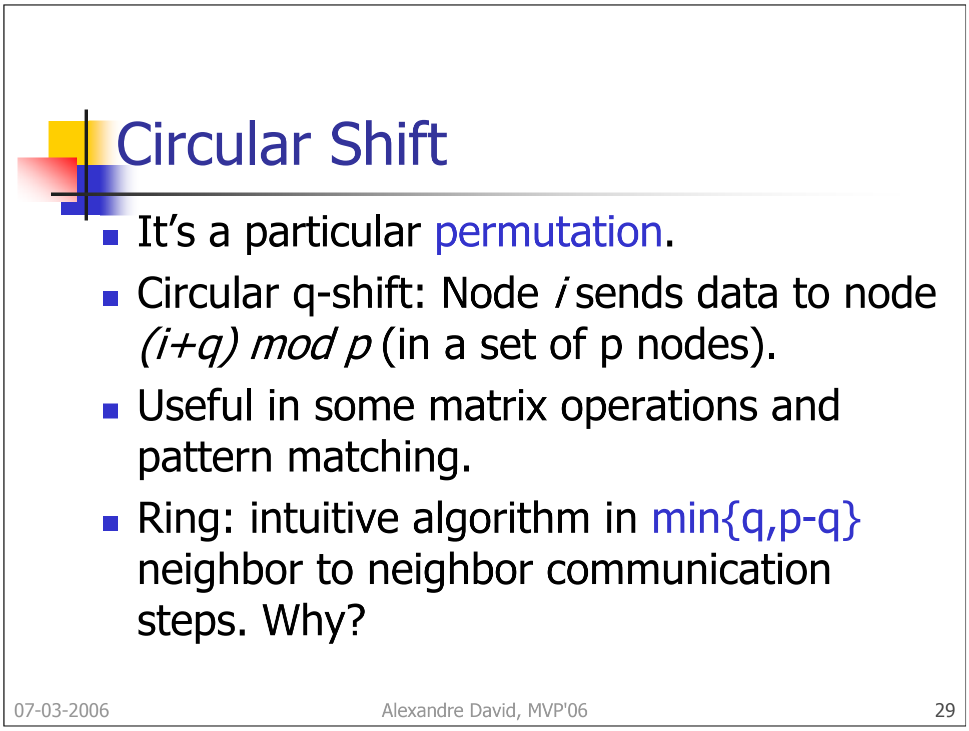
Point: Transmit less, only to the needed node, and avoid congestion with E-cube routing.



Cost Analysis

- Remark: Transmit less, only what is needed, but more steps.
- Number of steps: $p-1$.
- Transmission: size m per step.
- Cost: $T=(t_s+t_w m)(p-1)$.
- Compared with $T=(t_s+t_w mp/2)\log p$.
- Previous algorithm better for small messages.

This algorithm is now optimal: It reaches the lowest bound.

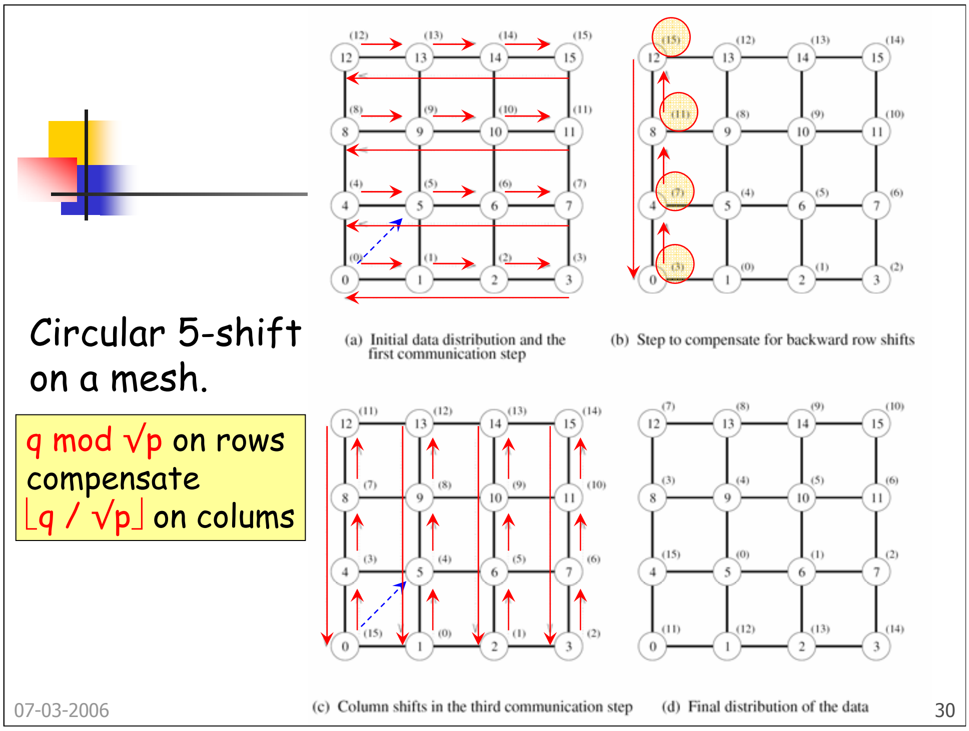


Circular Shift

- It’s a particular permutation.
- Circular q-shift: Node *i* sends data to node *(i+q) mod p* (in a set of p nodes).
- Useful in some matrix operations and pattern matching.
- Ring: intuitive algorithm in min{q,p-q} neighbor to neighbor communication steps. Why?

A permutation = a redistribution in a set.

You can call the shift a rotation in fact.



Circular 5-shift on a mesh.

$q \bmod \sqrt{p}$ on rows compensate $\lfloor q / \sqrt{p} \rfloor$ on colums

(a) Initial data distribution and the first communication step

(b) Step to compensate for backward row shifts

(c) Column shifts in the third communication step

(d) Final distribution of the data

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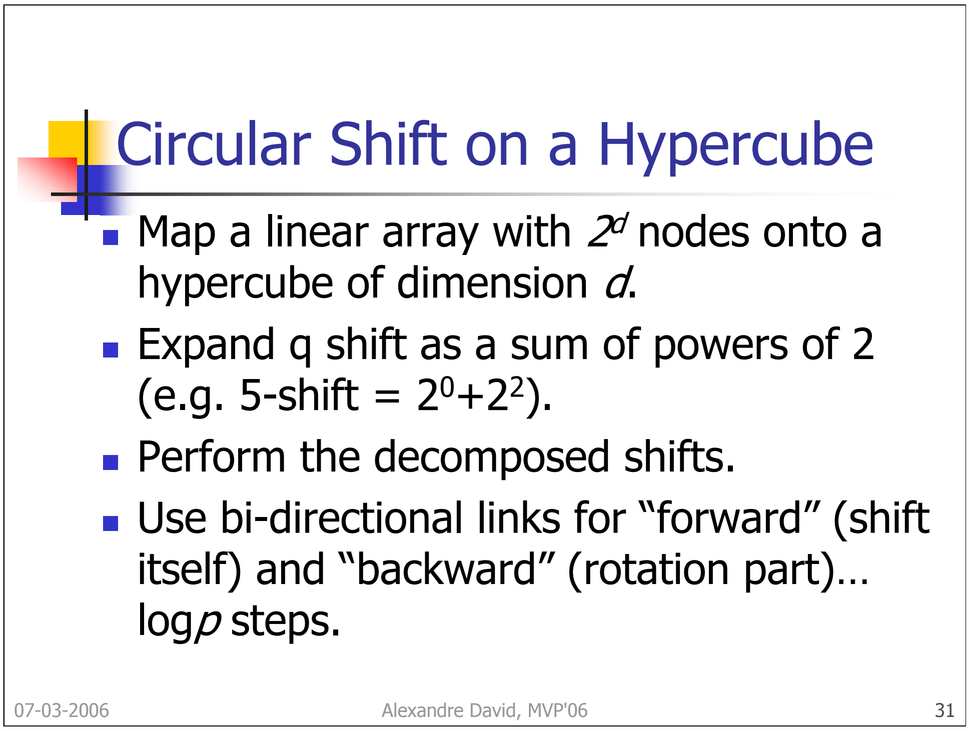
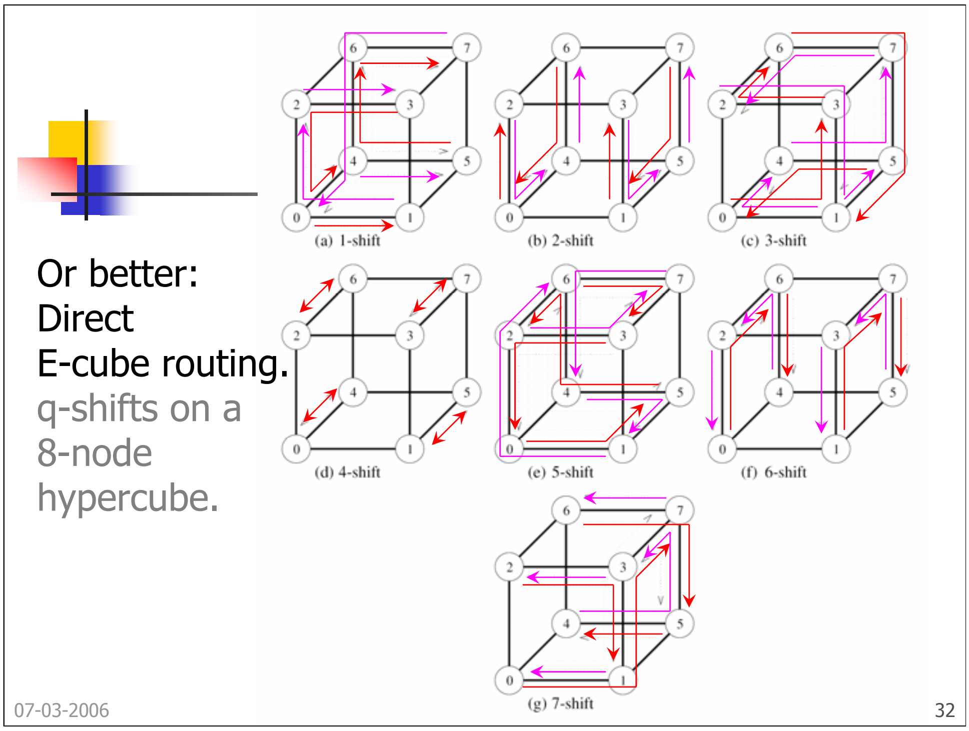


Circular Shift on a Hypercube

- Map a linear array with *2^d* nodes onto a hypercube of dimension *d*.
- Expand q shift as a sum of powers of 2 (e.g. 5-shift = 2^0+2^2).
- Perform the decomposed shifts.
- Use bi-directional links for “forward” (shift itself) and “backward” (rotation part)… log*p* steps.

Backward and forward my be misleading in the book.

Interesting but not best solution, no idea why it’s mentioned if the optimal solution is simpler.



Or better: Direct E-cube routing.

q-shifts on a 8-node hypercube.

(a) 1-shift

(b) 2-shift

(c) 3-shift

(d) 4-shift

(e) 5-shift

(f) 6-shift

(g) 7-shift

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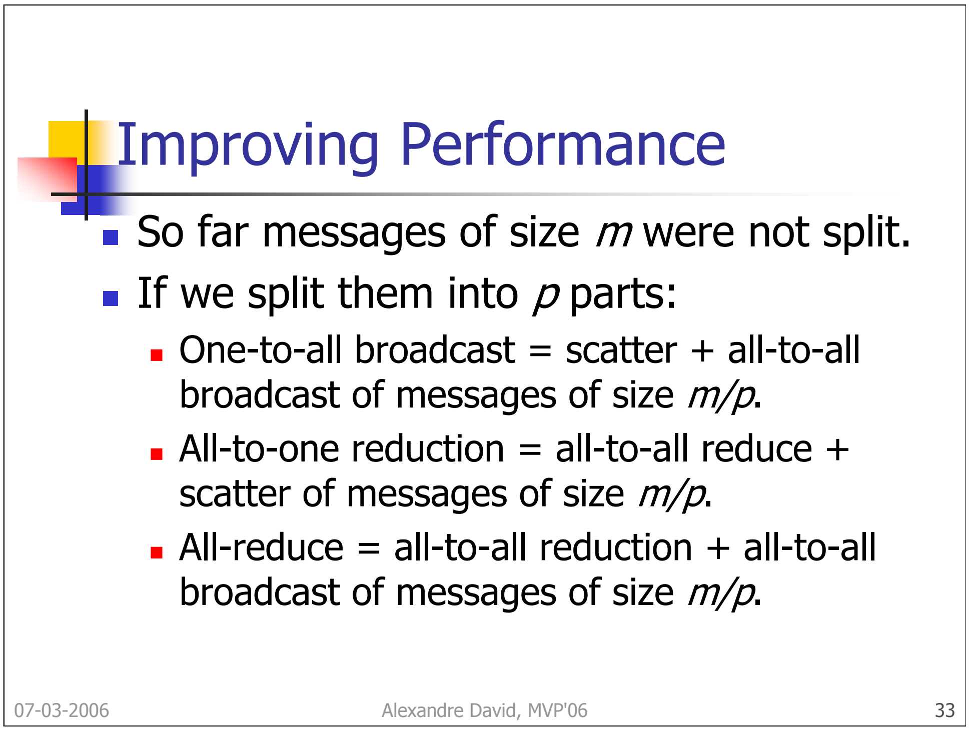
Exercise: Check the E-cube routing and convince me that there is no congestion.

Communication time = $t_s + t_w m$ in one step.



Improving Performance

- So far messages of size *m* were not split.
- If we split them into *p* parts:
  - One-to-all broadcast = scatter + all-to-all broadcast of messages of size *m/p*.
  - All-to-one reduction = all-to-all reduce + scatter of messages of size *m/p*.
  - All-reduce = all-to-all reduction + all-to-all broadcast of messages of size *m/p*.