Principle Of Parallel Algorithm Design (cont.)

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B2-206

Today

- Characteristics of Tasks and Interactions (3.3).
- Mapping Techniques for Load Balancing (3.4).
- Methods for Containing Interaction Overhead (3.5).
- Parallel Algorithm Models (3.6).

So Far...

- Decomposition techniques.
  - Identify tasks.
  - Analyze with task dependency & interaction graphs.
  - Map tasks to processes.
- Now properties of tasks that affect a good mapping.
  - Task generation, size, and size of data.

Task Generation

- Static task generation.
  - Tasks are known beforehand.
  - Apply to well-structured problems.
- Dynamic task generation.
  - Tasks generated on-the-fly.
  - Tasks & task dependency graph not available beforehand.

The well-structured problem can typically be decomposed using data or recursive decomposition techniques.

Dynamic tasks generation: Exploratory or speculative decomposition techniques are generally used, but not always. Example: quicksort.

Task Sizes

- Relative amount of time for completion.
  - Uniform – same size for all tasks.
    - Matrix multiplication.
  - Non-uniform.
    - Optimization & search problems.

Typically the size of non-uniform tasks is difficult to evaluate beforehand.

Size of Data Associated with Tasks

- Important because of locality reasons.
- Different types of data with different sizes
  - Input/output/intermediate data.
- Size of context – cheap or expensive communication with other tasks.

Example of 15-puzzle has a small context: easy to communicate the tasks to different processes.

Characteristics of Task Interactions

- Static interactions.
  - Tasks and interactions known beforehand.
  - And interaction at pre-determined times.
- Dynamic interactions.
  - Timing of interaction unknown.
  - Or set of tasks not known in advance.

Static vs. dynamic.

Static or dynamic interaction pattern.

Dynamic harder to code, more difficult for MPI.

Characteristics of Task Interactions

- Regular interactions.
  - The interaction graph follows a pattern.
- Irregular interactions.
  - No pattern.

Regular vs. irregular.

Regular patterns can be exploited for efficient implementations.

Dynamic harder to code, more difficult for MPI.

Example: Image Dithering

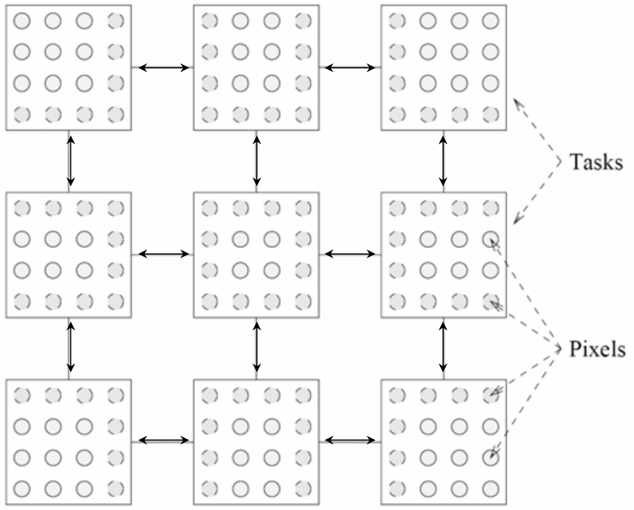


Figure 3.22 The regular two-dimensional task-interaction graph for image dithering. The pixels with dotted outline require color values from the boundary pixels of the neighboring tasks.

The color of each pixel is determined as the weighted average of its original value and the values of the neighboring pixels. Decompose into regions, 1 task/region. Pattern is a 2-D mesh. Regular pattern.

Example: Sparse Matrix*Vector

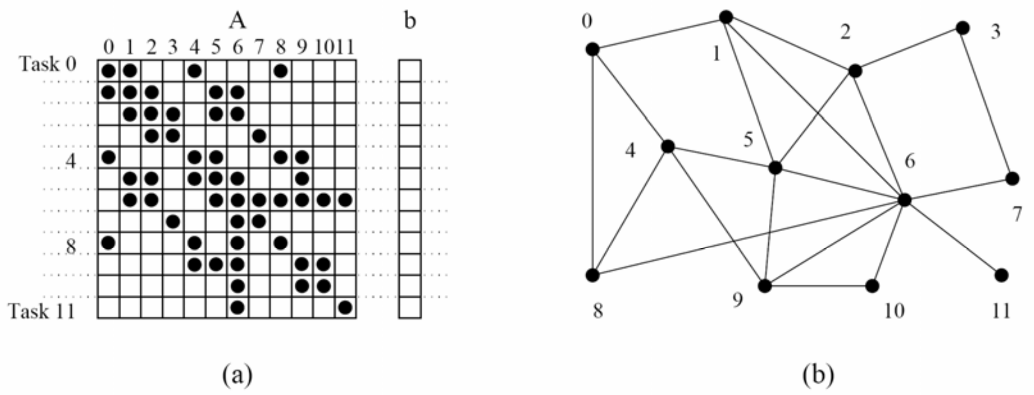


Figure 3.6 A decomposition for sparse matrix-vector multiplication and the corresponding task-interaction graph. In the decomposition Task i computes $\sum_{0 \leq j \leq 11, A[i,j] \neq 0} A[i, j].b[j]$.

Irregular pattern. Interaction pattern depends on the values in A.

Characteristics of Task Interactions

- Data sharing interactions:
  - Read-only interactions.
    - Read only data associated with *other* tasks.
  - Read-write interactions.
    - Read & modify data of *other* tasks.

Read-only vs. read-write.

Read-only example: matrix multiplication (share input). Read-write example: 15-puzzle with shared priority list of states to be explored; Priority given by some heuristic to evaluate the distance to the goal.

Characteristics of Task Interactions

- One-way interactions.
  - Only one task initiates and completes the communication ***without*** interrupting the other one.
- Two-way interactions.
  - Producer – consumer model.

One-way vs. two-way.

One-way more difficult with MPI since MPI has an explicit send & receive set of calls. Conversion one-way to two-way with polling or another thread waiting for communication.

Mapping Techniques for Load Balancing

- Map tasks onto processes.
- Goal: minimize overheads.
  - Communication.
  - Idling.
- Uneven load distribution may cause idling.
  - Constraints from task dependency $\rightarrow$ wait for other tasks.

Minimizing communication may contradict minimizing idling. Put tasks that communicate with each other on the same process but may unbalance the load -> distribute them but increase communication.

Load balancing is not enough to minimize idling.

Example

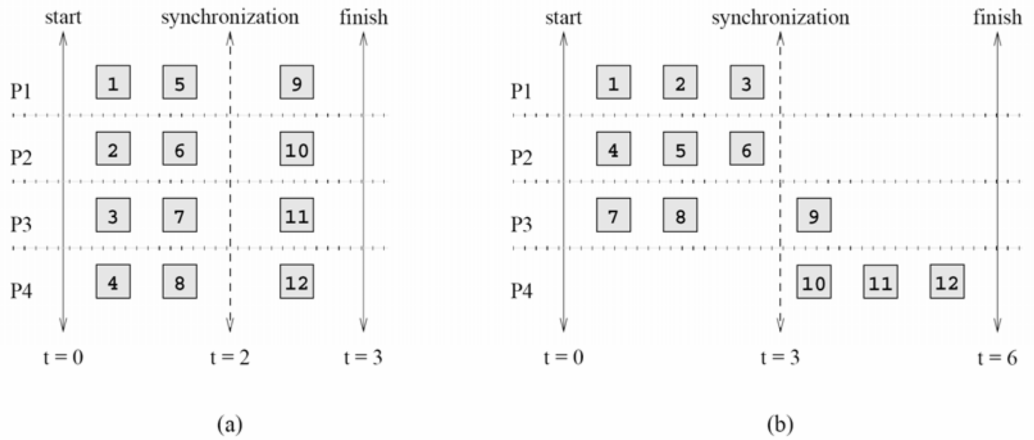


Figure 3.23 Two mappings of a hypothetical decomposition with a synchronization.

Global balancing OK but due to task dependency P4 is idling.

Mapping Techniques

- Static mapping.
  - NP-complete problem for non-uniform tasks.
  - Large data compared to computation.
- Dynamic mapping.
  - Dynamically generated tasks.
  - Task size unknown.

Even static mapping may be difficult: The problem of obtaining an optimal mapping is an NP-complete problem for non-uniform tasks. In practice simple heuristics provide good mappings.

Cost of moving data may out-weight the advantages of dynamic mapping.

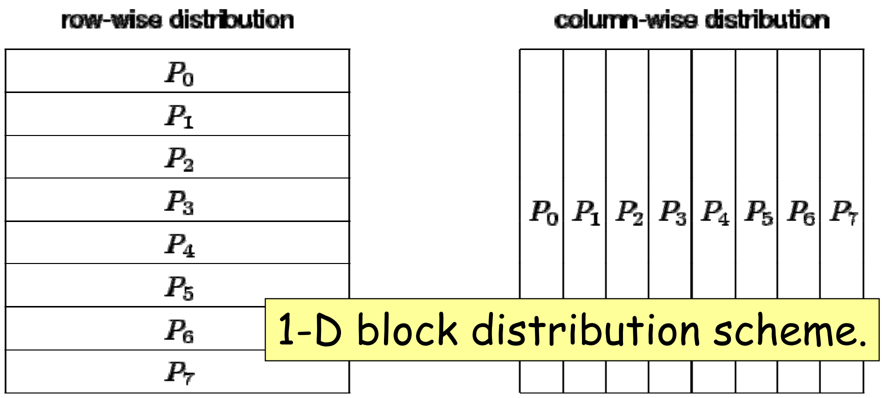
In shared address space dynamic mapping may work well even with large data, but be careful with the underlying architecture (NUMA/UMA) because data may be moved physically.

Schemes for Static Mapping

- Mappings based on data partitioning.
- Mappings based on task graph partitioning.
- Hybrid mappings.

Array Distribution Scheme

- Combine with “owner computes” rule to partition into sub-tasks.



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Data partitioning mapping.

Mapping data = mapping tasks.

Simple block-distribution.

Block Distribution cont.

P_0	P_1	P_2	P_3
P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	P_{11}
P_{12}	P_{13}	P_{14}	P_{15}

(a)

P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}

(b)

Generalize to higher dimensions: 4x4, 2x8.

Example: Matrix*Matrix

- Partition output of C=A*B.
- Each entry needs the same amount of computation.
- Blocks on 1 or 2 dimensions.
- Different data sharing patterns.
- *Higher dimensional* distributions
  - means we can use *more processes*.
  - sometimes *reduces* interaction.

In the case of matrix n*n multiplication, 1-D -> n processes at most, 2-D n^2 processes at most.

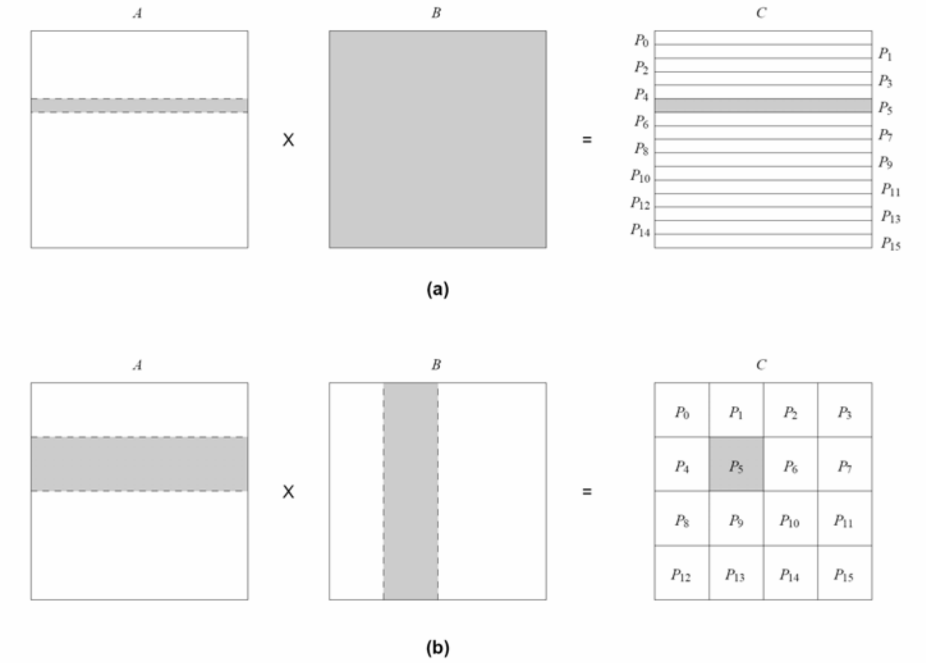

Figure 3.26 Data sharing needed for matrix multiplication with (a) one-dimensional and (b) two-dimensional partitioning of the output matrix. Shaded portions of the input matrices A and B are required by the process that computes the shaded portion of the output matrix C.

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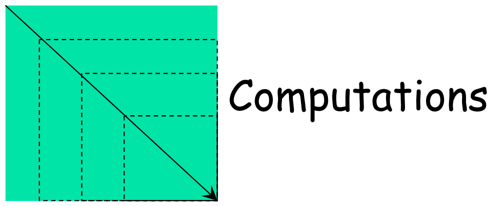
$O(n^2/\mathrm{sqrt}(p))$ vs. $O(n^2)$ shared data.

Imbalance Problem

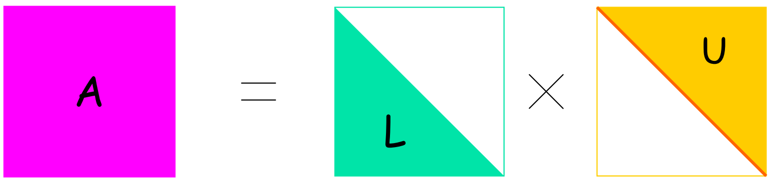
- If the amount of *computation* associated with data *varies* a lot then *block decomposition* leads to *imbalances*.
- Example: LU factorization (or Gaussian elimination).



Exercise on LU-decomposition.

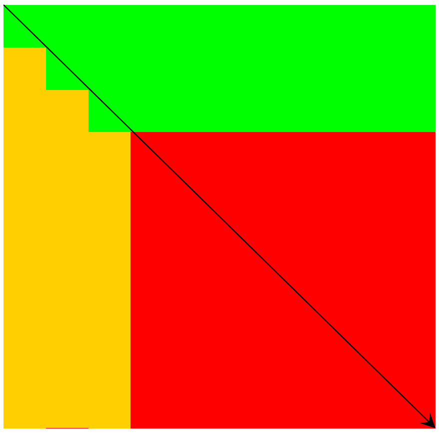
LU Factorization

- Non singular square matrix A (invertible).
- A = L*U.
- Useful for solving linear equations.



LU Factorization

In practice we work on A.



LU Algorithm

```
Proc LU(A)
begin
  for k := 1 to n-1 do
    for j := k+1 to n do
      A[j,k] := A[j,k]/A[k,k]
    endfor
    for j := k+1 to n do
      for i := k+1 to n do
        A[i,j] := A[i,j] - A[i,k]*A[k,j]
      endfor
    endfor
  endfor
end
```

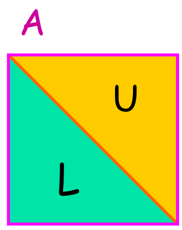
U[k,k]

L[j,k]

L[i,k] U[k,j]

Normalize L

U[k,j] := A[k,j]/L[k,k]



Another Variant

```
for k := 1 to n-1 do
  for j := k+1 to n do
    A[k,j] := A[k,j]/A[k,k]
    for i := k+1 to n do
      A[i,j] := A[i,j] - A[i,k]*A[k,j]
    endfor
  endfor
endfor
```


More common in the litterature.

Decomposition

$$\begin{pmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{pmatrix} \rightarrow \begin{pmatrix} L_{1,1} & 0 & 0 \\ L_{2,1} & L_{2,2} & 0 \\ L_{3,1} & L_{3,2} & L_{3,3} \end{pmatrix} \cdot \begin{pmatrix} U_{1,1} & U_{1,2} & U_{1,3} \\ 0 & U_{2,2} & U_{2,3} \\ 0 & 0 & U_{3,3} \end{pmatrix}$$

1: $A_{1,1} \rightarrow L_{1,1}U_{1,1}$
2: $L_{2,1} = A_{2,1}U_{1,1}^{-1}$
3: $L_{3,1} = A_{3,1}U_{1,1}^{-1}$
4: $U_{1,2} = L_{1,1}^{-1}A_{1,2}$
5: $U_{1,3} = L_{1,1}^{-1}A_{1,3}$
6: $A_{2,2} = A_{2,2} - L_{2,1}U_{1,2}$
7: $A_{3,2} = A_{3,2} - L_{3,1}U_{1,2}$
8: $A_{2,3} = A_{2,3} - L_{2,1}U_{1,3}$
9: $A_{3,3} = A_{3,3} - L_{3,1}U_{1,3}$
10: $A_{2,2} \rightarrow L_{2,2}U_{2,2}$
11: $L_{3,2} = A_{3,2}U_{2,2}^{-1}$
12: $U_{2,3} = L_{2,2}^{-1}A_{2,3}$
13: $A_{3,3} = A_{3,3} - L_{3,2}U_{2,3}$
14: $A_{3,3} \rightarrow L_{3,3}U_{3,3}$

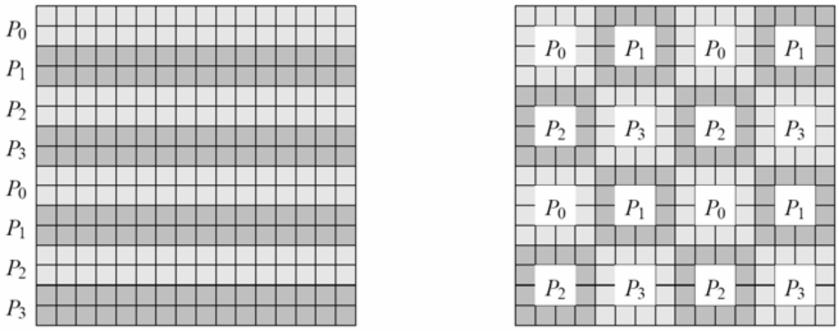
Figure 3.27 A decomposition of LU factorization into 14 tasks.

Load imbalance for individual tasks. Load imbalance from dependencies.

Cyclic and Block-Cyclic Distributions

- Idea:
  - Partition an array into many *more blocks than available processes*.
  - Assign partitions (tasks) to processes in a round-robin manner.
- $\rightarrow$ each process gets several non adjacent blocks.

Block-Cyclic Distributions



(a) (b)

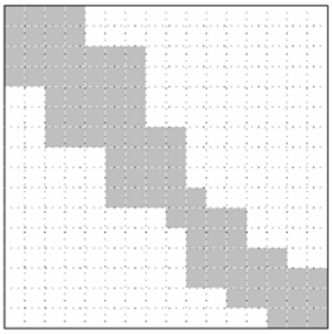
a) Partition 16x16 into 2*4 groups of 2 rows.
   αp groups of $n/\alpha p$ rows.

b) Partition 16x16 into square blocks of size 4*4 distributed on 2*2 processes.
   $\alpha^2 p$ groups of $n/\alpha^2 p$ squares.

Reduce the amount of idling because all processes have a sampling of tasks from *all parts* of the matrix.

But lack of locality may result in performance penalties + leads to high degree of interaction. Good value for α to find a compromise.

Randomized Distributions



(a)

P_0	P_1	P_2	P_3	P_0	P_1	P_2	P_3
P_4	P_5	P_6	P_7	P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	P_{11}	P_8	P_9	P_{10}	P_{11}
P_{12}	P_{13}	P_{14}	P_{15}	P_{12}	P_{13}	P_{14}	P_{15}
P_0	P_1	P_2	P_3	P_0	P_1	P_2	P_3
P_4	P_5	P_6	P_7	P_4	P_5	P_6	P_7
P_8	P_9	P_{10}	P_{11}	P_8	P_9	P_{10}	P_{11}
P_{12}	P_{13}	P_{14}	P_{15}	P_{12}	P_{13}	P_{14}	P_{15}

(b)

Irregular distribution with regular mapping!
Not good.

1-D Randomized Distribution

$$V = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$$

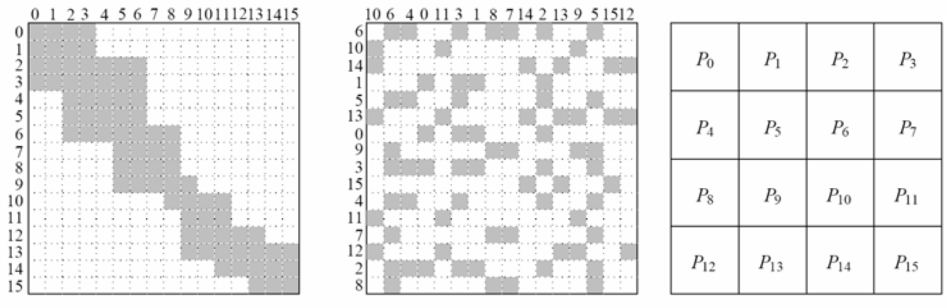
Permutation

$$\text{random}(V) = [8, 2, 6, 0, 3, 7, 11, 1, 9, 5, 4, 10]$$

$$\text{mapping} = \underbrace{8\ \ 2\ \ 6}_{P_0}\ \underbrace{0\ \ 3\ \ 7}_{P_1}\ \underbrace{11\ \ 1\ \ 9}_{P_2}\ \underbrace{5\ \ 4\ \ 10}_{P_3}$$

Figure 3.32 A one-dimensional randomized block mapping of 12 blocks onto four process (i.e., $\alpha = 3$).

2-D Randomized Distribution



2-D block random distribution.

Block mapping.

Graph Partitioning

- For sparse data structures and data dependent interaction patterns.
  - Numerical simulations. Discretize the problem and represent it as a mesh.
- Sparse matrix: assign equal number of nodes to processes & minimize interaction.
- Example: simulation of dispersion of a water contaminant in Lake Superior.

Discretization

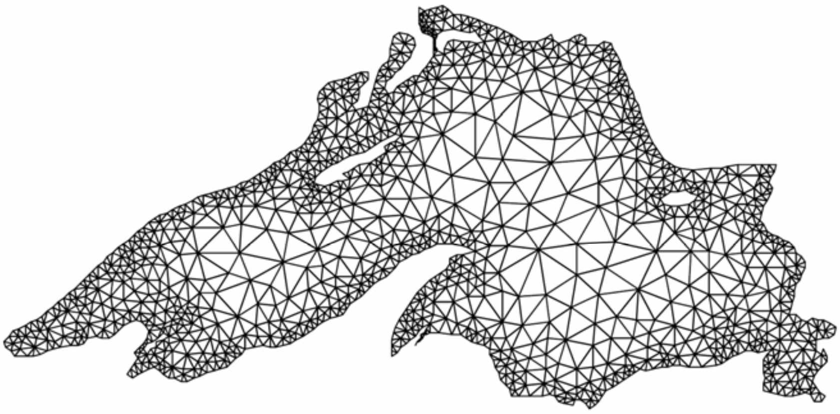
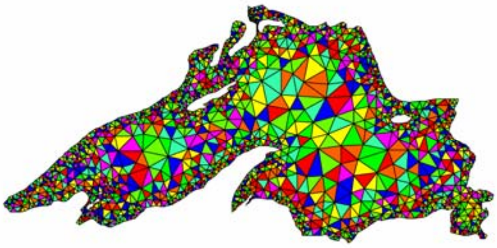
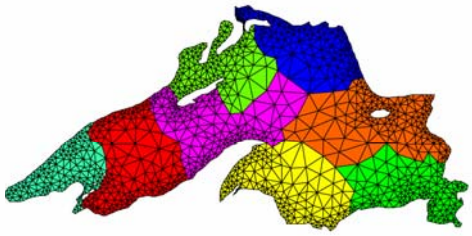


Figure 3.34 A mesh used to model Lake Superior.

Partitioning Lake Superior



Random partitioning.



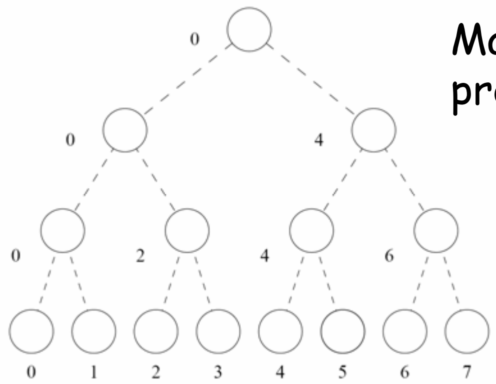
Partitioning with minimum edge cut.

Finding an exact optimal partitioning is an NP-complete problem.

Minimum edge cut from a graph point of view. Keep locality of data with processes to minimize interaction.

Mappings Based on Task Partitioning

- Partition the task dependency graph.
  - Good when static task dependency graph with known task sizes.



Mapping on 8 processes.

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Determining an optimal mapping is NP-complete. Good heuristics for structured graphs.

Binary tree task dependency graph: occurs in recursive decompositions as seen before. The mapping minimizes interaction. There is idling but it is inherent to the task dependency graph, we do not add more.

This example good on a hypercube. See why?

Sparse Matrix*Vector

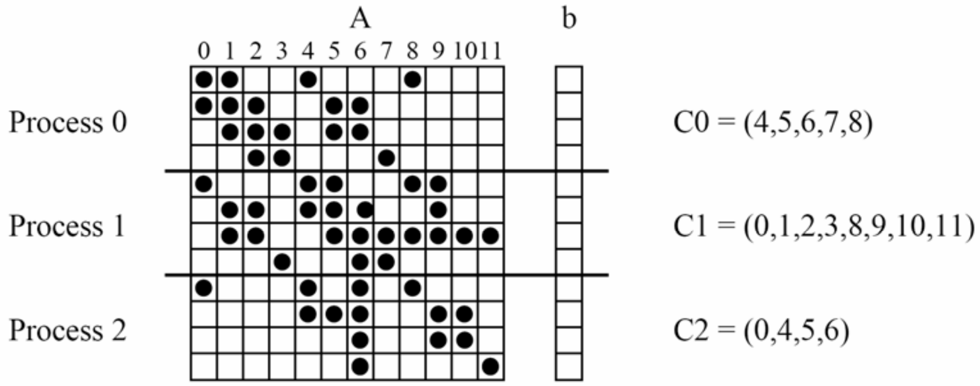

Figure 3.38 A mapping for sparse matrix-vector multiplication onto three processes. The list Ci contains the indices of b that Process i needs to access from other processes.

Example seen before.

Sparse Matrix*Vector

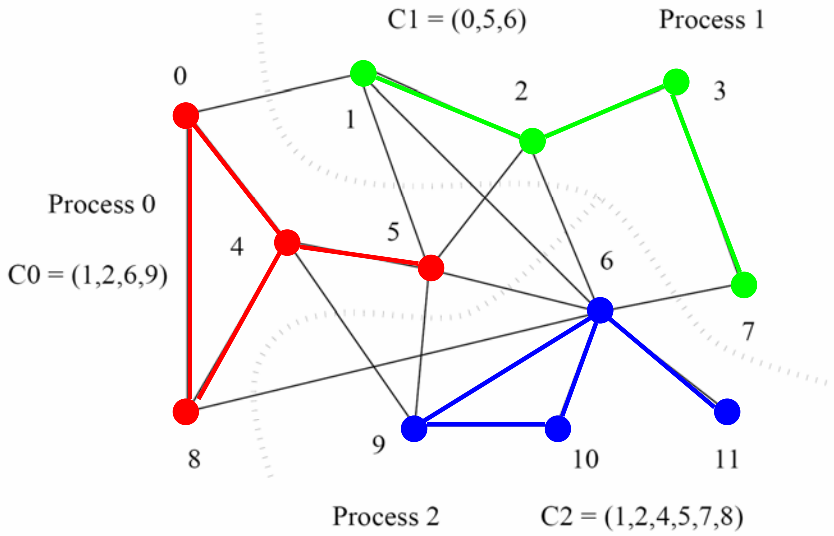

Figure 3.39 Reducing interaction overhead in sparse matrix-vector multiplication by partitioning the task-interaction graph.

Hierarchical Mappings

- Combine several mapping techniques in a structured (hierarchical) way.
- Task mapping of a binary tree (quicksort) does not use all processors.
  - Mapping based on task dependency graph (hierarchy) & block.

Binary Tree -> Hierarchical Block Mapping

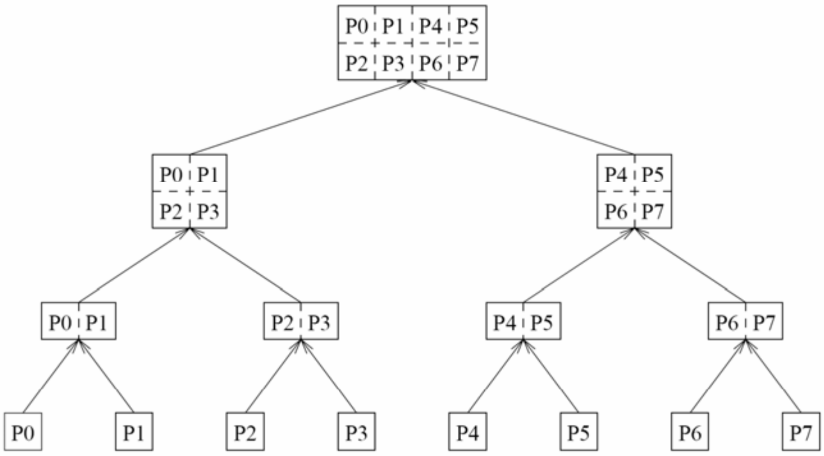

Figure 3.40 An example of hierarchical mapping of a task-dependency graph. Each node represented by an array is a supertask. The partitioning of the arrays represents subtasks, which are mapped onto eight processes.

Schemes for Dynamic Mapping

- Centralized Schemes.
  - Master manages pool of tasks.
  - Slaves obtain work.
  - Limited scalability.
- Distributed Schemes.
  - Processes *exchange tasks* to balance work.
  - Not simple, many issues.

Centralized schemes are easy to implement but present an obvious bottleneck (the master).

Self-scheduling: slaves pick up work to do whenever they are idle.

Bottleneck: tasks of size M, it takes t to assign work to a slave → at most M/t processes can be kept busy.

Chunk-scheduling: a way to reduce bottlenecks by getting a group of tasks. Problem for load imbalances.

Distributed schemes more difficult to implement.

How do you choose sender & receiver? i.e. if A is overloaded, which process gets something?

Initiate transfer by sender or receiver? i.e. A overloaded sends work or B idle requests work?

How much work to transfer?

When to transfer?

Answers are application specific.

Minimizing Interaction Overheads

- Maximize data locality.
  - Minimize volume of data-exchange.
  - Minimize frequency of interactions.
- Minimize contention and hot spots.
  - Share a link, same memory block, etc...
  - Re-design original algorithm to change the interaction pattern.

Minimize volume of exchange → maximize temporal locality. Use higher dimensional distributions, like in the matrix multiplication example. We can store intermediate results and update global results less often.

Minimize frequency of interactions → maximize spatial locality.

Related to the previously seen cost model for communications.

Changing the interaction pattern: For the matrix multiplication example, the sum is commutative so we can re-order the operations modulo sqrt(p) to remove contention.

Minimizing Interaction Overheads

- Overlapping computations with interactions – to reduce idling.
  - Initiate interactions in advance.
  - Non-blocking communications.
  - Multi-threading.
- Replicating data or computation.
- Group communication instead of point to point.
- Overlapping interactions.

Replication is useful when the cost of interaction is greater than replicating the computation. Replicating data is like caching, good for read-only accesses. Processing power is cheap, memory access is expensive – also apply at larger scale with communicating processes.

Collective communication such as broadcast. However, depending on the communication pattern, a custom collective communication may be better.

Overlapping Interactions

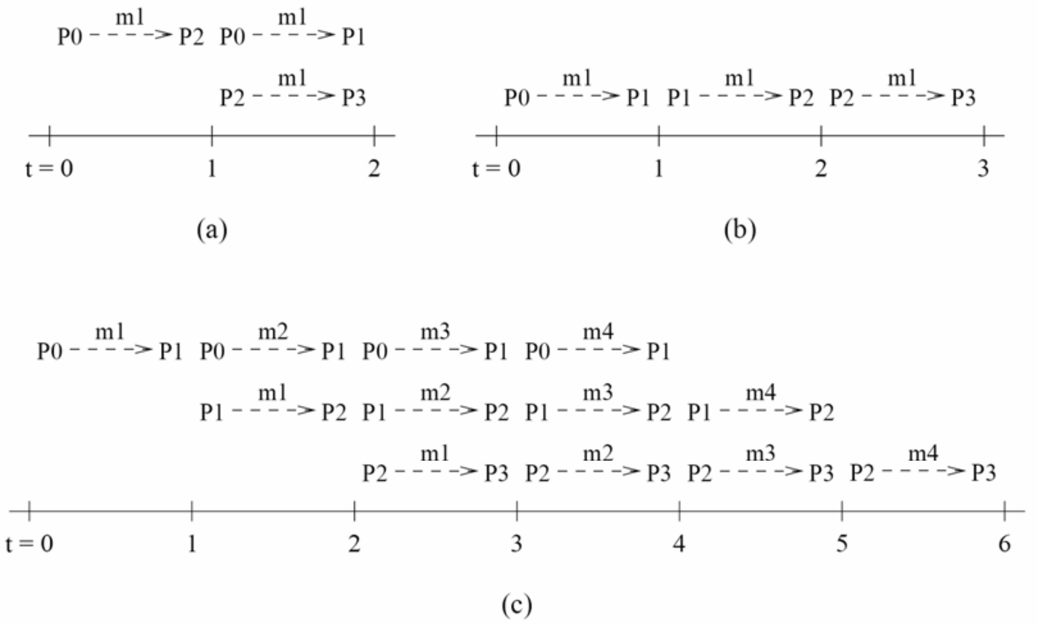

Figure 3.41 Illustration of overlapping interactions in broadcasting data from one to four processes.

Parallel Algorithm Models

- Data parallel model.
  - Tasks statically mapped.
  - Similar operations on different data.
    - SIMD.
- Task graph model.
  - Start from task dependency graph.
  - Use task interaction graph to promote locality.

An algorithm model is a *way of structuring a parallel algorithm* by selecting a *decomposition and mapping* technique and applying the appropriate strategy to minimize interactions.

Parallel Algorithm Models

- Work pool (or task pool) model.
  - No pre-mapping – centralized or not.
- Master-slave model.
  - Master generates work for slaves – allocation static or dynamic.
- Pipeline or producer – consumer model.
  - Stream of data traverses processes – stream parallelism.

Pipeline model heavily used in GPUs. Load balancing is a function of task granularity.

+ hybrid models:

•Multiple models applied hierarchically.

•Multiple models applied sequentially to different phases.