# Searching

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B2-206

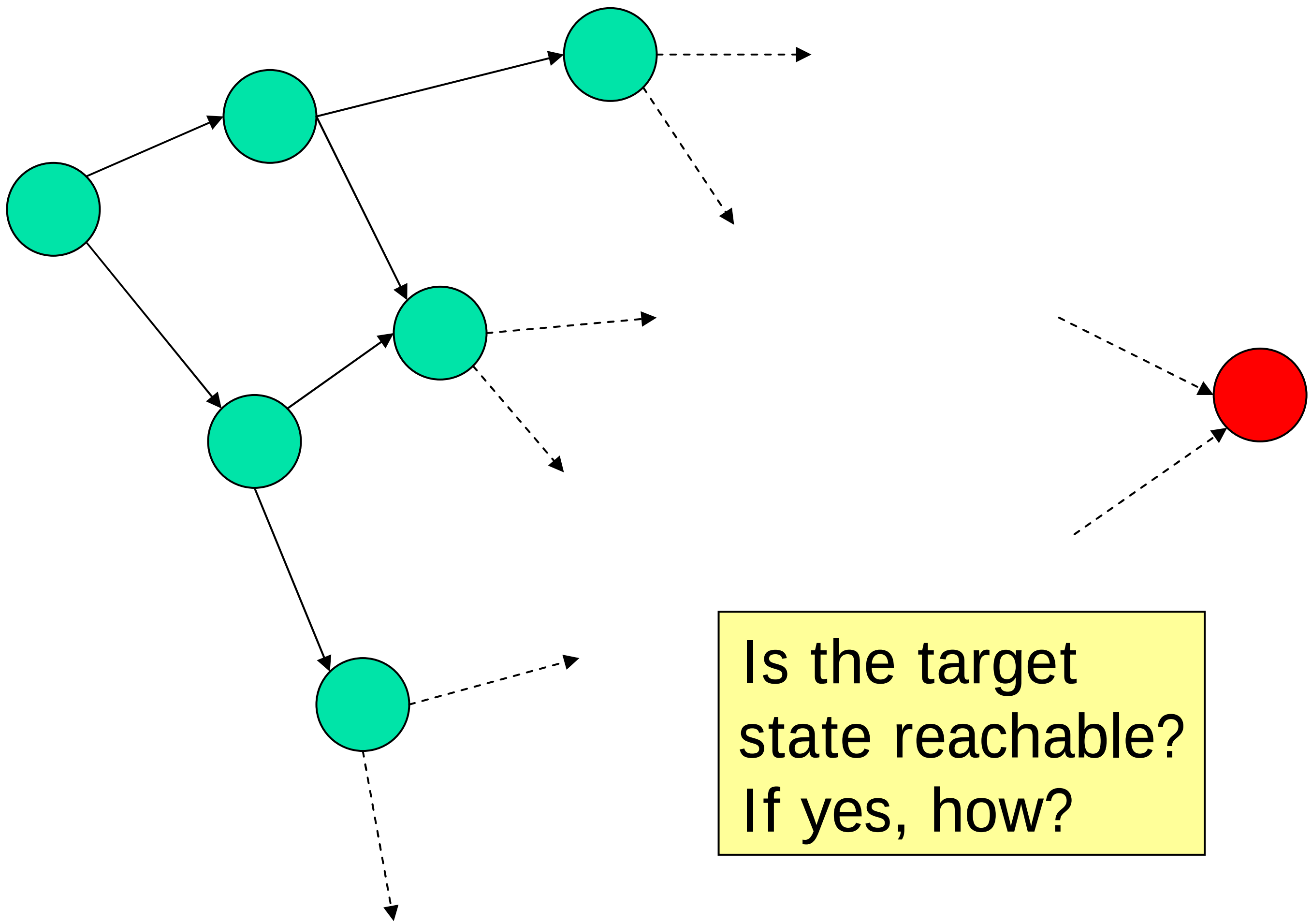
# The Problem

- Your system is a given state and you want it to reach another state.
  - You have a set of rules that tell you how the system may evolve.
  - You don't know how to get to the target state trivially.
- General problem, typically in the field of planning and AI.
- Classification: Graph algorithm.

# Definitions

- A state is the snapshot configuration of a system, typically a tuple with the values of all the variables of the system.
- The system changes state by taking transitions. The rules are given by a transition relation.
- The set of all states is called the state-space.
- A state S is reachable if there exists a sequence of transitions from the initial state to S.
  - The sequence of transition is called trace, path, or witness, depending on the field.

# Searching, a.k.a. State-space Exploration



# Exploration Algorithm

white = not explored yet.
black = explored.
~ = equivalence relation.
→ = transition.

**search**(init,target):
$S = \{(init, white)\}$
**while** $\{(a, white) \mid (a, white) \in S\} \neq \emptyset$ **do**
    pick $(a, white) \in S$
    **if** $a \sim target$ **then return true**
    $S = S[(a, black)/(a, white)]$
    **forall** $a \rightarrow a'$ **do**
        **if** $\{(b, color) \mid b \sim a'\} = \emptyset$ **then**
            $S = S \cup (a', white)$
        **fi**
    **done**
**done**
**return false**

# Correctness

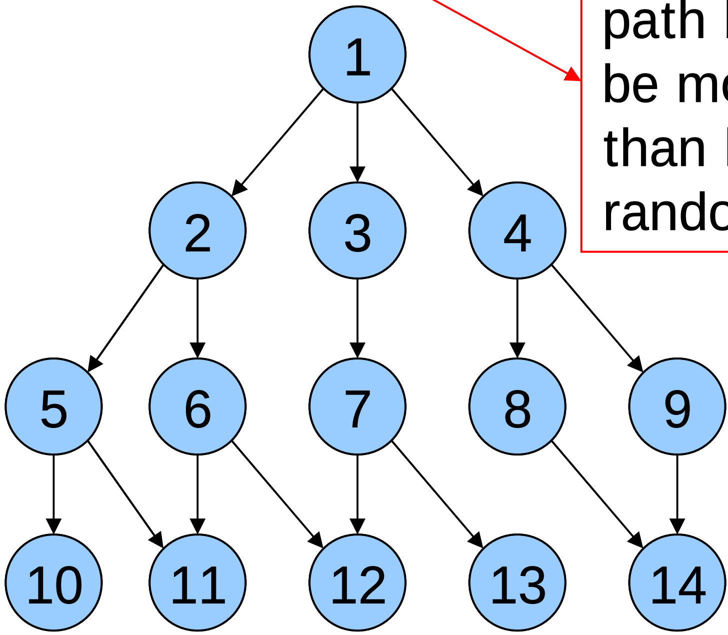
- The algorithm explores all possible reachable states.
  - It will terminate if the state-space is finite. This is often the case, you can argue that.
  - When it terminates, it proves that a state is reachable or not.
  - You can add simple information to keep track of predecessors to generate a trace.

# Technicalities

- How to represent S?
  - Hash table.
  - Compute a hash on a canonical representant of the equivalence class of your state.
- How to pick-up the next state to be explored?
  - FIFO: Breadth-first-search.
  - LIFO: Depth-first search.
  - Priority queue: Guided search with heuristics.

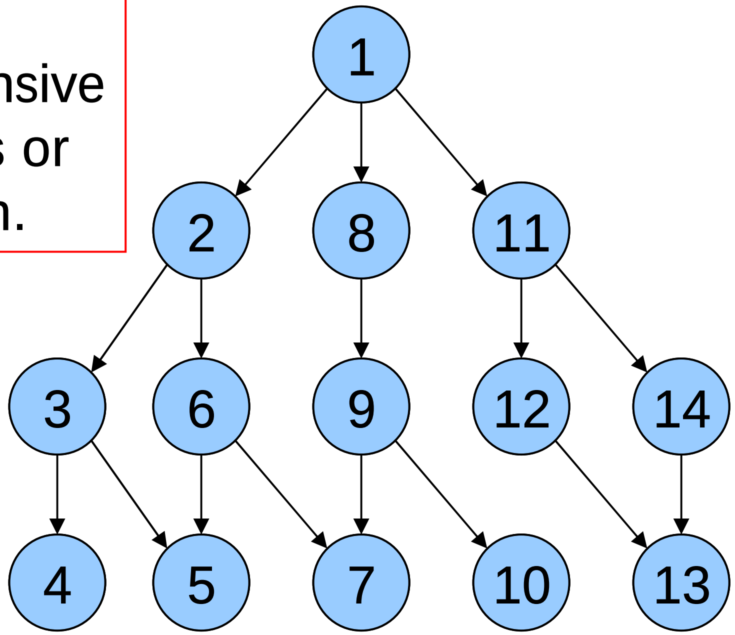
# Search Orderings

Breadth-first-search (BFS)




Gives shortest path but may be more expensive than heuristics or random search.

Depth-first-search (DFS)



# Application to Your Project

- Given a chess-board with pieces on it, move a piece from a position to another.
  - All the pieces may move by 1 in any direction if the target position is empty.
  - Not trivial to get a simple algorithm “guess” the solution, but this is an instance of a more general search problem.
- The initial state is given by the initial configuration of the board.
- The final state is given by the configuration of the board with the piece moved to the wanted position.

# Formalizing the Problem

- The state is a board (array) B[8][8] of pieces.
- 2 states B and B' are said equivalent (noted B~B') iff $\forall$i,j : B[i][j] = B'[i][j].
  - You can try to code the fact that we don't care about the nature of the pieces, *except* for the initial and final states, which is the problem
- Transition relation:

$$\frac{a = B[i][j] \neq \bot,\ B[i'][j'] = \bot,\ i' = i \pm 1 \ xor \ j' = j \pm 1}{B \longrightarrow B\left[a / B[i'][j'], \bot / B[i][j]\right]}$$

# Practice

- Hash table for S.
- Write a function to generate the successor states (transition relation).
- The successor states are looked-up in S.
- Have a queue (FIFO,LIFO,priority) to keep references to the “white” states.
- BIG PROBLEM: State-space explosion, so use a heuristic to guide the search.
  - ⊥, P, Bishop, Knight, Rook, Knight, Queen, on any 8x8: $7^{64}$ states.
- Extension of the transition relation: Allow diagonal moves.