AA1 - Introduction & Growth of Functions

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1 Euclid's Algorithm

1 Prove the correctness of the algorithm. Hint: Do it in two steps, (1) if n divides m then *gcd(m,n)=n*, and (2), *gcd(m,n)=gcd(n, m mod n)*.

2 Euclid's algorithm, as presented in Euclid's treatise, uses subtractions rather than integer divisions. Write a pseudocode for this version of Euclid's algorithm.

3 - For fun Euclid's game starts with two unequal positive numbers on the board. Two players move in turn. On each move, a player has to write on the board a positive number equal to the difference of two numbers already on the board; this number must be new, i.e., different from all the numbers already on the board. The player who cannot move loses the game. Should you choose to move first or second in this game?

2 Algorithm Problem Solving

1 Design a simple algorithm for the string-matching problem.

2 Icosian game: a century after Euler's discovery, another famous puzzle – invented by the renown Irish mathematician Sir William Hamilton (1805-1865) – was presented to the world under the name of the Icosian Game. The game was played on a circular wodden board on which the graph of Fig.1 was carved. Find a *Hamiltonian circuit* – a path that visits all the graph's vertices exactly once before returning to the starting vertex – for this graph.

3 Design an algorithm for the following problem: Given a set of n points in the $x - y$ coordinate plane, determine whether all of them lie on the same circumference.

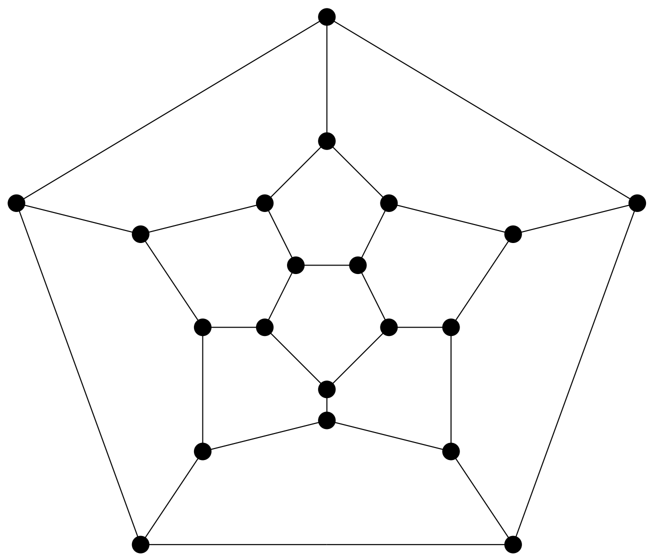

Figure 1: Icosian game.

4 For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes $f(n)$ microseconds. Dispatch the work within your group so you don't do everything yourself.

	1 sec	1 min	1 hour	1 day	1 month	1 year	1 century
$lg(n)$							
$\sqrt{n}$							
n							
$n * lg(n)$							
n^2							
n^3							
2^n							
$n!$							

3 Numerical Example

Let us compute $q = a/b$ using Newton-Raphson's method. Find an algorithm to compute q by searching the solution of $f(x) = 0$ for $f(x) = 1/x - b$.

4 Asymptotic Notations

1 Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of Θ-notation, prove that $max(f(n), g(n)) \in \Theta(f(n) + g(n))$.

2 Show that for any real constants a and b, where $b > 0$, $(n+a)^b = \Theta(n^b)$.

3 Explain why the statement "The running time of algorithm A is at least $O(n^2)$ is meaningless".

5 Fibonacci Numbers

Prove by induction that the ith Fibonacci number satisfies the equality

$$F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}},$$

where ϕ is the golden ratio and $\hat{\phi}$ is its conjugate: $\phi = \frac{1+\sqrt{5}}{2}$ and $\hat{\phi} = \frac{1-\sqrt{5}}{2}$.