Introduction to Algorithms – Exercises

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1 Euclid's Algorithm

1 Prove the correctness of the algorithm: (i) if n divides m then *gcd(m,n)=n*, and (ii), *gcd(m,n)=gcd(n, m mod n)*.

2 What is the smallest number of divisions made by Euclid's algorithm among all inputs $1 \leq m, n \leq 10$?

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4 Euclid's algorithm, as presented in Euclid's treatise, uses substractions rather than integer divisions. Write a pseudocode for this version of Euclid's algorithm.

5 Euclid's game starts with two unequal positive numbers on the board. Two players move in turn. On each move, a player has to write on the board a positive number equal to the difference of two numbers already on the board; this number must be new, i.e., different from all the numbers already on the board. The player who cannot move loses the game. Should you choose to move first or second in this game?

2 Algorithm Problem Solving

1 Design a simple algorithm for the string-matching problem.

2 Icosian game: a century after Euler's discovery, another famous puzzle – invented by the renown Irish mathematician Sir William Hamilton (1805-1865) – was presented to the world under the name of the Icosian Game. The game was played on a circular wodden board on which the graph of Fig.1 was carved. Find a *Hamiltonian circuit* – a path that visits all the graph's vertices exactly once before returning to the starting vertex – for this graph.

2 Design an algorithm for the following problem: Given a set of n points in the $x - y$ coordinate plane, determine whether all of them lie on the same circumference.

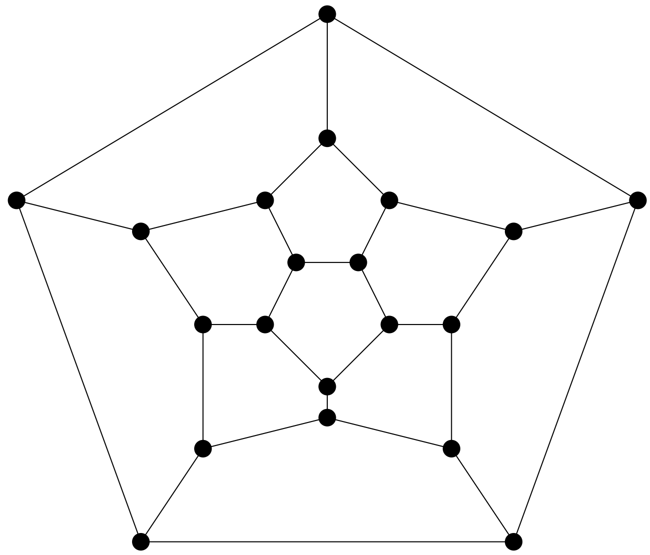

Figure 1: Icosian game.

3 Write a program that reads as its inputs the (x, y) coordinates of endpoints of two line segments P_1Q_1 and P_2Q_2 and determines whether the segments have a common point.

4 For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes $f(n)$ microseconds.

	1 sec	1 min	1 hour	1 day	1 month	1 year	1 century
$lg(n)$							
$\sqrt{n}$							
n							
$n * lg(n)$							
n^2							
n^3							
2^n							
$n!$							

3 Numerical Example

Let us compute $q = a/b$ using Newton-Raphson's method. Find an algorithm to compute q by searching the solution of $f(x) = 0$ for $f(x) = 1/x - b$.