

Growth of Functions – Exercises

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1 Asymptotic Notations

1 Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ -notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

2 Show that for any real constants a and b , where $b > 0$, $(n + a)^b = \Theta(n^b)$.

3 Explain why the statement “The running time of algorithm A is at least $O(n^2)$ ” is meaningless.

2 Fibonacci Numbers

Prove by induction that the i th Fibonacci number satisfies the equality

$$F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}},$$

where ϕ is the golden ratio and $\hat{\phi}$ is its conjugate: $\phi = \frac{1+\sqrt{5}}{2}$ and $\hat{\phi} = \frac{1-\sqrt{5}}{2}$.